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# Generation of the reciprocal-binomial state

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**Abstract**

We present an experimentally feasible scheme to generate in a cavity the recently suggested reciprocal-binomial state of the radiation field. For a running wave, this state would play the role of a reference field for measuring the quantum optical phase probability distribution by projection synthesis as proposed by Barnett and Pegg [Phys. Rev. Lett. 76 (1996) 4148]. The same scheme can also be used to generate the binomial and the negative binomial states. © 1998 Elsevier Science B.V.

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In a recent Letter [1], Barnett and Pegg have presented an ingenious technique for experimental determination of the canonical quantum optical phase probability distribution. Their scheme involves synthesizing a suitable projection onto a phase state which allows us to find the probability density $P_\Psi(\theta)$, for a field in the state $|\Psi\rangle$ to have a phase $\theta$. It is achieved by coherently mixing the field in state $|\Psi\rangle$ with a reference field in state $|\Phi\rangle$, by means of a beam splitter. The main result in Ref. [1] consists of showing that by finding a suitable reference field state $|\Phi\rangle$, the photocounting in the two outputs of the beam splitter leads to the required probability distribution $P_\Psi(\theta)$. The authors have shown that the probability for registering $N$ photocounts in detector $D_\Psi$ (in the direction of the transmitted photons from $|\Psi\rangle$), and no counts in detector $D_\Phi$ (transmitted photons from $|\Phi\rangle$), is proportional to $P_\Psi(\theta)$. The reference field supporting this result has to be what the authors called the reciprocal-binomial state

$$|\Phi\rangle = \frac{1}{\mathcal{N}} \sum_{k=0}^{N} \binom{N}{k}^{-1/2} e^{ik(\theta-\pi/2)} |k\rangle, \tag{1}$$

where $\mathcal{N}$ is a normalization constant and $\binom{N}{k}$ is the usual binomial coefficients.

The projection synthesis technique has thus shown how the (seemingly exotic) states of the radiation field that have been suggested [2–4], can be useful for measuring quantum properties of light. Such a technique has also anticipated the necessity of improvement in what has been called the quantum state engineering of the radiation field [5]. In this report we present an experimentally feasible scheme for preparing a cavity mode in the reciprocal-binomial state required for measuring the phase probability distribution by projection synthesis [1]. We hope that our scheme stands as a first step to produce a running wave field in the reciprocal-binomial state, as required in Ref. [1]. The

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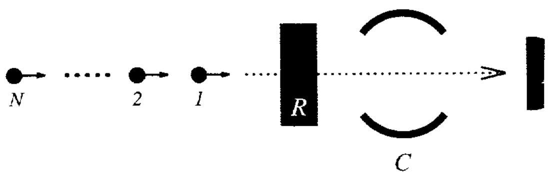


Fig. 1. Sketch of the experimental setup for generating the reciprocal-binomial state.

present technique can also be considered for preparing the binomial state of Stoler, Saleh and Teich [2], and the negative binomial state of Joshi and Lawande [3].

As shown in Fig. 1, our experimental setup consists of $N$ identical two-level atoms (levels $|g\rangle$ and $|e\rangle$), a Ramsey zone $R$, an initially empty high-$Q$ cavity $C$, and a state-selective field ionization detector $D$. The two-level atoms are considered as circular Rydberg atoms which are suitable for preparing and detecting atom-field long-lived correlations [6,7]. They present a strong coupling to microwaves and a very long radiative decay time of the order of cavity lifetimes for high-$Q$ superconducting cavities. Moreover, Rydberg atomic states are counted with high efficiency by state selective field ionization detectors [8]. Each atom is prepared by the microwave field $R$ in a given $g, e$ superposition state $c_g^k|g\rangle_k + c_e^k|e\rangle_k$ ($k$ labeling the $k$th atom), at a given time with a well-defined velocity. The atoms are thus sent, one-by-one, through cavity $C$, and after their on-resonant interaction with the field in $C$, they are counted in $D$. The on-resonant interaction is described by the Jaynes-Cummings Hamiltonian $H_{\text{on}} = \hbar\Omega_1(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$, where $\hat{\sigma}_+ = |e\rangle\langle g|$, $\hat{\sigma}_- = |g\rangle\langle e|$, and $\Omega_1$ is the one-photon Rabi frequency. Once the $N$ atoms have crossed the cavity, a straightforward calculation permits us to verify that when all of them are detected in their ground state, leaving their photon in the cavity, the pure field state of the cavity reads

$$|\phi\rangle = \frac{1}{\mathcal{N}}\sum_{k=0}^{N} \Lambda_k^N \, e^{-ik\pi/2}|k\rangle, \tag{2}$$

where $\mathcal{N}$ is a normalization constant and the coefficients $\Lambda_k^N$ are given by the recurrence formula

$$\Lambda_k^N = (1-\delta_{k,0})\Lambda_{k-1}^{N-1} c_e^N \sin(\sqrt{k}\Omega_1\tau_N) + (1-\delta_{k,N})\Lambda_k^{N-1} c_g^N \cos(\sqrt{k}\Omega_1\tau_N), \tag{3}$$

with $\Lambda_0^0 = 1$. The probability for detecting the whole series of atoms in the ground state is of the order of $1/2^N$, restricting the present technique to the preparation of cavity fields with a small number of atoms.

