30 June 1997

PHYSICS LETTERS A

ELSEVIER

Physics Letters A 231 (1997) 23-28

Preparation of the Bell operator basis through a restricted multiplying device

M.H.Y. Moussa [1]

Departamento de Física, Universidade Federal de São Carlos, Via Washington Luis, km 235, São Carlos 13565-905, SP, Brazil

Received 2 December 1996; revised manuscript received 8 April 1997; accepted for publication 9 April 1997
Communicated by P.R. Holland

Abstract

We present a restricted multiplying device, realizable with nowadays technology, that is able to clone quantum states which are not superposition states. Through such a device we show how to prepare all the n-particle Bell operator basis vectors (BOBV). © 1997 Elsevier Science B.V.

PACS: 03.65.Bz; 32.80.-t; 42.50.Dv

Since the past decade it is well known that quantum-mechanical states cannot be cloned [1,2]. The linearity of the quantum-mechanical evolution operator prevents such a multiplying device, i.e., a quantum copying machine, from working. Such a device was first proposed by Herbert [3] to achieve faster-than-light communication by associating it with an EPR-type experimental setup. However, despite the "no-cloning theorem", the linearity of quantum mechanics does not forbid the replication of quantum states which are not superposition states [1]. In this paper we present a restricted multiplying device (RMD) wich is able to clone one or the other state of a two-level atom (levels $|g\rangle$ and $|e\rangle$).

The RMD, based on cavity QED techniques and realizable with current technology, permits that coherent atom-field interactions dominate over dissipative processes due to cavity losses and atomic spontaneous emission [4]. The two-level atoms are considered as circular Rydberg atoms wich are suitable for preparing and detecting atom-field long-lived correlations [5]. They present a strong coupling to microwaves and a very long radiative decay time of the order of cavity lifetimes for high-Q superconducting cavities. Moreover, Rydberg atomic states, which can be prepared at a given time with a well defined velocity, present a near-unity detection efficiency when counted by state selective field ionization detection [4]. Such properties make the entanglemets between circular Rydberg atoms suitable for improved tests that challenge local realistic theories [6], and to examine related phenomena such as teleportation [7] and quantum computation [8].

To prepare atomic entangled states which could be used for testing Bell's inequalities, Cirac and Zoller [6] have proposed a technique based on the interaction of an atom with a cavity mode. In particular, the authors show how to prepare a pair of two-level atoms in the singlet state, which was considered in the original derivation of Bell's theorem [9]. They also show

[1] E-mail: miled@power.ufscar.br.

PII S0375-9601(97)00284-3

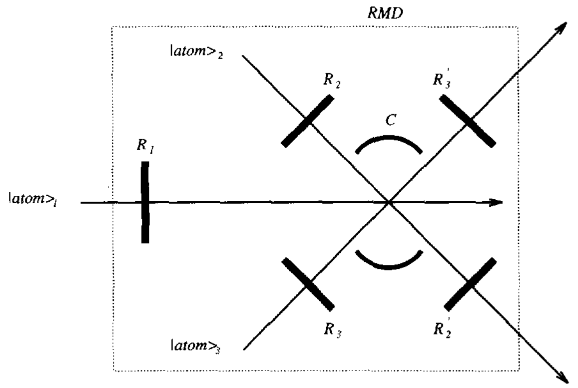

Fig. 1. Sketch of the restricted multiplying device.

how to prepare the *three*-particle states proposed by GHZ [10]. Finally, Cirac and Zoller have generalized their scheme for the preparation of macroscopic superposition in many-atom systems. Here it will be shown that the proposed RMD can be used to prepare all the possible BOBV [11].

The setup of the RMD, sketched in Fig. 1, consists of a cavity C, Ramsey zones R, and a set of identical two-level atoms. The cavity field can be described through the basis of zero- and one-photon states ($|0\rangle$ and $|1\rangle$, respectively), since there is never more than one photon in the cavity. Two different kinds of atom-field interactions are used in our scheme: "on-ressonant" and "off-ressonant" interactions. The former is used to transfer quantum states between atom and cavity, while the latter is used to produce conditional phase shifts in the atomic states, controlled by the photon number of the cavity field. These two types of interactions can be produced from a single atomic species by appropriate Stark shifting of the atomic levels, which is achieved by applying timed sequences of pulsed electric field on the cavity mirrors [7,8].

