ARTICLES

Teleportation of a cavity-radiation-field state: An alternative scheme

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We present an alternative and experimentally feasible scheme to realize the teleportation of a cavity mode in a coherent superposition of zero-, one-, and two-photon field states. Particular nonmaximally entangled states of two-level atoms and particular cavity modes are employed and their preparation is discussed. A Ramsey-type arrangement, together with pointer atoms, is required to prepare the atomic entangled states as well as to read out the cavity system. Basically, the pointer atoms induce the state-vector collapse of the field in the cavity system and indicate, through their detection, the measurement result. [S1050-2947(96)05412-1]

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I. INTRODUCTION

The phenomenon of quantum nonlocality observed [1,2] in experiments on Einstein-Podolsky-Rosen (EPR) [3] states, through the violation of the Bell's inequalities [4], has been considered to investigate the possibility of a variety of recent striking predictions in quantum mechanics. Different quantum cryptographic protocols [5] and quantum computers models [6] have been suggested and Bennett *et al.* [7] have outlined a scheme for quantum-state teleportation that has been discussed by several authors.

By teleportation Bennett *et al.* mean that, via dual classical and quantum channels, an unknown state $|\Phi\rangle_1$ of a particle 1 is exactly replicated into the state of a particle 3 far away from 1. To do so, particle 1 is given to a sender, Alice, who shares a maximally entangles state concerning particles 2 and 3, $|\Psi\rangle_{23}$, with a receiver, Bob. By performing a joint measurement of the von Neumann type on her two particles, 1 and 2, Alice is able to automatically project Bob's particle 3 into a pure state that differs from $|\Phi\rangle_1$ just by an irrelevant phase factor or a rotation around the x, y, or z axes. Information on Alice's measurement, transmitted to Bob through the classical channel, allows him to apply the appropriate rotation on particle 3, if necessary, converting it into a replica of particle 1. The original particle-1 state is destroyed in the process, obeying the no-cloning theorem [8].

Schemes employing cavity QED phenomena to realize the teleportation of quantum superpositions with two orthogonal states, following the ideas of Bennett *et al.*, have been proposed. Davidovich *et al.* [9] have presented an experimentally feasible scheme for the teleportation of an unknown two-level atomic state between two high-Q cavities containing a nonlocal microwave field. Recently developed methods to build and measure nonclassical coherent superpositions of states of the electromagnetic field [10,11] (Schrödinger cat states) have been used in Ref. [9]. Cirac and Parkins [12] have presented a scheme to realize teleportation by employing a technique to prepare two or more atoms in certain entangled states [13], which is based on the interaction of the atoms with a cavity mode. Such a scheme can also be used for the teleportation of a cavity mode in a superposition of the zero- and one-photon Fock states besides a two-level atomic state. In both cases [9,12], circular Rydberg atoms and field ionization detectors are considered.

Bennett *et al.* have also addressed the question of the teleportation of a system having $N>2$ orthogonal states. As a generalization of their scheme outlined for $N=2$, the EPR spin pair in the singlet state is replaced by a pair of N-state particles in a completely entangled state. By writing this state as $\Sigma_j|j\rangle\otimes\langle j|/\sqrt{N}$, where $j=0,1,\ldots,N-1$ labels the N elements of an orthonormal basis for each of the N-state systems, the process of teleportation turns out to be exactly the same as explained above. The result of Alice's joint measurement on particles 1 and 2 is now described by the eigenstates

$$|\Psi\rangle_{nm}=\sum_j e^{2\pi ijn/N}|j\rangle\otimes|(j+m)\mathrm{mod}N\rangle/\sqrt{N}. \tag{1}$$

The outcome nm is communicated by classical means to Bob, who performs, on his previously entangled particle, the unitary transformation

$$U_{nm}=\sum_k e^{2\pi ikn/N}|k\rangle\otimes\langle(k+m)\mathrm{mod}N|, \tag{2}$$

which converts Bob's particle 3 to the original state of Alice's particle 1.

In this paper we propose an alternative and more realistic scheme to realize the teleportation of a cavity mode in a coherent superposition of zero-, one-, and two-photon field states,

$$|\Phi\rangle=c_0|0\rangle+c_1|1\rangle+c_2|2\rangle. \tag{3}$$

In place of a pair of three-state particles as required in the scheme of Bennett *et al.*, the quantum channel takes place through two pairs of two-level atoms prepared in the nonmaximally entangled states

$$|\Psi\rangle_{2j-1,2j}=\gamma_{2j-1}|e\rangle_{2j-1}|g\rangle_{2j}+\gamma_{2j}|g\rangle_{2j-1}|e\rangle_{2j}, \quad j=1,2. \tag{4}$$

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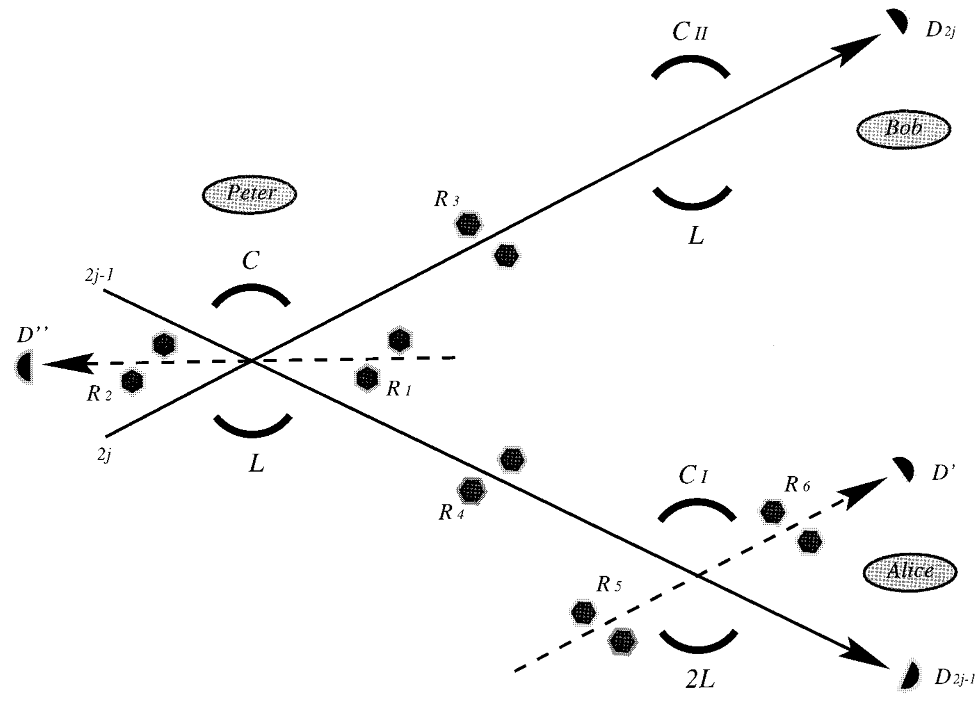

FIG. 1. Sketch of the experimental setup for teleportation of a cavity mode.

