

# Teleportation of a zero- and one-photon running-wave state by projection synthesis

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We show how to teleport a running-wave superposition of zero- and one-photon field states through the projection-synthesis technique. The fidelity of the scheme is computed taking into account the noise introduced by dissipation and the efficiency of the detectors. These error sources have been introduced through a single general relationship between input and output operators. [S1050-2947(99)04110-4]

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## I. INTRODUCTION

About five years after the proposition of quantum teleportation by Bennett *et al.* [1], this phenomenon has recently been demonstrated in a couple of experiments [2,3] through photon polarized states. The required quantum channel, an Einstein-Podolsky-Rosen state [4], has been generated, in both experimental realizations, by photons emerging from a type-II degenerate parametric down-conversion [5]. Basically, a superposition of horizontally ( $|h\rangle$ ) and vertically ( $|v\rangle$ ) polarized states of photon 1, i.e.,  $\alpha|h\rangle + \beta|v\rangle$ , is teleported to another photon, say 3, which is part of an entangled quantum channel  $(|v\rangle|h\rangle + |h\rangle|v\rangle)/\sqrt{2}$ .

Despite the fact that the experimental demonstration of teleportation of an atomic state has not yet been realized, there are a number of proposals on this subject [6,7]. There is even a proposal that considers two particles whose states are simultaneously teleported one to the other, namely the identity interchange process [8]. Experimental proposals for teleporting a Schrödinger cat state of the radiation field, both trapped in a high- $Q$  cavity [9] and as a running wave [10], have been reported. In Ref. [10] the authors analyze teleportation of continuous quantum variables and calculate the fidelity of the process. Teleportation of  $N$ -dimensional states has also been proposed in the cavity QED domain and regarding other physical systems [11].

In the present paper we show how to teleport a running wave superposition of zero- and one-photon field states. The teleportation machine is based on a recently reported scheme for optical state truncation by projection synthesis [12]. Such a “quantum-scissors” device is suitable for teleportation since both the processes rely exactly on the same principles: the possibility of entanglement, and so nonlocality, and the projection postulate. The technique of projection synthesis has been proposed originally by Barnett and Pegg [13] for the experimental determination of the canonical quantum optical phase probability distribution, and has also been applied for  $Q$ -function measurement [14].

As sketched in Fig. 1(a), the teleportation experiment consists of a couple of 50/50 symmetric beam splitters,  $BS_1$  and  $BS_2$ , and a couple of photodetectors,  $D_b$  and  $D_c$ . As in the original Bennett teleportation scheme, Bob is supposed to share a quantum channel with Alice, here an entanglement

composed by the output fields  $a$  and  $b$ . While the output field  $a$ , which is expected to receive the teleported state, is given to Bob, the output field  $b$  is given to Alice. She is expected to realize a Bell-type measurement on field  $b$  itself and the one injected through the input mode  $c$  in the state to be teleported. Obviously, such a state in mode  $c$  is supposedly unknown to both Alice and Bob. As the output modes  $a$  and  $b$  had been previously entangled, the phenomenon of nonlocality plus the projection postulate lead to the achievement of the teleportation process [1].

Let us stress that for the desires of Alice and Bob regarding the teleportation of the unknown qubit, they need to consider just the apparatus in Fig. 1(a). The preparation of the state to be teleported through the additional “projection-synthesis” step depicted in Fig. 1(b), was introduced here uniquely for further computation of the fidelity of the whole process, i.e., preparation and teleportation of a given running-wave superposition state. This is actually important because to *measure* the fidelity of the teleportation, it is necessary for the prepared state and the teleported state to be subjected to similar quantum tests.