The Ramsey zones consist in microwave fields that play the role of atomic state "polarizers" and "analyzers", and are mainly used in the present scheme to set up the Ramsey-type arrangements (RTA). A RTA consists of two Ramsey zones sandwiching a cavity where an off-resonant interaction is supposed to occur. Basically, as shown in Fig. 1, the RMD here proposed consists of two RTA's sharing the same cavity. When passing across a RTA, an atom undergoes a transformation such that the probability for an $|e\rangle \to |g\rangle$ transition is characteristic of the photon number in the cavity. Such a transition probability depends also on a given setting of the Ramsey zones besides depending on the off-resonant atom-cavity interaction. For the present purpose we assume that the microwave zones are set so that an atom undergoes exactly a $\pi/2$ pulse, on the $|e\rangle \to |g\rangle$ transtion, in each zone. The cavity detuning is set so that the atom undergoes a phase shift per photon exactly equal to π. In this way, as obtained in Ref. [12], the $|e\rangle \to |g\rangle$ transfer probability is one when the cavity field is $|0\rangle$, and therefore, zero when the cavity field is $|1\rangle$. Due to the off-resonant atom-cavity interaction the photon number in the cavity remains unchanged and the atom-cavity system undergoes the transformations:

$$|g\rangle \begin{cases} |0\rangle \to |e\rangle|0\rangle \\ |1\rangle \to |g\rangle|1\rangle \end{cases}, \qquad |e\rangle \begin{cases} |0\rangle \to -|g\rangle|0\rangle \\ |1\rangle \to -|e\rangle|1\rangle \end{cases}. \tag{1}$$

In the following we outline the two steps involved in the operation of the RMD.

Step 1. Preparation of the cavity mode

Atom 1, initially prepared in the ground *or* excited state, which is supposed to be cloned, is first sent accross a Ramsey zone R_1, which produces a $\pi/2$ rotation about the x axis in the atomic space. After such a rotation, described by the operator $\sigma_x = |e\rangle\langle g|+|g\rangle\langle e|$, atom 1 is sent accross the initially empty cavity C. This atom is tuned to have an on-resonance interaction with C, undergoing a π pulse and leaving the cavity in the one (zero)-photon field state if it was initially prepared in the ground (excited) state. Step 1 can be summarized by the transformation:

$$\begin{pmatrix} |g\rangle_1 \\ |e\rangle_1 \end{pmatrix} \xrightarrow{R_1} \begin{pmatrix} |e\rangle_1 \\ |g\rangle_1 \end{pmatrix}, \qquad \begin{pmatrix} |e\rangle_1 \\ |g\rangle_1 \end{pmatrix} |0\rangle \xrightarrow{C} |g\rangle_1 \begin{pmatrix} -|1\rangle \\ |0\rangle \end{pmatrix}, \tag{2}$$

where the subscript 1 labels atom 1.

Step 2. Replication

Once cavity C has been prepared, atoms 2 and 3, initially in their ground states, are sent accross the RTA's $R_2 - C - R_2'$ and $R_3 - C - R_3'$, respectively.

According to transformation (1), depending on the state of cavity C ($|0\rangle$ or $|1\rangle$, which is related to the initial state of atom 1, $|e\rangle$ or $|g\rangle$, respectively), atoms 2 and 3 undergo the transformation:

$$|g\rangle_2|g\rangle_3 \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} \overset{\text{RTA's}}{\rightarrow} \begin{cases} |1\rangle|e\rangle_2|e\rangle_3 \\ |0\rangle|g\rangle_2|g\rangle_3 \end{cases}. \tag{3}$$

Therefore, the final states of atoms 2 and 3 constitute clones of the initial state of atom 1. By considering the notation of Wooters and Zurek [1], we can summarize the operation of the RMD through the transformations

$$|g\rangle_1|0\rangle \overset{\text{RMD}}{\rightarrow} |1\rangle|g\rangle_2|g\rangle_3,$$
$$|e\rangle_1|0\rangle \overset{\text{RMD}}{\rightarrow} |0\rangle|e\rangle_2|e\rangle_3, \tag{4}$$

where on the left-hand side of Eq. (4) we have the atomic state to be cloned (atom 1) and the initial state of the RMD, basically represented by the field state in cavity C. On the right-hand side of Eq. (4) we have the final state of the RMD and the cloned states carried by atoms 2 and 3. Evidently, we can even produce n copies of the state of atom 1 by sending n atoms, initially prepared in their ground state, accross cavity C.