The atom-field interaction is described using the resonant Jaynes-Cummings Hamiltonian [14].

Similarly to the result in [7], it is shown that the first pair of two-level atoms described in (4), $j=1$, is able to teleport the superposition of the first two Fock states in the cavity mode (3) (with their respective probability amplitudes $c_0|0\rangle + c_1|1\rangle$). However, the teleportation of the whole cavity mode (3) will only be accomplished through the subsequent pair of two-level atoms, $j=2$, which is able to complete the superposition with the remained two-photon field state (and its respective probability amplitude $c_2|2\rangle$). In this way, the whole cavity mode is supposed to be teleported from one to another cavity, step by step, each one accomplished by a pair of two-level atoms in a particular nonmaximally entangled state.

Differently from the procedure outlined in [7], where, in principle, the teleportation process is achieved with probability 1, this alternative scheme presents a probability less than 1 associated to each of the above-mentioned steps. So each step must be repeatedly performed until the desired result has been achieved. Another difference, the complete measurement of von Neumann type required in the scheme of Bennett *et al.* to couple the system whose state is to be teleported with the system constituting the quantum channel, is here substituted by the two steps of the above-mentioned atom-cavity interactions.

Concerning the classical channel, in the present scheme it has to be extended to the experimenter who is supposed to prepare the nonmaximally entangled states of two-level atoms. It will be shown that Alice, Bob, and the preparer (henceforth ''Peter'') previously have to decide about the relation between the coefficients γ_{2j-1} and γ_{2j} of the entangled states of two-level atoms to be considered.

Pointer atoms are considered in the present scheme to prepare the required superposition states. In addition, as their main role in the teleportation process, the pointer atoms induce the state-vector collapse of the field in the cavity system and simultaneously indicate the measurement result. A Ramsey-type arrangement is also required for the preparation of the atomic entangled state.

A sketch of the experimental setup for the present alternative scheme for teleportation is displayed in Fig. 1. The setup contains three microwave cavities C, C_{I}, and C_{II}. Through a Ramsey-type arrangement, where cavity C is placed between the auxiliary microwave zones R_1 and R_2, Peter is supposed to prepare the nonmaximally entangled states of two-level atoms $2j-1$ and $2j$ (solid-line trajectories). Pointer atoms (dashed-line trajectories) are required to prepare both the entangled states of two-level atoms and, in C_{I}, the cavity mode to be teleported. Additional microwaves zones R_3 and R_4 have to be considered for the preparation of the atomic entangled states. Once atoms $2j-1$ and $2j$ have successively crossed cavities C_{I} and C_{II} (to where the cavity mode in C_{I} is supposed to be teleported), pointer atoms are also required, together with microwave zones R_5 and R_6, to read the field in the cavity system, inducing its state-vector collapse and indicating the measurement result. All the atomic beams are made of identical two-level atoms that are counted by state-selective field ionization detectors D, D', and D''. For the experimental proposal we must consider, as will be discussed further, atomic Rydberg states with adjacent principal quantum numbers.

In Sec. II we describe the two steps necessary for the teleportation process of a cavity mode (3). The rules of the dual classical and quantum channel are discussed in Sec. III and the preparation of the entangled states of two-level atoms and of the cavity mode to be teleported is presented in

Sec. IV. In Sec. V we discuss the teleportation of a cavity mode in a coherent superposition with more than three Fock states. We conclude in Sec. VI by discussing the experimental feasibility of this alternative teleportation process with present techniques.

II. SCHEME

A. Step 1

As a first step of the present scheme for teleportation of a cavity mode, a pair of two-level atoms in a nonmaximally entangled state is required. To prepare such an entangled state, as described by (4), atom $2j-1$, initially prepared in the state $|\psi\rangle_{2j-1}$, is send across a cavity C, initially in the vacuum state. Subsequently, atom $2j$, initially prepared in the state $|\psi\rangle_{2j}$, is sent across the same cavity, as represented schematically in Fig. 1. The definition of the states $|\psi\rangle_{2j-1}$ and $|\psi\rangle_{2j}$, as well as the detailed preparation of the required entangled state, will be discussed further, after information about the relation between γ_{2j-1} and γ_{2j} has been obtained.

Once atoms 1 and 2 ($j=1$) have been prepared in the entangled state

$$|\Psi\rangle_{12}=\gamma_1|e\rangle_1|g\rangle_2+\gamma_2|g\rangle_1|e\rangle_2, \tag{5}$$

atom 1 is sent across cavity C_{I}, initially prepared in the state $|\Phi\rangle_{\mathrm{I}}$, which is supposed to be teleported to a cavity C_{II}, initially in the vacuum state $|0\rangle_{\mathrm{II}}$. By sending atom 2 across C_{II} we set up the nonlocal quantum channel required for the teleportation process.

Let us suppose that C_{I} has been prepared in the superposition of zero-, one-, and two-photon field states in Eq. (3). It will be later explained how to prepare this cavity mode. Thus, before the atom-field interactions, the complete state vector of the system composed by atoms 1 and 2 plus cavities C_{I} and C_{II} reads

$$|\Psi\rangle_{12}|\Phi\rangle_{\mathrm{I}}|\Phi\rangle_{\mathrm{II}}=(\gamma_1|e\rangle_1|g\rangle_2+\gamma_2|g\rangle_1|e\rangle_2)\times(c_0|0\rangle_{\mathrm{I}}+c_1|1\rangle_{\mathrm{I}}+c_2|2\rangle_{\mathrm{I}})|0\rangle_{\mathrm{II}}. \tag{6}$$

The interactions between atoms 1 and 2 and their respective cavities are described by the resonant Jaynes-Cummings Hamiltonian in the interaction picture

$$\hat{H}=\hbar\Omega(\hat{\sigma}_+\hat{a}+\hat{\sigma}_-\hat{a}^\dagger), \tag{7}$$

where the vacuum Rabi frequency Ω determines the coupling strength between atoms and quantized cavity fields. $\hat{a}^\dagger$ and $\hat{a}$ denote, respectively, the creation and annihilation operators for the cavity mode, while the atomic operators $\hat{\sigma}_+$ and $\hat{\sigma}_-$ describe the transitions of the two-level system.