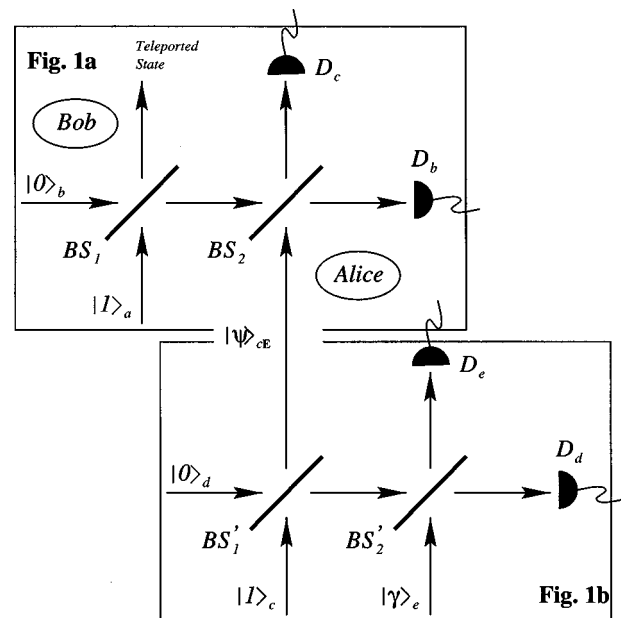


FIG. 1. Sketch of the experimental setup for teleportation by projection synthesis.

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## II. IDEAL PROCESS

Let us briefly consider the situation where no losses are introduced. The state to be teleported, injected through the input mode  $c$ , and expected to be described as

$$|\psi\rangle_c = c_0|0\rangle_c + c_1|1\rangle_c, \quad (1)$$

is prepared through a quantum-scissors device [12], which is a replica of the teleportation machine as depicted in Fig. 1(b). To prepare the state (1), a single-photon field is injected through the input mode  $c$  on  $BS_1$  resulting in the entangled field  $|\psi\rangle_{cd} = (|10\rangle_{cd} + i|01\rangle_{cd})/\sqrt{2}$ . Next, a coherent field  $|\gamma\rangle_e = \sum_{n=0}^{\infty} \gamma_n |n\rangle_e = \gamma_0|0\rangle_e + \gamma_1|1\rangle_e + \dots$  is sent through the input mode  $e$  with its vacuum and one-photon amplitudes satisfying  $\gamma_0/c_0 = \gamma_1/c_1 = C$  [ $C^2 = (|\gamma_0|^2 + |\gamma_1|^2)$ ]. As shown in [12], when registering a single photon in detector  $D_d$  and no counts in  $D_e$ , one synthesizes the projection of the entanglement resulting from  $BS'_1$  on a given state of the output field  $d$ , i.e.,  ${}_d\langle\phi|\psi\rangle_{cd}$ , leading to the prepared state (1). Thus, when also accounting for the preparation of the state to be teleported, the whole machine consists of a double quantum-scissors device: the output mode  $c$ , where the state to be teleported is supposed to be prepared [Fig. 1(b)], and the input mode  $c$  of the teleportation device [Fig. 1(a)].

In Ref. [12] the authors have pointed out that the quantum-scissors device is a limited form of teleportation, in that part of the coherent state  $|\gamma\rangle_e$ , formed from the vacuum and one-photon states, is “teleported” to mode  $c$ . However, we show below that it is possible to teleport a given quantum superposition of zero- and one-photon field state from mode  $c$  to mode  $a$ .

Simultaneously to the preparation of the state to be teleported, the quantum channel is prepared through a single-photon field  $a$  incident on  $BS_1$  in a way to superpose on  $BS_2$  both the state to be teleported and the output field  $b$  entangled to  $a$  as  $|\psi\rangle_{ab} = (|10\rangle_{ab} + i|01\rangle_{ab})/\sqrt{2}$ . The product of Alice’s state to be teleported and the quantum channel can be expanded, apart from an irrelevant phase factor, as

$$\begin{aligned} & \frac{1}{2} [ |\Psi^-\rangle_{bc} (c_0|0\rangle_a + c_1|1\rangle_a) + |\Phi^-\rangle_{bc} (c_1|0\rangle_a + c_0|1\rangle_a) \\ & - |\Psi^+\rangle_{bc} (c_0|0\rangle_a - c_1|1\rangle_a) - |\Phi^+\rangle_{bc} (c_1|0\rangle_c - c_0|1\rangle_c) ], \end{aligned} \quad (2)$$

where we have introduced the complete set of eigenstates of Bell operators,

$$|\Psi^\pm\rangle_{bc} = \frac{1}{\sqrt{2}} (|01\rangle_{bc} \pm i|10\rangle_{bc}), \quad (3a)$$

$$|\Phi^\pm\rangle_{bc} = \frac{1}{\sqrt{2}} (|00\rangle_{bc} \pm i|11\rangle_{bc}). \quad (3b)$$