Preparation of the BOBV

Two-particle BOBV. Referring to the RMD in Fig. 1, let us consider that atom 1 is prepared in the excited state $|e\rangle_1$. The Ramsey zone R_1 is now adjusted so that a $\pi/2$ pulse is applied to the atom which leaves R_1 in the superposition $(|e\rangle_1 + |g\rangle_1)/\sqrt{2}$. The atom is thus tuned to interact resonantly with C, undergoing a π pulse and leaving the cavity in the coherent superposition $(|0\rangle - |1\rangle)/\sqrt{2}$, while it is left in the ground state $|g\rangle_1$. Next, atom 2, initially prepared in the ground state $|g\rangle_2$, is sent across the RTA $R_2 - C - R_2'$, so that the "atom 2+C" system evolves to the entangled state $(|e\rangle_2|0\rangle - |g\rangle_2|1\rangle)/\sqrt{2}$. Atom 3, initially prepared in the ground state $|g\rangle_3$, is thus sent accross cavity C. The field in R_3 is now switched off and atom 3 is tuned to resonance with C, undergoing a π pulse. After interacting with C, atom 3 goes accross the Ramsey zone R_3', and the system composed by atoms 2 and 3 ends up in the entanglement

$$|\Psi^{(\pm)}\rangle_{23} = (1/\sqrt{2})(|e\rangle_2|g\rangle_3 \pm |g\rangle_2|e\rangle_3), \tag{5a}$$

where the state $|\Psi^{(-)}\rangle$ ($|\Psi^{(+)}\rangle$, except for an irrelevant phase factor) is obtained when the field in R_3' is switched off (set to produce a $\pi/2$ rotation about the z axis, described by the operator $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, on atom 3).

Let us consider that atom 2 is now initially prepared in the excited state $|e\rangle_2$. Thus, after the preparation of the cavity mode $(|0\rangle - |1\rangle)/\sqrt{2}$, atom 2 is sent accross the RTA $R_2 - C - R_2'$, and the "atom 2 + C" system evolves to the entangled state $(|e\rangle_2|1\rangle - |g\rangle_2|0\rangle)/\sqrt{2}$. Atom 3 is again prepared in the ground state and sent accross C. The field in R_3' is switched off and the atom is tuned to interact resonantly with C, undergoing a π pulse. Finally, after interacting with C and R_3', atoms 2 and 3 end up in the entanglement

$$|\Phi^{(\pm)}\rangle_{23} = (1/\sqrt{2})(|e\rangle_2|e\rangle_3 \pm |g\rangle_2|g\rangle_3), \tag{5b}$$

where the signals $-$ or $+$ correspond to R_3' switched off or producing the above-mentioned $\pi/2$ rotation about the z axis, respectively. The entangled states $|\Psi^{(\pm)}\rangle$ and $|\Phi^{(\pm)}\rangle$, here prepared through a single cavity, constitute the *two*-particle BOBV which has received considerable attention with the recently introduced teleportation process [13].

Note that after preparation of the entanglements between atoms 2 and 3, the cavity mode setting up the RMD is left in its original state, the vacuum, which factorizes. So, there is no "projection noise" when one traces over the unobserved cavity field [6]. Cavity damping mechanisms before and after the interaction will not affect the feasibility of the *two*-particle BOBV. The dissipation mechanism of the cavity must be negligible only during the time the entanglement is prepared. On the other hand it have to be noted that in order to adjust the instantaneous microwave field phase one needs precise information on the arrival time for any individual atom entering the Ramsey zones, which will be hard to obtain in an actual experiment. Actually, in the interaction of atom 2 with the experimental setup, two phases (namely that of the cavity field produced by atom 1 and that related to the above-mentioned arrival time) must be carefully controlled to justify the result (5a) with the special values of the phase factor, $+1$ or -1. The situation for the state (5b) is still more serious, since they lack stationarity in the sense that the relative phase between the two terms in the respective sum oscillates strongly with time [12].

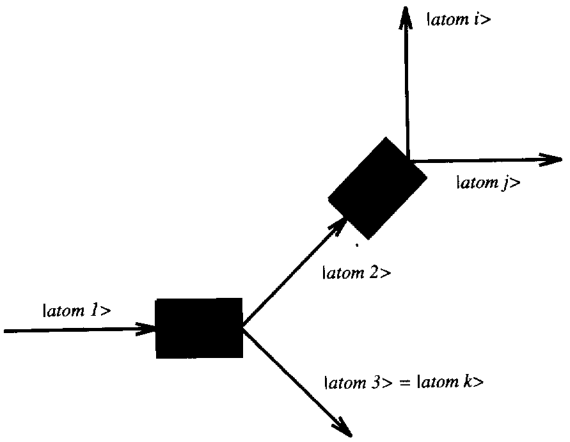

Fig. 2. Sketch of the arrangement used for the preparation of the three-particle BOBV.