In order to adjust the atom-field interaction times to our convenience, we consider that the atomic velocities have been selected by means of a velocity-selective chopping [13,12]. By considering the interaction time of atom 2 with cavity C_{II} given by $\tau_2=\pi/2\Omega$, after the interaction the whole system in Eq. (6) will be left in the entangled state

$$\begin{aligned}e^{-i/\hbar\hat{H}_1\tau_1}e^{-i/\hbar\hat{H}_2\tau_2}|\Psi\rangle_{12}|\Phi\rangle_{\mathrm{I}}|\Phi\rangle_{\mathrm{II}}
&=\sum_{n=0}^{2}c_n\{-i|g\rangle_1[\gamma_1\sin(\sqrt{n+1}\Omega\tau_1)|n+1\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\
&\quad+\gamma_2\cos(\sqrt{n}\Omega\tau_1)|n\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}]\\
&\quad+|e\rangle_1[\gamma_1\cos(\sqrt{n+1}\Omega\tau_1)|n\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\
&\quad-\gamma_2\sin(\sqrt{n}\Omega\tau_1)|n-1\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}]\}|g\rangle_2,\end{aligned} \tag{8}$$

where τ_1 stands for the interaction time of atom 1 with cavity C_{I}. As shown in Fig. 1, after the atom-field interactions, the detections of atoms $2j-1$ and $2j$ are performed by state-selective field ionization detectors D_{2j-1} and D_{2j}, respectively [9]. From (8) we see that atom 2 is left in the ground state, while atom 1 is left in a superposition of the ground and excited states. It does not matter, for the following argument, in which state atom 1 is detected. However, for simplicity, let us consider that atom 1 has been detected in the ground state, so the pure field state of the cavity systems now reads

$$\begin{aligned}|\Phi\rangle_{\mathrm{I,II}}=\mathcal{N}\sum_{n=0}^{2}c_n[\gamma_1\sin(\sqrt{n+1}\Omega\tau_1)|n+1\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\
+\gamma_2\cos(\sqrt{n}\Omega\tau_1)|n\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}],\end{aligned} \tag{9}$$

with the normalization

$$\mathcal{N}=\left\{\sum_{n=0}^{2}\left|c_n\right|^2[|\gamma_1|^2\sin^2(\sqrt{n+1}\Omega\tau_1)+|\gamma_2|^2\cos^2(\sqrt{n}\Omega\tau_1)]\right\}^{-1/2}.$$

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B. Teleportation of the cavity mode $c_0|0\rangle+c_1|1\rangle$

In view of Eq. (9), let us consider for a while that the cavity mode to be teleported from C_{I} to C_{II} presents just the Fock states $|0\rangle_{\mathrm{I}}$ and $|1\rangle_{\mathrm{I}}$, $|\Phi\rangle_{\mathrm{I}}'=c_0|0\rangle_{\mathrm{I}}+c_1|1\rangle_{\mathrm{I}}$, instead of the original one added by the Fock state $|2\rangle_{\mathrm{I}}$. So Eq. (9) turns out to be

$$\begin{aligned}|\Phi\rangle'_{\mathrm{I,II}}=\mathcal{N}'\{\gamma_2c_0|0\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}+|1\rangle_{\mathrm{I}}[\gamma_1\sin(\Omega\tau_1)c_0|0\rangle_{\mathrm{II}}\\
+\gamma_2\cos(\Omega\tau_1)c_1|1\rangle_{\mathrm{II}}]\\
+\gamma_1\sin(\sqrt{2}\Omega\tau_1)c_1|2\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\}.\end{aligned} \tag{10}$$

Therefore, from Eq. (10) we observe that, to achieve the teleportation of the state $|\Phi\rangle_{\mathrm{I}}'$ from C_{I} to C_{II}, we have to consider $\gamma_1=\gamma_2=1/\sqrt{2}$ and $\tau_1=(m+1/4)\pi/\Omega$ (with m an integer). In this way, by factorizing the quantity $\sin[(m+1/4)\pi]/\sqrt{2}$ in Eq. (10), cavity C_{II} is left in the state $|\Phi\rangle_{\mathrm{I}}'$ as soon as cavity C_{I} has been detected in the one-photon field state. The normalization constant now reads $\mathcal{N}'=\{1+\sin^2[(m+1/4)\pi]-|c_1|^2\cos^2[(m+1/4)\sqrt{2}\pi]\}^{-1/2}$ and the probability to achieve the teleportation is given by $|\mathcal{N}'|^2\sin^2[(m+1/4)\pi]/2$. By considering $\tau_1=17\pi/4\Omega$, i.e., $m=4$, the probability for the accomplishment of the teleportation process is maximized, around 0.5.

To induce the collapse of the entangled state (10) and simultaneously indicate the measurement result, we have to send a pointer atom, initially prepared in the excited state $|e\rangle^p$ (the superscript p labeling the pointer atom), across the Ramsey-type arrangement $R_5-C_{\mathrm{I}}-R_6$ (see Fig. 1). First, the pointer atom is tuned to have a dispersive interaction with the field in C_{I} [15]. The effect of such an interaction is probed by two separated oscillatory fields applied in microwave zones R_5 and R_6, which are sandwiching C_{I}. Equivalently to a recently demonstrated Ramsey atomic interferometry [16], the probability for the pointer atom to undergo an $|e\rangle^p\rightarrow|g\rangle^p$ transition exhibits a fringe pattern that is characteristic of the photon number in C_{I}. Such a transition probability depends on a given setting of the microwave fields in R_5 and R_6, in addition to depending on the atom-cavity interaction. For the present purpose we assume that the microwave zones R_5 and R_6 are set so that the pointer atom undergoes exactly a $\pi/2$ pulse, on the $|e\rangle^p\rightarrow|g\rangle^p$ transition, in each zone. The cavity detuning is also set so that the atom undergoes a phase shift per photon exactly equal to π. In this way, as obtained in Ref. [11], the system composed by the pointer atom plus a cavity containing n photons ends up in the superposition state

$$|\Phi_{\mathrm{final}}^{\mathrm{pointer\ atom\ +\ field}}\rangle=\tfrac{1}{2}[(e^{-in\pi}-1)|e\rangle^p-(e^{-in\pi}+1)|g\rangle^p]|n\rangle. \tag{11}$$

After crossing the Ramsey-type arrangement, the probability for detecting the pointer atom in $|g\rangle^p$ ($|e\rangle^p$) is thus zero for n odd (even) and the whole system evolves to the final state

$$|e\rangle^p|\Phi\rangle'_{\mathrm{I,II}}\rightarrow\mathcal{N}'\{c_0|g\rangle^p|0\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}+\sin[(m+1/4)\pi]|e\rangle^p|1\rangle_{\mathrm{I}}(c_0|0\rangle_{\mathrm{II}}+c_1|1\rangle_{\mathrm{II}})+\sin[(m+1/4)\sqrt{2}\pi]c_1|g\rangle^p|2\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\}. \tag{12}$$

So the detection of the pointer atom in the excited state $|e\rangle^p$ causes the fields in C_{I} and C_{II} to collapse into their eigenstates $|1\rangle_{\mathrm{I}}$ and $c_0|0\rangle_{\mathrm{I}}+c_1|1\rangle_{\mathrm{II}}$, successfully completing the teleportation process. The probability for this occurrence remains $|\mathcal{N}'|^2\sin^2[(m+1/4)\pi]/2$ (≈ 0.5 when $m=4$). As indicated in Fig. 1, the detection of the pointer atom is achieved with the help of the state-selective ionization detector D'. It is worth noting that, in the present scheme, only the teleportation of a cavity mode with two orthogonal states requires a maximally entangled state ($\gamma_1=\gamma_2$) as the quantum channel.