Hence, a measurement using Bell-state analyzers on fields  $b$  and  $c$  projects the mode  $a$  on a superposition of zero- and one-photon field state as described in (2). This required joint measurement can be achieved through  $BS_2$  by the

projection-synthesis technique. In fact, by superposing the field state to be teleported with the output field  $b$  in  $BS_2$ , we get

$$\hat{U}|\Phi^\pm\rangle_{bc} \propto \begin{cases} |01\rangle_{bc} \\ |10\rangle_{bc}, \end{cases} \quad (4a)$$

$$\hat{U}|\Psi^\pm\rangle_{bc} \propto \begin{cases} |00\rangle_{bc} \mp \frac{1}{\sqrt{2}} (|20\rangle_{bc} + |02\rangle_{bc}), \end{cases} \quad (4b)$$

where  $\hat{U} = \exp[i(\pi/4)(\hat{b}^\dagger\hat{c} + \hat{c}^\dagger\hat{b})]$  is the unitary operator describing the action of an ideal  $BS_2$ . We thus see from Eqs. (4a) and (4b) that a measurement of the field state  $|10\rangle_{bc}$ , which requires the incoming Bell state  $|\Psi^-\rangle_{bc}$ , projects the output field  $a$  exactly on the original state of field  $c$ . Otherwise, a joint measurement of the Bell state  $|\Psi^+\rangle_{bc}$  is achieved by measuring the field state  $|10\rangle_{bc}$  leaving the output field  $a$  in the original state  $|\psi\rangle_c$  but phase-shifted through  $\pi$ . However, the whole process, beginning with the preparation of the state to be teleported, will be developed below in a noise environment and taking account of detector inefficiency.

## III. LOSSES IN THE $BS$ 'S

With the inclusion of errors due to photoabsorption in the beam splitters, the general relationships between the input and output operators  $\hat{\alpha}, \hat{\beta}$ , i.e.,  $\hat{a}, \hat{b}$  ( $\hat{d}, \hat{c}$ ) in  $BS_1(BS'_1)$  or  $\hat{b}, \hat{c}$  ( $\hat{d}, \hat{e}$ ) in  $BS_2(BS'_2)$  are [15]

$$\hat{\alpha}_{\text{out}} = t\hat{\alpha}_{\text{in}} + r\hat{\beta}_{\text{in}} + \hat{\mathcal{L}}_\alpha, \quad (5a)$$

$$\hat{\beta}_{\text{out}} = t\hat{\beta}_{\text{in}} + r\hat{\alpha}_{\text{in}} + \hat{\mathcal{L}}_\beta, \quad (5b)$$

where  $t$  and  $r$  are the beam-splitter transmission and reflection coefficients, respectively. In fact, such coefficients, and so the operators, depend on the frequency of the fields and here a monochromatic source is considered. The input fields and the noise sources are required to be independent so that the input operators must commute with the output Langevin operators:

$$[\hat{\alpha}_{\text{in}}, \hat{\mathcal{L}}_\alpha] = [\hat{\alpha}_{\text{in}}, \hat{\mathcal{L}}_\beta] = [\hat{\alpha}_{\text{in}}, \hat{\mathcal{L}}_\alpha^\dagger] = [\hat{\alpha}_{\text{in}}, \hat{\mathcal{L}}_\beta^\dagger] = 0, \quad (6)$$

with similar relations for the  $\beta$  operators. Imposition of the bosonic commutation relations on the output mode operators then leads to the requirements on the noise-operator commutation relations:

$$[\hat{\mathcal{L}}_\alpha, \hat{\mathcal{L}}_\alpha^\dagger] = [\hat{\mathcal{L}}_\beta, \hat{\mathcal{L}}_\beta^\dagger] = \Gamma, \quad (7a)$$

$$[\hat{\mathcal{L}}_\alpha, \hat{\mathcal{L}}_\beta^\dagger] = [\hat{\mathcal{L}}_\beta, \hat{\mathcal{L}}_\alpha^\dagger] = -\Omega, \quad (7b)$$

where  $\Gamma = 1 - |t|^2 - |r|^2$  is the damping constant and  $\Omega = tr^* + rt^*$ . For optical frequencies the state of the environment can be very well approximated by the vacuum state even at room temperature, so that

$$\hat{\mathcal{L}}_\alpha|0\rangle = \hat{\mathcal{L}}_\beta|0\rangle = \hat{\alpha}_{\text{in}}|0\rangle = \hat{\beta}_{\text{in}}|0\rangle = 0, \quad (8)$$

and, from the input-output relations [(5a) and (5b)], it also follows that

$$\hat{\alpha}_{\text{out}}|0\rangle = \hat{\beta}_{\text{out}}|0\rangle = 0. \quad (9)$$