These problems have to be taken into account in the preparation of the three-, four-, and n-particle BOBV.

Three-, four- and n-particle BOBV.

From the BOBV consisting of $|\Psi^{(\pm)}\rangle$ and $|\Phi^{(\pm)}\rangle$, and with the help of an additional RMD (an additional cavity), we show in the following how to generate the *three*-particle BOBV

$$|\Psi_1^{(\pm)}\rangle_{123} = \frac{1}{\sqrt{2}}(|e\rangle_1|e\rangle_2|e\rangle_3 \pm |g\rangle_1|g\rangle_2|g\rangle_3), \quad (6a)$$

$$|\Psi_2^{(\pm)}\rangle_{123} = \frac{1}{\sqrt{2}}(|e\rangle_1|e\rangle_2|g\rangle_3 \pm |g\rangle_1|g\rangle_2|e\rangle_3), \quad (6b)$$

$$|\Psi_3^{(\pm)}\rangle_{123} = \frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_2|e\rangle_3 \pm |g\rangle_1|e\rangle_2|g\rangle_3), \quad (6c)$$

$$|\Psi_4^{(\pm)}\rangle_{123} = \frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_2|g\rangle_3 \pm |g\rangle_1|e\rangle_2|e\rangle_3). \quad (6d)$$

The state $|\Psi_1^{(\pm)}\rangle_{123}$ is the one considered in GHZ's proof.

By considering first the states $|\Phi^{(\pm)}\rangle_{23}$ and sending either atom 2 or 3 accross another RMD (RMD_2 in Fig. 2), we obtain the three-particle BOBV $|\Psi_1^{(\pm)}\rangle_{ijk}$ (Eq. (6a)), where the subscripts i, j, k label the outcome states as shown in Fig. 2. To prepare the remaining three-particle BOBV (Eqs. (6b,c,d)), we have now to use the states $|\Psi^{(\pm)}\rangle_{23}$. Similarly to the preparation of $|\Psi_1^{(\pm)}\rangle_{ijk}$, either atom 2 or 3 is sent accross the RMD_2, sketched in Fig. 2, so that the resulting entangled state (except for an irrelevant phase factor when sending atom 3), reads

$$|\Psi^{(\pm)}\rangle_{ijk} = \frac{1}{\sqrt{2}}(|e\rangle_i|e\rangle_j|g\rangle_k \pm |g\rangle_i|g\rangle_j|e\rangle_k). \quad (7)$$

Therefore, through an appropriate label of the outcomes i, j, k, each of the BOBV in Eqs. (6b,c,d) can be generated. The choice $i = 1$, $j = 2$, $k = 3$ gives us the Bell states $|\Psi_2^{(\pm)}\rangle_{123}$, while the choice $i = 1$, $j = 3$, $k = 2$ gives us $|\Psi_3^{(\pm)}\rangle_{123}$, and finally, $i = 2$, $j = 3$, $k = 1$ leads to $|\Psi_4^{(\pm)}\rangle_{123}$.

As in Eqs. (5a,b), in Eq. (7) we have omitted the cavity mode, the vacuum, which factorizes together with the ground state of the atom (2 or 3) which was sent accross the RMD_2.

The four-particle BOBV is similarly generated just from the states $|\Psi_1^{(\pm)}\rangle_{123}$ and $|\Psi^{(\pm)}\rangle_{ijk}$ (one of the remaining three-particle BOBV in Eqs. (6b,c,d)). Now a third RMD has to be considered as displayed in Fig. 3 (indicated by RMD_3).