C. Step 2

Returning to the teleportation process of the original cavity mode $|\Phi\rangle_{\mathrm{I}}$, as the second step another pair of two-level atoms, 3 and 4, must be prepared, as will be shown later, in the nonmaximally entangled state

$$|\Psi\rangle_{34}=\gamma_3|e\rangle_3|g\rangle_4+\gamma_4|g\rangle_3|e\rangle_4. \tag{13}$$

After the preparation of the state (13), atoms 3 and 4 travel across C_{I} and C_{II}, respectively, exactly as in the first step. Since the fields in C_{I} and C_{II} have been left in the entangled state (9), after the interactions we obtain

$$\begin{aligned}
e^{-i/\hbar\hat{H}_3\tau_3}e^{-i/\hbar\hat{H}_4\tau_4}|\Psi\rangle_{34}|\Phi\rangle_{\mathrm{I,II}}=\mathcal{N}\sum_{n=0}^{2}c_n\{&-i|g\rangle_3|g\rangle_4[\gamma_1\gamma_3\sin(\sqrt{n+1}\Omega\tau_1)\sin(\sqrt{n+2}\Omega\tau_3)|n+2\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\
&+\gamma_1\gamma_4\sin(\sqrt{n+1}\Omega\tau_1)\cos(\sqrt{n+1}\Omega\tau_3)|n+1\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}\\
&+\gamma_2\gamma_4\sin(\sqrt{2}\pi/2)\cos(\sqrt{n}\Omega\tau_1)\cos(\sqrt{n}\Omega\tau_3)|n\rangle_{\mathrm{I}}|2\rangle_{\mathrm{II}}]\\
&-|g\rangle_3|e\rangle_4[\gamma_2\gamma_3\cos(\sqrt{n}\Omega\tau_1)\sin(\sqrt{n+1}\Omega\tau_3)|n+1\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\
&+\gamma_2\gamma_4\cos(\sqrt{2}\pi/2)\cos(\sqrt{n}\Omega\tau_1)\cos(\sqrt{n}\Omega\tau_3)|n\rangle_{\mathrm{I}}|1\rangle]\\
&+|e\rangle_3|g\rangle_4[\gamma_1\gamma_3\sin(\sqrt{n+1}\Omega\tau_1)\cos(\sqrt{n+2}\Omega\tau_3)|n+1\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\
&-\gamma_1\gamma_4\sin(\sqrt{n+1}\Omega\tau_1)\sin(\sqrt{n+1}\Omega\tau_3)|n\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}\\
&-\gamma_2\gamma_4\sin(\sqrt{2}\pi/2)\cos(\sqrt{n}\Omega\tau_1)\sin(\sqrt{n}\Omega\tau_3)|n-1\rangle_{\mathrm{I}}|2\rangle_{\mathrm{II}}]\\
&-i|e\rangle_3|e\rangle_4[\gamma_2\gamma_3\cos(\sqrt{n}\Omega\tau_1)\cos(\sqrt{n+1}\Omega\tau_3)|n\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\
&+\gamma_2\gamma_4\cos(\sqrt{2}\pi/2)\cos(\sqrt{n}\Omega\tau_1)\sin(\sqrt{n}\Omega\tau_3)|n-1\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}]\}.
\end{aligned} \tag{14}$$

Similarly to the interaction of atom 2 with cavity C_{II} in the first step, the atom-field interaction time of atom 4 with cavity C_{II} has been assumed to be $\tau_4=\pi/2\Omega$. Now two conditions have to be satisfied for the accomplishment of the teleportation process of the cavity mode $|\Phi\rangle_{\mathrm{I}}$ from C_{I} to C_{II}.

(i) By explicitly writing the sum in Eq. (14) we observe that only when atom 4 is left in the ground state $|g\rangle_4$, no matter which state atom 3 is in, C_{II} presents a linear combination of the elements $c_0|0\rangle_{\mathrm{II}}$, $c_1|1\rangle_{\mathrm{II}}$, and $c_2|2\rangle_{\mathrm{II}}$, which compose the state of the original C_{I} cavity mode. Since the Jaynes-Cummings Hamiltonian conserves the excitation number, by detecting atoms 3 and 4 in their ground states, $|g\rangle_3$ and $|g\rangle_4$, C_{II} is left in the two-photon field state, and by detecting them successively in the excited and ground states, $|e\rangle_3$ and $|g\rangle_4$, C_{II} is left in the one-photon field state. Supposing that both atoms have been detected in the ground state, we obtain the pure field state of the cavity system as

$$\begin{aligned}|\Phi\rangle''_{\mathrm{I,II}}=\mathcal{N}''\sum_{n=0}^{2} c_n[&\gamma_1\gamma_3\sin(\sqrt{n+1}\Omega\tau_1)\\ &\times\sin(\sqrt{n+2}\Omega\tau_3)|n+2\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\\ &+\gamma_1\gamma_4\sin(\sqrt{n+1}\Omega\tau_1)\\ &\times\cos(\sqrt{n+1}\Omega\tau_3)|n+1\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}\\ &+\gamma_2\gamma_4\sin(\sqrt{2}\pi/2)\cos(\sqrt{n}\Omega\tau_1)\\ &\times\cos(\sqrt{n}\Omega\tau_3)|n\rangle_{\mathrm{I}}|2\rangle_{\mathrm{II}}],\end{aligned} \tag{15}$$

with the normalization

$$\begin{aligned}\mathcal{N}''=\Bigg\{\sum_{n=0}^{2}&|c_n|^2[|\gamma_1\gamma_3|^2\sin^2(\sqrt{n+1}\Omega\tau_1)\sin^2(\sqrt{n+2}\Omega\tau_3)\\ &+|\gamma_1\gamma_4|^2\sin^2(\sqrt{n+1}\Omega\tau_1)\cos^2(\sqrt{n+1}\Omega\tau_3)\\ &+|\gamma_2\gamma_4|^2\sin^2(\sqrt{2}\pi/2)\cos^2(\sqrt{n}\Omega\tau_1)\\ &\times\cos^2(\sqrt{n}\Omega\tau_3)]\Bigg\}^{-1/2}.\end{aligned}$$