Finally, the quantum averages of the Langevin operators vanish,

$$\langle \hat{\mathcal{L}}_\alpha \rangle = \langle \hat{\mathcal{L}}_\beta \rangle = \langle \hat{\mathcal{L}}_\alpha^\dagger \rangle = \langle \hat{\mathcal{L}}_\beta^\dagger \rangle = 0, \quad (10)$$

and the only nonzero ground-state expectation values for the products of pairs of noise operators are

$$\langle \hat{\mathcal{L}}_\alpha \hat{\mathcal{L}}_\alpha^\dagger \rangle = \langle \hat{\mathcal{L}}_\beta \hat{\mathcal{L}}_\beta^\dagger \rangle = \Gamma, \quad (11a)$$

$$\langle \hat{\mathcal{L}}_\alpha \hat{\mathcal{L}}_\beta^\dagger \rangle = \langle \hat{\mathcal{L}}_\beta \hat{\mathcal{L}}_\alpha^\dagger \rangle = -\Omega. \quad (11b)$$

As noted in Ref. [15], the above relations for the averages of the Langevin operators may also be derived from a canonical one-dimensional theory applied to a dielectric slab.

Next, it is easy to conclude that, similar to the relations (5a) and (5b), the transformations leading from the output to the input operators preserving the above-mentioned properties for the Langevin operators read

$$\hat{\alpha}_{\text{in}} = t^* \hat{\alpha}_{\text{out}} + r^* \hat{\beta}_{\text{out}} + \hat{\mathcal{L}}_\alpha, \quad (12a)$$

$$\hat{\beta}_{\text{in}} = t^* \hat{\beta}_{\text{out}} + r^* \hat{\alpha}_{\text{out}} + \hat{\mathcal{L}}_\beta, \quad (12b)$$

where the bosonic commutation relations on the input mode operators are satisfied.

#### IV. EFFICIENCY OF THE DETECTORS

To deal with the efficiency of the detectors, we again take advantage of the Langevin operators. Introducing output operators accounting for the detection of a given input field  $\alpha$  (modes  $b, c(d, e)$  reaching the detectors in Fig. 1(a) [Fig. 1(b)]), we write

$$\hat{\alpha}_{\text{out}} = \sqrt{\eta} \hat{\alpha}_{\text{in}} + \hat{\mathcal{L}}_\alpha, \quad (13)$$

considering the case in which the detectors have the same efficiency  $\eta$ . Obviously, different from the  $BS$ 's, the detectors do not couple different modes in a way that the Langevin operators  $\hat{\mathcal{L}}_\alpha$ , despite satisfying all the properties of those introduced above, obey the commutation relations

$$[\hat{\mathcal{L}}_\alpha, \hat{\mathcal{L}}_\alpha^\dagger] = 1 - \eta, \quad (14a)$$

$$[\hat{\mathcal{L}}_\alpha, \hat{\mathcal{L}}_\beta^\dagger] = 0, \quad (14b)$$

and the ground-state expectation values for the products of pairs are

$$\langle \hat{\mathcal{L}}_\alpha \hat{\mathcal{L}}_\alpha^\dagger \rangle = 1 - \eta, \quad (15a)$$

$$\langle \hat{\mathcal{L}}_\alpha \hat{\mathcal{L}}_\beta^\dagger \rangle = 0. \quad (15b)$$

#### V. GENERAL RELATIONS FOR THE ERRORS

We next introduce an algebra accounting for both the error sources due to photoabsorption in the  $BS$ 's [Eqs. (5a) and (5b)] and the efficiency of detectors [Eq. (13)]. One can check that in such an algebra, the output operators  $\hat{\alpha}, \hat{\beta}$ , those describing the fields reaching the detectors  $\hat{b}, \hat{c}$  ( $\hat{d}, \hat{e}$ ) in  $BS_2(BS'_2)$ , are

$$\hat{\alpha}_{\text{out}} = \mathbf{t} \hat{\alpha}_{\text{in}} + \mathbf{r} \hat{\beta}_{\text{in}} + \hat{\mathbf{L}}_\alpha, \quad (16a)$$