By considering the state $|\Psi_1^{(\pm)}\rangle_{123}$ and sending the atom leaving the RMD_2, whatever it may be, through the RMD_3 (Fig. 3a), or even sending the atom leaving the RMD_1 through the RMD_3, we get the four-particle BOBV

$$|\Psi^{(\pm)}\rangle_{lmno} = (1/\sqrt{2})(|e\rangle_l|e\rangle_m|e\rangle_n|e\rangle_o \pm|g\rangle_l|g\rangle_m|g\rangle_n|g\rangle_o), \quad (8a)$$

with l, m, n, o labeling the outcome states. However, when we consider the states $|\Psi^{(\pm)}\rangle_{ijk}$, the arrangement in Fig. 3a gives us the BOBV

$$|\Psi^{(\pm)}\rangle_{lmno} = (1/\sqrt{2})(|e\rangle_l|e\rangle_m|e\rangle_n|g\rangle_o \pm|g\rangle_l|g\rangle_m|g\rangle_n|e\rangle_o), \quad (8b)$$

while the arrangement in Fig. 3b leads to the Bell states

$$|\Psi^{(\pm)}\rangle_{lmno} = (1/\sqrt{2})(|e\rangle_l|e\rangle_m|g\rangle_n|g\rangle_o \pm|g\rangle_l|g\rangle_m|e\rangle_n|e\rangle_o). \quad (8c)$$

From the states in Eq. (8b) we are able to prepare, by labeling appropriately the outcomes l, m, n, o, eight of the four-particle BOBV. From the states in Eq. (8c) we get another six vectors that, added to the two vectors in Eq. (8a), perform the set of sixteen vectors composing the four-particle BOBV.

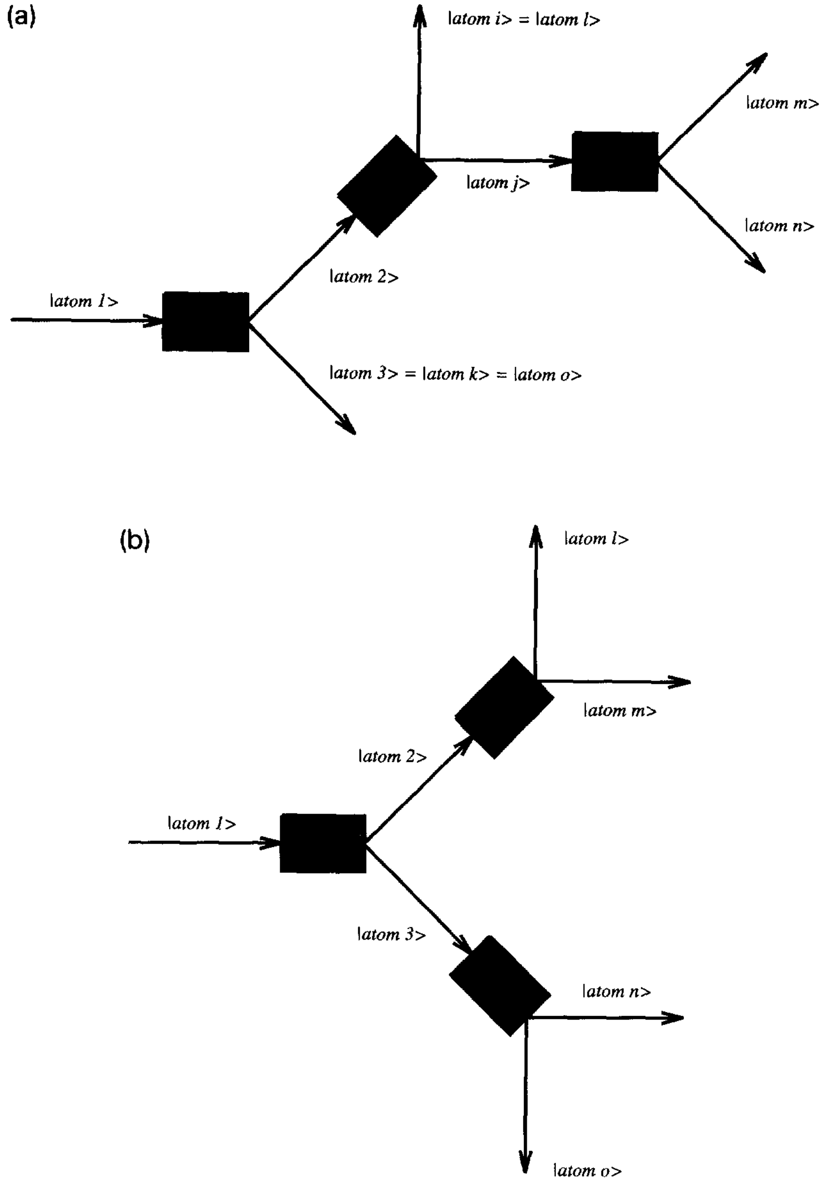

Fig. 3. Sketch of the arrangements used to generate the four-particle BOBV.