(ii) From Eq. (15) we conclude that the coefficients appearing in the nonmaximally entangled states of two-level atoms in Eqs. (5) and (13), namely, γ_1,γ_2 and γ_2,γ_4, respectively, must satisfy the equality

$$\begin{aligned}&\gamma_1\gamma_3\sin(\sqrt{n+1}\Omega\tau_1)\sin(\sqrt{n+2}\Omega\tau_3)|_{n=0}\\ &\quad=\gamma_1\gamma_4\sin(\sqrt{n+1}\Omega\tau_1)\cos(\sqrt{n+1}\Omega\tau_3)|_{n=1}\\ &\quad=\gamma_2\gamma_4\sin(\sqrt{2}\pi/2)\cos(\sqrt{n}\Omega\tau_1)\cos(\sqrt{n}\Omega\tau_3)|_{n=2}.\end{aligned}$$

The values of n $(=0,1,2)$ correspond to finding C_{I} in the two-photon field state as mentioned above. So, for the accomplishment of the teleportation process the following relations must be satisfied:

$$\gamma_2=\gamma_1\frac{\tan(\sqrt{2}\Omega\tau_1)}{\sin(\sqrt{2}\pi/2)}, \tag{16}$$

$$\gamma_4=\gamma_3\frac{\sin(\Omega\tau_1)\tan(\sqrt{2}\Omega\tau_3)}{\sin(\sqrt{2}\Omega\tau_1)}. \tag{17}$$

With the above relations the pure field states of the cavity system Eq. (15) turns out to be

$$\begin{aligned}|\Phi\rangle''_{\mathrm{I,II}}=\mathcal{N}''\gamma_1\gamma_3\sin(\Omega\tau_1)\sin(\sqrt{2}\Omega\tau_3)\Bigg[&\frac{1}{\cos(\sqrt{2}\Omega\tau_1)\cos(\sqrt{2}\Omega\tau_3)}c_0|0\rangle_{\mathrm{I}}|2\rangle_{\mathrm{II}}+\frac{\cos(\Omega\tau_3)}{\cos(\sqrt{2}\Omega\tau_3)}|1\rangle_{\mathrm{I}}\Bigg(\frac{\sin(\Omega\tau_1)}{\sin(\sqrt{2}\Omega\tau_1)}c_0|1\rangle_{\mathrm{II}}\\ &+\frac{\cos(\Omega\tau_1)}{\cos(\sqrt{2}\Omega\tau_1)}c_1|2\rangle_{\mathrm{II}}\Bigg)+|2\rangle_{\mathrm{I}}(c_0|0\rangle_{\mathrm{II}}+c_1|1\rangle_{\mathrm{II}}+c_2|2\rangle_{\mathrm{II}})+|3\rangle_{\mathrm{I}}\Bigg(\frac{\sin(\sqrt{2}\Omega\tau_1)\sin(\sqrt{3}\Omega\tau_3)}{\sin(\Omega\tau_1)\sin(\sqrt{2}\Omega\tau_3)}c_1|0\rangle_{\mathrm{II}}\\ &+\frac{\sin(\sqrt{3}\Omega\tau_1)\cos(\sqrt{3}\Omega\tau_3)}{\sin(\sqrt{2}\Omega\tau_1)\cos(\sqrt{2}\Omega\tau_3)}c_2|1\rangle_{\mathrm{II}}\Bigg)+\frac{\sin(\sqrt{3}\Omega\tau_1)\sin(2\Omega\tau_3)}{\sin(\Omega\tau_1)\sin(\sqrt{2}\Omega\tau_3)}c_2|4\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}\Bigg],\end{aligned} \tag{18}$$

presenting the probability $|\mathcal{N}''\gamma_1\gamma_3|^2\sin^2(\Omega\tau_1)\sin^2(\sqrt{2}\Omega\tau_3)$ that the cavity mode $|\Phi\rangle_{\mathrm{I}}$ has been transferred from C_{I} to C_{II}.

To induce the collapse of the entangles state (18) and simultaneously indicate the measurement result, we now have to send two pointer atoms 1 and 2, initially prepared in the excited and ground states $|e\rangle_1^p$ and $|g\rangle_2^p$, successively across the Ramsey-type arrangement $R_5-C_{\mathrm{I}}-R_6$. The microwave zones and the detuning between pointer atom and cavity are first set in the same way as explained in Sec. II A, so that the pointer atom 1 plus cavity C_{I} end up in the superposition state (11). However, fields in R_5 and R_6 are switched off for the passage of pointer atom 2, which is selected in such a way that $\tau_2^p=(2m+1)\pi/2\Omega$ (with m with integer), and tuned to interact resonantly with C_{I}. Through these two pointer atoms we are able to know if the teleportation has succeeded, which demands that they have been detected successively in the states $|g\rangle_1^p$ and $|e\rangle_2^p$, causing the fields in C_{I} and C_{II} to collapse into their eigenstates $|2\rangle_{\mathrm{I}}$ and $c_0|0\rangle_{\mathrm{II}}+c_1|1\rangle_{\mathrm{II}}+c_2|2\rangle_{\mathrm{II}}$. After a little algebra we can also see that the interaction of these pointer atoms with cavity C_{I} changes the probability for the accomplishment of the teleportation process to $|\mathcal{N}''\gamma_1\gamma_3|^2\sin^2(\Omega\tau_1)\sin^2(\sqrt{2}\Omega\tau_3)\sin^2[(2m+1)\pi/\sqrt{2}]$.

So the probability that the cavity mode $|\Phi\rangle_{\mathrm{I}}$ has been transferred from C_{I} to C_{II} is maximized, with respect to m, when the interaction time of pointer atom 2 with C_{I} is fixed on $\tau_2^p = 5\pi/2\Omega$ ($m=2$), so that $\sin^2(5\pi/\sqrt{2}) \approx 1$. By considering $\tau_2^p = 5\pi/2\Omega$ and $\tau_1 = \tau_3 = \pi/2\Omega$, we obtain a probability around 0.25. Finally, given the interaction times specified above, both the relations (16) and (17) reduce to

$$\gamma_{2j} = \frac{\gamma_{2j-1}}{\cos(\sqrt{2}\pi/2)}, \quad j=1,2. \tag{19}$$

III. DUAL CLASSICAL AND QUANTUM CHANNELS

Following the ideas outlined by Bennett *et al.* [7], the teleportation process of a system having N orthogonal states is realized by coupling this system with a pair of N-state particles in a completely entangled state. The entanglement of the whole system is achieved through a joint measurement whose eigenstates satisfy Eq. (1). In the scheme presented above for the teleportation of the cavity mode $c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$, the quantum channel takes place through two independent pairs of two-level atoms prepared in particular nonmaximally entangled states. Thus the entanglement of the whole system—the cavity mode to be teleported plus the quantum channel—is accomplished step by step, each one realized by a pair of two-level atoms. Besides the pairs of two-levels atoms composing the quantum channel, the required pointer atoms contribute to the entangled state of the whole system as well.