$$\hat{\beta}_{\text{out}} = \mathbf{t} \hat{\beta}_{\text{in}} + \mathbf{r} \hat{\alpha}_{\text{in}} + \hat{\mathbf{L}}_\beta, \quad (16b)$$

where  $\mathbf{t} = \sqrt{\eta}t$ ,  $\mathbf{r} = \sqrt{\eta}r$ , and  $\hat{\mathbf{L}}_\alpha = \hat{\mathcal{L}}_\alpha + \hat{\mathcal{L}}_\alpha$ . In fact, from all the above-mentioned properties of the operators in relations (16a) and (16b), we get

$$[\hat{\mathbf{L}}_\alpha, \hat{\mathbf{L}}_\alpha^\dagger] = [\hat{\mathbf{L}}_\beta, \hat{\mathbf{L}}_\beta^\dagger] = \eta\Gamma + (1 - \eta), \quad (17a)$$

$$[\hat{\mathbf{L}}_\alpha, \hat{\mathbf{L}}_\beta^\dagger] = [\hat{\mathbf{L}}_\beta, \hat{\mathbf{L}}_\alpha^\dagger] = -\eta\Omega. \quad (17b)$$

When considering  $\eta = 1$  in Eqs. (17a) and (17b), we recover the relations (7a) and (7b), while for  $\Gamma = 0$ , which also implies  $\Omega = 0$ , we recover the relations (15a) and (15b), respectively.

#### VI. ENGINEERING THE STATE TO BE TELEPORTED

Back to the apparatus in Fig. 1(b), when engineering the state to be teleported in a noise environment by sending a single-photon field  $c$  on  $BS'_1$ , Eq. (12a) leads the  $c$  and  $d$  output fields, together with the environment, in the entanglement

$$(t|10\rangle_{cd} + r|01\rangle_{cd} + |00\rangle_{cd} \hat{\mathcal{L}}_c) |\mathbf{0}\rangle_{\mathbf{E}}. \quad (18)$$

Next, on  $BS'_2$  the field in mode  $d$  is coupled to an additional field in mode  $e$  prepared in a coherent state  $|\gamma\rangle_e = \sum_n \gamma_n |n\rangle_e$ . As mentioned above, the synthesized projection onto the state in mode  $d$  leading to the engineered state (1) results when a single photon is registered in  $D_d$  and no counts are in  $D_e$ , in a way that the output state  $c$  plus environment reads

$${}_d \langle 1| {}_c \langle 0| (t|1\rangle_c + r|0\rangle_c \hat{d}_{\text{in}}^\dagger + |0\rangle_c \hat{\mathcal{L}}_c^\dagger) \sum_n \gamma_n \frac{\hat{e}_{\text{in}}^{\dagger n}}{\sqrt{n!}} |00\rangle_{de} |\mathbf{0}\rangle_{\mathbf{E}}. \quad (19)$$

We note that the environmental states due to both beam splitters have been put together. Since all the output operators composing  $\hat{e}_{\text{in}}^\dagger$  commute to each other, using the binomial formula

$$\hat{e}_{\text{in}}^{\dagger n} = \sum_{k=0}^n \sum_{l=0}^{n-k} \binom{n}{k} \binom{n-k}{l} \mathbf{t}^k \mathbf{r}^l \hat{d}_{\text{out}}^\dagger \hat{e}_{\text{out}}^{\dagger k} \hat{\mathbf{L}}_e^{\dagger n-k-l}, \quad (20)$$

we end up with

$$|\psi\rangle_{c\mathbf{E}} = \mathcal{N} [\gamma_0 |0\rangle_c |\mathbf{A}_0(t, r)\rangle_{\mathbf{E}} + \gamma_1 |1\rangle_c |\mathbf{A}_1(t, r)\rangle_{\mathbf{E}}], \quad (21)$$

where the environmental states read

$$|\Lambda_0(t,r)\rangle_{\mathbf{E}} = \frac{\mathbf{r}}{\gamma_0} \sum_n \frac{\gamma_n}{\sqrt{n!}} (t\hat{\mathbf{L}}_e^\dagger + nr\hat{\mathbf{L}}_d^\dagger + n\hat{\mathbf{L}}_c^\dagger)\hat{\mathbf{L}}_e^{\dagger n-1} |\mathbf{0}\rangle_{\mathbf{E}}, \quad (22a)$$

$$|\Lambda_1(t,r)\rangle_{\mathbf{E}} = \frac{t\mathbf{r}}{\gamma_1} \sum_n \frac{n\gamma_n}{\sqrt{n!}} \hat{\mathbf{L}}_e^{\dagger n-1} |\mathbf{0}\rangle_{\mathbf{E}}. \quad (22b)$$