In summary, concerning the preparation of an n-particle BOBV, when atom 2 is initially prepared in the excited state, after carrying the above described process through the now required $n-1$ cavities, we automatically generate the n-particle entangled states

$$|\Psi^{(\pm)}\rangle_{1,2,\ldots,n} = (1/\sqrt{2})(|e\rangle_1|e\rangle_2\cdots|e\rangle_n \pm|g\rangle_1|g\rangle_2\cdots|g\rangle_n). \tag{9a}$$

On the other hand, if atom 2 is initially prepared in the ground state, we automatically generate, depending on the arrangements of the $n-1$ RMD's, the entangled states

$$|\Psi^{(\pm)}\rangle_{1,\ldots,r,\ldots,n} = (1/\sqrt{2})(|e\rangle_l\cdots|e\rangle_r|g\rangle_{r+1}\cdots|g\rangle_n \pm|g\rangle_l\cdots|g\rangle_r|e\rangle_{r+1}\cdots|e\rangle_n), \tag{9b}$$

where $r = n - s$, with $s \in [1, \mathrm{int}[(n+1)/2]]$ the total number of possible arrangement from which we obtain all the n-particle BOBV.

It worth noting that the preparation of the states (8a and b) can be achieved without considering a third RMD. By considering the states $|\Phi^{(\pm)}\rangle_{23}$ and sending an additional atom, together with either atoms 2 or 3, accross the RMD_2, we obtain the states (8a). If considering the states $|\Psi^{(\pm)}\rangle_{23}$, the utilization of the additional atom, which is sent accross the RMD_2, gives us the states (8b). However, for the preparation of the states (8c) we have necessarilly to consider the RMD_3. Such an argument holds also for the preparation of certain n-particle BOBV.

The preparation of macroscopic superpositions in many-atoms systems is mainly limited by the dissipative mechanism in both the two-level atoms and in the cavity mode during the preparation procedure. The cavity lifetimes for high-Q superconducting cavities, as well as the radiative decay times of Rydberg atoms in circular states $l = n - 1$, are of the order of 10^{-2} s [4]. The typical atom-cavity interaction times in present experiments are of the order of 10^{-5} s [4], three orders of magnitude smaller than the abovementioned decoherence times. Finally, we note that the presented scheme requires controll over atomic velocities just for the preparation of the two-particle BOBV. For the preparation of an $n(>2)$-particle BOBV, we do not need to care about atomic velocities.

Acknowledgments

I wish to thank W.B. da Costa, G.P. Berman, M.C. Oliveira, V. Dodonov and S.S. Mizrahi for interesting discussions, and R.J. Napolitano for enlightening discussions and a critical reading of the manuscript.

References

[1] W.K. Wootters and W.H. Zurek, Nature (London) 299 (1982) 802.
[2] D. Dieks, Phys. Lett. A 92 (1982) 271.
[3] N. Herbert, Found. Phys. 12 (1982) 1171.
[4] S. Haroche, in: Fundamental Systems in Quantum Optics (Les Houches, 1990) eds. J. Dalibard, J.M. Raimond and J. Zinn-Justin (Elsevier, New York, 1992) p. 771.
[5] M. Brune, P. Nussenzveig, F. Schmidt-Kaler, F. Bernadot, A. Maali, J.M. Raimond and S. Haroche, Phys. Rev. Lett. 72 (1994) 3339.
[6] J.I. Cirac and P. Zoller, Phys. Rev. A 50 (1994) R2799.
[7] L. Davidovich, N. Zagury, M. Brune, J.M. Raimond and S. Haroche, Phys. Rev. A 50 (1994) R895.
[8] T. Sleator and H. Weinfurter, Phys. Rev. Lett. 74 (1995) 4087.
[9] J.S. Bell, Physics (NY) 1 (1964) 195.
[10] D.M. Geenberger, M.A. Horne and A. Zeilinger, in: Bell's Theorem, Quantum Theory and Conceptions of the Universe, ed. M. Kafatos (Kluwer, Dordrecht, 1989).
[11] S.L. Braunstein, A. Man and M. Revzen, Phys. Rev. Lett. 68 (1992) 3259.
[12] H. Paul, Quantum Opt. 3 (1991) 169;
M. Brune, S. Haroche, J.M. Raimond, L. Davidovich and N. Zagury, Phys. Rev. A 45 (1992) 5193.
[13] C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. Wootters, Phys. Rev. Lett. 70 (1993) 1895.