There are four possibilities for the relations between the coefficients γ_{2j-1} and γ_{2j} of the entangled states in Eq. (4). Equations (16) and (17) represent one of these four possibilities. Returning to Sec. II C, if we consider that Alice has found atom 3 in the excited state (instead of the ground state as we have considered before), the relations (16) and (17) change, giving us the second possibility. Returning to the first step, if we consider that Alice has detected atom 1 in the excited state (instead of the ground state, as considered before), two other possibilities for the relations (16) and (17) take place, the third and fourth possibilities, depending on the state that Alice will find atom 3 in the second step $|g\rangle_3$ or $|e\rangle_3$.

In view of these four possibilities for the relations between the coefficients of the entangled states in Eq. (4), in this alternative scheme the classical channel has to be extended to the experimenter who is supposed to prepare these entangled states of two-level atoms. Alice, Bob, and Peter have to decide previously about which of the four possibilities to consider. Let us suppose that they have decided to prepare the entangled states $|\Psi\rangle_{12}$ and $|\Psi\rangle_{34}$ satisfying the relations (16) and (17). Thus, in the first step, by considering that Peter has already prepared the entangled state $|\Psi\rangle_{12}$, Alice and Bob necessarily have to find atoms 1 and 2 in their ground states. Once Alice and Bob have succeeded and reported their findings to each other and to Peter, they are ready for the second step. Otherwise, the first step must be repeated. In the second step, once Peter has prepared the entangled state $|\Psi\rangle_{34}$ and sent the atoms across their respective cavities, Bob necessarily has to find atom 4 in the ground state. Finally, the teleportation process of the cavity mode $|\Phi\rangle_{\mathrm{I}}$ will be achieved only if Alice finds both atom 4 in the ground state and the pointers atoms 1 and 2 in the states $|g\rangle_1^p$ and $|e\rangle_2^p$. Next, it is shown how to prepare the cavity mode $|\Phi\rangle_{\mathrm{I}}$ and the entangled states $|\Psi\rangle_{2j-1,2j}$ with their coefficients satisfying the relation (19) (assuming that $\tau_1 = \tau_3 = \pi/2\Omega$).

IV. PREPARATION OF THE REQUIRED SUPERPOSITIONS

In a recent work, Cirac and Zoller [13] have proposed a technique to prepare two or more atoms in certain entangled states based on the interaction of the atoms with a cavity mode. Here, the principle outlined in [13] is considered and extended by using a Ramsey-type arrangement, together with a pointer atom, which help us to prepare nonmaximally entangled states that coefficients satisfy the relation (18). However, for the preparation of such entangled states, the required cavity mode evolves to a superposition state which is entangled with atoms $2j-1$ and $2j$ and the pointer atom as well. So the decoherence process introduced by cavity dissipation must be negligible during and after the atom-field interactions, until the pointer atom has been detected. It will be shown that the detection of the pointer atom leaves the cavity and the atomic system in product states, so that there is no ‘‘projection noise’’ [13] when one traces over the unobserved cavity field.

A. Entangled state $|\Psi\rangle_{2j-1,2j}$

Atom $2j-1$, initially prepared in the state $|e\rangle_{2j-1}$, is sent across cavity C, initially in the vacuum state. Subsequently, atom $2j$, initially prepared in the state $|e\rangle_{2j}$, is sent across the same cavity, as shown in Fig. 1. Supposing that the atomic velocities have been selected so that $\tau_{2j-1} = \tau_{2j}/2 = \pi/4\Omega$, after the interactions the state of the system composed of both atoms $2j-1$ and $2j$ plus cavity C reads

$$\begin{aligned}|\Psi\rangle_{2j-1,2j,C} = \frac{-i}{\sqrt{2}}\{&[|e\rangle_{2j-1}|g\rangle_{2j} \\ &+ \cos(\sqrt{2}\pi/2)|g\rangle_{2j-1}|e\rangle_{2j}]|1\rangle \\ &- i\sin(\sqrt{2}\pi/2)|g\rangle_{2j-1}|g\rangle_{2j}|2\rangle\}.\end{aligned} \tag{20}$$

Cavity C is thus left in a superposition of one- and two-photon field states. Now a pointer atom, initially prepared in the excited state $|e\rangle^p$, has to be sent across the Ramsey-type arrangement $R_1 - C - R_2$ (see Fig. 1) to induce, through its measurement, the collapse of the superposition (20). The cavity detuning and the auxiliary microwave zones are set so that the pointer atom plus cavity C ends up in the superposition state (11), resulting from Eq. (20),

$$\begin{aligned}|e\rangle^p|\Psi\rangle_{2j-1,2j,C} \to \frac{i}{\sqrt{2}}\{&[|e\rangle_{2j-1}|g\rangle_{2j} \\ &+ \cos(\sqrt{2}\pi/2)|g\rangle_{2j-1}|e\rangle_{2j}]|e\rangle^p|1\rangle \\ &- i\sin(\sqrt{2}\pi/2)|g\rangle_{2j-1}|g\rangle_{2j}|g\rangle^p|2\rangle\}.\end{aligned} \tag{21}$$

Therefore, the detection (trough D'') of the pointer atom in the state $|e\rangle^p$ (presenting the probability $\frac{1}{2}[1+\cos^2(\sqrt{2}\pi/2)]$), leaves cavity C and atoms $2j-1$ and $2j$ in a product state, so that the damping mechanism of the cavity does not affect the generated atomic entangled state

$$|\Psi\rangle_{2j,2j-1}=\gamma_{2j}|e\rangle_{2j-1}|g\rangle_{2j}+\gamma_{2j-1}|g\rangle_{2j-1}|e\rangle_{2j}, \quad (22)$$

with γ_{2j-1} and γ_{2j} satisfying the relation (19). To obtain the state $|\Psi\rangle_{2j-1,2j}$ designated by Eq. (4), we finally must apply a 180° rotation around the x axis to both atoms $2j-1$ and $2j$, corresponding to changes γ_{2j-1} into γ_{2j} and vice versa. These rotations are performed by means of appropriated microwave pulses applied in zones R_3 and R_4, as indicated in Fig. 1.

Finally, after the preparation of the first entangled state ($j=1$), another reference atom is sent through the same arrangement R_1-C-R_2. Fields in R_1 and R_2 are now switched off and the atom, selected in such a way as to present the interaction time $\pi/2\Omega$, is tuned to interact resonantly with C. As a consequence, cavity C is left in its initial vacuum state, so that it is ready to prepare the next entangled state ($j=2$).

It is worth noting that the velocity of atom $2j-1$ across cavity C has been selected in such a way that $\tau_{2j-1}=\pi/4\Omega$. However, we have considered above, in Sec. II, that the interaction time of atom $2j-1$ with cavity C_{I} is given by $\tau_{2j-1}=\pi/2\Omega$. So the length of cavity C_{I} must be two times the length of cavities C and C_{II}, that is, C_{I} must contain two times the number of wavelength inside C and C_{II}. That is why in Fig. 1 we have labeled cavity C_{I} by $2L$ and the other cavities by L. In the Appendix it is shown how to prepare the required atomic entangled states when atoms 3 and 4 are expected to be detected in the excited and ground states, respectively (instead of being detected in their ground states, as considered in Sec. II).