Now we use the Wick's theorem for boson operators,

$$\langle \hat{\mathbf{L}}_\alpha^n \hat{\mathbf{L}}_\beta^{\dagger m} \rangle = \delta_{\alpha\beta} \delta_{nm} n! \langle \hat{\mathbf{L}}_\alpha \hat{\mathbf{L}}_\beta^\dagger \rangle^n = \delta_{\alpha\beta} \delta_{nm} n! [\eta\Gamma + (1-\eta)]^n, \quad (23)$$

where for  $\eta=1$  it follows that  $\hat{\mathbf{L}}_\alpha = \hat{\mathcal{L}}_\alpha$  and for  $\Gamma=0$ ,  $\hat{\mathbf{L}}_\alpha = \hat{\mathcal{L}}_\alpha$ . From relation Eq. (23) we get, for the normalization factor  $\mathcal{N}$  in (21), the result

$$\mathcal{N} = \left\{ e^{[\eta\Gamma + (1-\eta)]|\alpha|^2} \eta |r|^2 \left[ |C|^2 |t|^2 + \left( \eta\Gamma + \frac{\Gamma}{|r|^2} + (1-\eta) \right) |r|^2 |\gamma_1|^2 \right] \right\}^{-1}. \quad (24)$$

The fidelity of the optical state truncation scheme leading to the engineered field  $|\psi\rangle_{c\mathbf{E}}$ , expected to be  $|\Psi\rangle_c = (\gamma_0|0\rangle_c + \gamma_1|1\rangle_c)/\mathcal{C}$  when 50/50 BS's are considered (i.e.,  $|t|=|r|=|\xi|$ ), also results from Wick's theorem as

$$\mathcal{F} = \|\langle \Psi | \psi \rangle_{c\mathbf{E}}\|^2 = 1 - \frac{1 - \eta \left( \frac{1+\Gamma^2}{1-\Gamma} \right)}{(1+\mathcal{R}) \left\{ 1 + \mathcal{R} \left[ 1 - \eta \left( \frac{1+\Gamma^2}{1-\Gamma} \right) \right] \right\}}, \quad (25)$$

where  $\mathcal{R} = (|\gamma_0|/|\gamma_1|)^2$ . As expected, when considering ideal detectors ( $\eta=1$ ) and disregarding the losses in the BS's ( $\Gamma=0$ ), we find  $\mathcal{F}=1$ . Moreover, for finite  $\eta$  and  $\Gamma$ , the larger the ratio  $\mathcal{R}$ , making the probability to find a photon negligible, the closer to unity is the fidelity. We note that, unlike other situations, when measuring  $|10\rangle_{bc}$ , as we have done above, we do not need to consider 50/50 BS's in order for the relation  $\gamma_0/c_0 = \gamma_1/c_1 = \mathcal{C}$  to be required.

## VII. TELEPORTATION PROCESS

As mentioned above, simultaneously to the preparation of the state to be teleported, which is given to Alice, the quantum channel has to be prepared by sending a one-photon field state through BS<sub>1</sub>. The quantum channel is exactly described by Eq. (18), except that we must change the output symbols  $c$  and  $d$  by  $a$  and  $b$ , respectively, while the state to be teleported comprehends Eq. (21), as indicated in Fig. 1. For simplicity, the state to be teleported will be rewritten as

$$|\psi\rangle_{c\mathbf{E}} = \mathcal{N} (|\gamma_0\rangle_{\mathbf{E}} |0\rangle_c + |\gamma_1\rangle_{\mathbf{E}} |1\rangle_c), \quad (26)$$

with  $|\gamma_0\rangle_{\mathbf{E}} = \gamma_0 |\Lambda_0(t,r)\rangle_{\mathbf{E}}$  and  $|\gamma_1\rangle_{\mathbf{E}} = \gamma_1 |\Lambda_1(t,r)\rangle_{\mathbf{E}}$ . Alice is thus supposed to realize the joint measurement on fields  $b$  and  $c$ , which is accomplished through BS<sub>2</sub> following exactly the steps outlined in Eqs. (19)–(22b), substituting the coherent state  $|\gamma\rangle_c$  by  $|\psi\rangle_{c\mathbf{E}}$  and the modes  $d, e$  by  $b, c$ , respec-

tively. As we see from Eq. (4a), in the ideal situation, when projecting the correlated output fields  $b$  and  $c$  on the state  $|10\rangle_{bc}$ , as done in Eq. (19), we are proceeding to a Bell measurement of the incoming state  $|\Psi^-\rangle_{bc}$ . Obviously, this is not the case when the loss mechanisms in the beam splitters are taken into account. After a straightforward calculation, we obtain, for the teleported field  $a$ , the state

$$|\psi\rangle_{a\mathbf{E}} = \mathbf{N} (|\lambda_0\rangle_{\mathbf{E}} |0\rangle_c + |\lambda_1\rangle_{\mathbf{E}} |1\rangle_c), \quad (27)$$

with the normalization constant

$$\mathbf{N} = \left\{ e^{-\eta(1-\Gamma)|\alpha|^2} \eta \left( \frac{1-\Gamma}{2} \right)^2 \times \left[ 1 + \frac{1}{\mathcal{R}} \left( \frac{4}{1-\Gamma} - 3\eta(1-\Gamma) \right) \right] \right\}^{-1} \quad (28)$$

and

$$|\lambda_0\rangle_{\mathbf{E}} = |\gamma_0\rangle_{\mathbf{E}} + |\gamma_1\rangle_{\mathbf{E}} \left( \hat{\mathbf{L}}_c^\dagger + \frac{r}{t} \hat{\mathbf{L}}_b^\dagger + \frac{1}{t} \hat{\mathbf{L}}_a^\dagger \right) |\mathbf{0}\rangle_{\mathbf{E}}, \quad (29a)$$

$$|\lambda_1\rangle_{\mathbf{E}} = |\gamma_1\rangle_{\mathbf{E}} |\mathbf{0}\rangle_{\mathbf{E}}. \quad (29b)$$

The fidelity of the teleported state expected to be  $|\Psi\rangle_a = 1/\mathcal{C} (\gamma_0|0\rangle_a + \gamma_1|1\rangle_a)$ , after computing losses from both the state engineering scheme and the teleportation process, results,

$$\mathbf{F} = \|\langle \Psi | \psi \rangle_{a\mathbf{E}}\|^2 = 1 - \frac{\frac{3+\Gamma}{1-\Gamma} - 3\eta(1-\Gamma)}{(1+\mathcal{R}) \left\{ 1 + \mathcal{R} \left[ \frac{4}{1-\Gamma} - 3\eta(1-\Gamma) \right] \right\}}, \quad (30)$$

where 50/50 BS's were considered. Evidently, for the ideal case  $\mathbf{F}=1$  and also the largest  $\mathcal{R}$ , the closest to unity is the fidelity.

It is straightforward to obtain the density matrix for both prepared (21) and teleported field state (26) by getting rid of the environmental degrees of freedom. By comparing the prepared field state density matrix when  $\Gamma=0$  we obtain exactly the result presented in Ref. [12].

## VIII. CONCLUDING REMARKS

For estimating the fidelities in Eqs. (25) and (30), we note that the efficiency for single-photon detectors is about 70%, while the damping constant for BS's is considerably small, less than 2% in BK7 crystals. Despite the nonunity quantum efficiency detectors and the absorptive beam splitters, there are two other experimental nonidealities that seem to be equally important. The first is the fact that we do not have perfect single-photon sources as required for optical state truncation and teleportation by projection synthesis. The commonly cited method of parametric fluorescence only approximates a single-photon source, and this approximation must be evaluated in its effect on the projection-synthesis technique. Second, the detectors are required to discriminate between zero, one and two (and even more) photon arrivals.

Since this is currently not possible, the analysis should also account for this deficiency, which, in a rough way, can also be estimated through Eqs. (13)–(15a) by including it in the parameter  $\eta$ .

As far as we know, until the present date there has been no other scheme for teleportation of a zero- and one-photon running-wave superposition. The present scheme becomes possible due to the recent proposals for state truncation of traveling optical fields [12,16]. Finally, we note that it is worth proceeding to the generalization of the present

scheme, for teleportation of  $N$ -dimensional states, possibly through the Dakna *et al.* engineering technique for an arbitrary quantum state of traveling fields [16].

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