B. Cavity modes

To prepare the cavity mode in Eq. (3) we introduce here a method that is a particularization of the ‘‘quantum state engineering of the radiation field’’ presented by Vogel *et al.* [17]. Here, the cavity mode (3) is prepared by sequentially by sending two two-level pointer atoms, initially prepared in the superpositions $c_g^j|g\rangle_j^p+c_e^j|e\rangle_j^p$ ($j=1,2$), across a cavity initially in the vacuum state. The atom-field interaction time is considered to be $\tau_j=\pi/2\Omega$. After crossing the cavity, the first atom is left in the ground state, while the cavity is left in the pure field state $c_g^1|0\rangle_1-ic_e^1|1\rangle_1$. If we had prepared atom $j=1$ in the excited state $|e\rangle_1$, the cavity would have been left in the one-photon field state (as required by cavity C', in the Appendix). After the interaction with a second pointer atom the cavity is left in the atom-field entangled state

$$|g\rangle_1^p\{|g\rangle_2^p[c_g^1c_g^2|0\rangle-ic_g^1c_e^2|1\rangle-c_e^1c_e^2\sin(\sqrt{2}\pi/2)|2\rangle]$$
$$-|e\rangle_2^p[c_e^1c_g^2|0\rangle+ic_e^1c_e^2\cos(\sqrt{2}\pi/2)|1\rangle]\} \quad (23)$$

and the detection of pointer atom 2 in the ground state leaves the cavity in the field state superposition $|\Phi\rangle=c_0|0\rangle+c_1|1\rangle+c_2|2\rangle$, with $c_0=c_g^1c_g^2$, $c_1=-ic_g^1c_e^2$, and $c_2=-c_e^1c_e^2\sin(\sqrt{2}\pi/2)$. The cavity mode $|\Phi\rangle$ is unknown to Alice, Bob and Peter, as well as the coefficients of the atomic state c_g^j and c_e^j.

Next, we discuss the teleportation of the cavity mode $\Sigma_{n=0}^N c_n|n\rangle$, with $N>2$. To prepare such a state N two-level pointer atoms, initially prepared in the superpositions $c_g^j|g\rangle_j^p+c_e^j|e\rangle_j^p$ ($j=1,2,\ldots,N$), must be sequentially sent across an initially empty cavity. By sequentially detecting these N two-level atoms in their ground states, after crossing the cavity, we obtain the required cavity radiation field state.

V. CAVITY MODES HAVING MORE THAN THREE FOCK STATES

In principle, the alternative scheme here presented, as well as the scheme of Bennett *et al.*, can be used to realize the teleportation of the cavity mode $\Sigma_{n=0}^N c_n|n\rangle$, with $N>2$. Now, N pairs of two-level atoms prepared in nonmaximally entangled states [Eq. (4)] have to be considered. The first pair $j=1$ is able to teleport the superposition $c_0|0\rangle+c_1|1\rangle$. Each of the subsequent pairs $j=2,3,\ldots,N$ is able to complete the superposition to be teleported by adding the remained Fock state $|j\rangle$, accompanied by its respective probability amplitude c_j. In other words, each of the subsequent pairs j is able to ‘‘switch on’’ its respective element in the sum $c_0|0\rangle+c_1|1\rangle+\Sigma_{j=2}^N c_j|j\rangle$, which represents the teleported superposition. In this way, the whole cavity mode is supposed to be teleported from one cavity to another through N steps, each one accomplished by a pair of two-level atoms in a particular nonmaximally entangled state.

We have obtained numerical values for the probability to achieve the teleportation process of particular cavity modes. These values correspond to particular choices of the atom-field interaction times and do not take the preparation of the atomic entangled states and the cavity modes into account. By considering that atoms 3 and 4 have been detected in their ground states (Sec. II), the probabilities for the accomplishment of the teleportation of cavity modes with $N=1$ and $N=2$ are around 0.5 and 0.25, respectively (the velocities of atoms $2j-1$ and $2j$ across C_{I} and C_{II}, respectively, have been fixed at $\tau_{2j-1}=\tau_{2j}=\pi/2\Omega$). In the Appendix, by considering that atoms 3 and 4 have been detected in the states $|e\rangle_3$ and $|g\rangle_4$, we have obtained a probability around 0.45 for the accomplishment of the teleportation of a cavity mode with $N=2$ (with $\tau_{2j-1}=\tau_{2j}/2=\pi/4\Omega$). In fact, it is expected that the probability to realize the teleportation of the cavity mode $\Sigma_{n=0}^N c_n|n\rangle$ becomes smaller as N increases. As we have seen, the larger N is, the larger the number of pairs of two-level atoms in a nonmaximally entangled state necessary to realize the teleportation process. Furthermore, there is just a finite probability for the successfully achievement of each step realized by each pair of two-level atoms. On account of the oscillatory functions coming from the Jaynes-Cummings interaction, it is also expected that a general scaling law (regarding N) for the probability to achieve the teleportation process results from the average of such oscillatory functions with respect to the interaction parameter $\Omega\tau$. The knowledge of such a general scaling law for the present scheme might be useful to determine to what extent one can teleport a given state. Furthermore, the dissipative processes due to cavity losses and atomic spontaneous emission, discussed below, represent additional problems to overcome.

VI. COMMENTS AND CONCLUSIONS

We have presented an alternative scheme to realize the teleportation of a cavity mode in a coherent superposition of zero-, one-, and two-photon field states. Basically, the nonlocal channel established by a pair of three-state particles in a completely entangled state, as required in the original teleportation scheme outlined by Bennett *et al.* [7], is substituted by two pairs of two-level atoms, each pair in a nonmaximally entangled state. The teleportation process is thus realized step by step, each step being accomplished through a nonmaximally entangled state of two-level atoms. In this way, the entanglement of the whole system, the system whose state is to be teleported and the quantum channel, established through a joint measurement in Ref. [7], is now accomplished through the two required steps. The atom-field interaction has been described by the resonant Jaynes-Cummings Hamiltonian and pointer atoms have been considered both to induce the state-vector collapse of the field in the cavity system and to indicate the measurement result. Since there is a probability associated with the achievement of each of the two steps, the pointer atoms turn out to be necessary to read out the cavities every single step.

The process of preparation of macroscopic superpositions in many-atoms systems, outlined by Cirac and Zoller [13], has been employed here for the preparation of the atomic entangled states. However, we have extended such a process by using a Ramsey-type arrangement, together with pointer atoms, which help us to prepare particular nonmaximally entangled states. It is important to stress that for the teleportation of a cavity mode having more than three Fock states, the process of preparing the required atomic entangled states becomes more complicated, demanding the use of additional cavities.

Certainly, there are some sensitive conditions required for successful implementation of the present scheme. The correct adjustment of the interaction parameter $\Omega\tau$ seems to be possible by means of controlled atomic velocities, which require velocity-selective chopping of the atomic beams before crossing the cavities. As pointed out by Cirac *et al.* [12,13], current experiments in the microwave regime [16,11] can reach the parameters necessary ($\Omega \approx 2\times 10^5\ \mathrm{s}^{-1}$ and atomic velocities $\approx 10^3$ m/s), in order for the dispersion in the atomic velocities not to depart the atom-field entangled state appreciably from the expected one.

The efficiency of the atomic detection is also a sensitive point. For the implementation of the scheme proposed in Ref. [7], Davidovich *et al.* [9] have examined the requirements on detection efficiency imposed by the quantum mechanical nonlocality constraint $\underline{I}>2/3$ [18], $I\ (=\langle\phi|\rho|\phi\rangle)$ being the teleportation fidelity coefficient and $\underline{I}$ its average over all possible states $|\phi\rangle$ of the particle to be teleported. The above constraint is satisfied if the detection efficiencies are over 0.7, actually a good enough condition as compared to other tests of quantum nonlocality. For the present step-by-step teleportation process the lower bounds both in the average fidelity and in the detection efficiency must be reexamined. The required number of detections of two-level atoms as well as the assimilation of the dispersion in the atomic velocities will certainly demand an improvement on the detection efficiency.

As mentioned above, a negligible cavity loss is required in the preparation of the non-maximally entangled state during and after the atom-field interactions, until the detection of the pointer atoms, which causes the state vector of the atom-cavity system to collapse, with a certain probability, in a desired product state. A negligible cavity loss is also required during the whole process of nonlocal correlation between cavities C_{I} and C_{II}. Presently, the cavity lifetime for high-Q superconducting cavities is approximately 10^{-2} s [10], which is three orders of magnitude longer than typical atom-field interaction. However, the whole teleportation process must be accomplished around the cavity lifetime period, which is clearly the main problem to overcome in the present scheme.

Finally, concerning to the spontaneous decay, for Rydberg atoms in circular states with principal quantum number $n\approx 50$ and maximum angular momentum $l=n-1$, atomic excited-state lifetimes are also of the order of 10^{-2} s [10]. So, assuming atomic velocities of approximately 10^3 m/s, the cavity system sketched in Fig. 1 must be confined to an area about 1 m^2 in order for the whole operation required for the accomplishment of each step to be realized around half the radiative lifetimes of circular Rydberg atoms.

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APPENDIX

Returning to Sec. II, let us consider that atoms 3 and 4 have been detected in the excited and ground states, respectively. One can show that, for the accomplishment of the teleportation process, the new coefficients γ_1 and γ_2 must satisfy exactly the relation (16), while the coefficients γ_3 and γ_4 must satisfy the relation (17) with $\tan(\sqrt{2}\Omega\tau_3)$ changed into $-\cot(\sqrt{2}\Omega\tau_3)$. So, by selecting the velocities of atoms 1 and 3 across cavity C_{I} in a way that $\tau_1=\tau_3=\pi/4\Omega$, we obtain $\gamma_2=\gamma_1/[2\cos^2(\sqrt{2}\pi/4)]$ and $\gamma_4=-\gamma_3\cos(\sqrt{2}\pi/4)/[\sqrt{2}\sin^2(\sqrt{2}\pi/4)]$. To prepare the atomic entangled state satisfying the relation between γ_1 and γ_2, similarly to what we have done previously (Sec. IV), we must send atom 1, initially prepared in the excited states, across cavity C, initially in the vacuum state. Subsequently, atom 2, initially prepared in the excited state, is sent across the same cavity. However, both atoms now have to be selected to present the same atom-field interaction time $\tau=\pi/4\Omega$. After the interaction of atom 2 with C, it has to be sent across an additional cavity, say C', initially prepared in the one-photon field state and placed between C and C_{II}. The interaction time between atom 2 and C' also has to be $\tau=\pi/4\Omega$. After this, two pointer atoms have to be considered, 1 and 2, together with an additional Ramsey-type arrangement $R_1'-C'-R_2'$, where R_j' $(j=1,2)$ represents auxiliary microwave zones that are sandwiching C'. Finally, the pointer atoms 1 and 2 are sent across the Ramsey-type arrangements $R_1 - C - R_2$ and $R_1' - C' - R_2'$, respectively. In both arrangements the microwave zones and the detuning between pointer atom and cavity are set so that the system ends up in the superposition state (11). By detecting both pointer atoms (and here an additional state-selective ionization detector has to be considered together with the arrangement $R_1{}' - C' - R_2'$) in their excited states, atoms 1 and 2 are left in the entangled state (22) ($j=1$), with γ_1 and γ_2 satisfying the relation $\gamma_2 = \gamma_1/[2\cos^2(\sqrt{2}\pi/4)]$. By applying a 180° rotation around the x axis to both atoms (through zones R_3 and R_4), we finally obtain the entangled state $|\Psi\rangle_{12}$, satisfying the required relation between γ_3 and γ_4.

To obtain the atomic entangled state obeying the relation between γ_3 and γ_4, atom 3, initially prepared in the excited state, is sent across C, initially prepared in the one-photon field state. Subsequently, atom 4, initially prepared in the ground state, is also sent across C. Both atoms have to be selected to present the same atom-field interaction time $\tau = \pi/4\Omega$. After the interaction of atoms 3 and 4 with C, a pointer atom has to be send across the Ramsey-type arrangement $R_1 - C - R_2$, with the auxiliary microwave zones and the detuning between pointer atom and cavity set so that the system ends up in the superposition state (11). By detecting this pointer atom in its excited state, atoms 3 and 4 are left in the entangled state (22) ($j=2$), with γ_3 and γ_4 satisfying the relation (19). Finally, we have to apply a 180° rotation around the x axis to both atoms to obtain the entangled state $|\Psi\rangle_{34}$, satisfying the required relation between γ_3 and γ_4.

It is worth noting that the velocity of atom 2 across cavities C and C' and the velocity of atom 4 across C have been selected so that the interaction times between the atoms and the cavity modes are $\tau = \pi/4\Omega$. However, as considered in Sec. II, the interaction times of atoms 2 and 4 with cavity C_{II} are given by $\tau = \pi/2\Omega$. So, as opposed to the situation discussed in Sec. IV, here cavity C_{II} is the one whose length must be two times the length of cavities C and C_{I}. Finally, by considering that atoms 3 and 4 have been detected in their excited and ground states, respectively, the probability to achieve the teleportation of the cavity mode $|\Phi\rangle_{\mathrm{I}}$ from C_{I} to C_{II} is around 0.45 (by choosing $\tau_3 = \tau_4 = \pi/4\Omega$).

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