After we have detected the $N$ atoms in their ground state, let us now consider an auxiliary atom which is sent across $C$, tuned to have an off-resonant interaction with the field in this cavity. The off-resonant interaction is used to produce conditional phase shift in the atomic states, controlled by the photon number in the cavity field. The off-resonant Hamiltonian can be modeled by [9] $H_{\text{off}} = \hbar(\Omega_2/2)\hat{a}^\dagger\hat{a}(|e\rangle\langle e| - |g\rangle\langle g|)$, where $\Omega_2$ is the change in atomic level spacing per photon in the cavity. For the present purpose we assume that the auxiliary atom is initially prepared in the ground state, an eigenstate of the operator $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$. Therefore, after the off-resonant atom-cavity interaction, each component $|k\rangle$ of the Fock states in superposition (2) gains a phase shift proportional to the photon number $k$:

$$|\Phi\rangle = \frac{1}{\mathcal{N}}\sum_{k=0}^{N} \Lambda_k^N \, e^{ik(\theta-\pi/2)}|k\rangle, \tag{4}$$

where $\theta = \Omega_2 T/2$, $T$ being the duration of the interaction.

Finally, the state $|\Phi\rangle$ turns out to be exactly the reciprocal-binomial state when requiring that its coefficients satisfy the condition

$$\Lambda_k^N = \binom{N}{k}^{-1/2}, \tag{5}$$

in such a way that the normalization constant turns out to be

$$\mathcal{N} = \left[\sum_{k=0}^{N}\binom{N}{k}^{-1}\right]^{1/2}.$$

As can be observed from the recurrence formula (3), the relation (5) is just a requirement on how we have to prepare the $N$ atoms which are supposed to build the cavity mode (4) by resonantly interacting, one-by-one, with the initially empty cavity $C$. As an example, when considering the preparation of the cavity mode (4) with $N = 2$, we get, from the recurrence formula (3) and the condition (5), the relations

$$c_g^1 c_g^2 = c_e^1 c_e^2 \sin(\Omega_1\tau_1)\sin(\sqrt{2}\Omega_1\tau_2),$$
$$= \sqrt{2}[c_e^1 c_g^2 \sin(\Omega_1\tau_1)\cos(\Omega_1\tau_2) + c_g^1 c_e^2 \sin(\Omega_1\tau_2)].$$

When choosing $\tau_1 = \tau_2 = \pi/2\Omega_1$, the coefficients $c_g^i$, $c_e^i$, $i = 1,2$ become real, leading to the relations $c_e^1/c_g^1 = \sqrt{2}/\sin(\pi/\sqrt{2})$, and $c_e^2/c_g^2 = 1/\sqrt{2}$. So, to prepare the cavity mode (4) with $N = 2$, both the atoms have to be prepared through the microwave field $R$ in such a way that their coefficients satisfy the above-described relations. For $N$ atoms we are faced with an algebraic equation of degree $N$ in the variable $c_e^N/c_g^N$, which has no algorithm yielding analytical solution, in general, for $N > 4$ [10]. In this case only numerical solutions can be found from a set of $N+1$ coupled equations.

It is worth mentioning that through the engineering of the cavity radiation field presented above we can also prepare the binomial and the negative binomial states [2]. For this purpose, instead of the condition (5), the preparation of the atomic states supposed to build the binomial and the negative binomial states has to be dictated, respectively, by the relations

$$A_k^N = \left[\binom{N}{k}\eta^k(1-\eta)^{N-k}\right]^{1/2}, \tag{6a}$$

$$A_k^N = \binom{N+k-1}{k}\eta^k(1-\eta)^N, \tag{6b}$$

with $0 \leqslant \eta \leqslant 1$. Relations (6a) and (6b) lead to the normalization constants

$$\mathcal{N} = \left[\sum_{k=0}^{N}\binom{N}{k}\eta^k(1-\eta)^{N-k}\right]^{1/2} = 1,$$
$$\mathcal{N} = \left\{\sum_{k=0}^{N}\left[\binom{N+k-1}{k}\eta^k(1-\eta)^N\right]^2\right\}^{1/2},$$

respectively.

It should be noted that the phase factor appearing in the state (4) can be removed by properly adjusting the detuning between the auxiliary atom and cavity $C$, so that the phase of this atom can be shifted by $\pi/2$ when the photon number varies by one unit. By considering the preparation of the binomial state when $N = 2$, we get from the condition (6a) and the recurrence formula (3), the equalities

$$c_g^1 c_g^2 = 1-\eta,$$
$$c_e^1 c_e^2 \sin(\Omega_1\tau_1)\sin(\sqrt{2}\Omega_1\tau_2) = \eta,$$
$$c_e^1 c_g^2 \sin(\Omega_1\tau_1)\cos(\Omega_1\tau_2) + c_g^1 c_e^2 \sin(\Omega_1\tau_2) = [2\eta(1-\eta)]^{1/2}.$$

So, through the previous choice $\tau_1 = \tau_2 = \pi/2\Omega_1$, the two atoms supposed to build the binomial state with $N = 2$ have to be prepared in such a way that their coefficients satisfy the relations

$$c_e^1/c_g^1 = [\eta/2(1-\eta)]^{1/2}/\sin(\pi/\sqrt{2}),$$
$$c_e^2/c_g^2 = [2\eta/(1-\eta)]^{1/2}.$$

At this point, one could take advantage of these relations to prepare a number state as a limit of the binomial state for $\eta \to 1$. To this end, both the atoms should be injected into the cavity initially prepared approximately in their excited states, i.e., $c_e^i \approx 1$, $i = 1,2$. This result is similar to a previous one obtained using trapping states procedure [11]. Concerning the negative binomial state, an interesting limit in this case would be the Susskind–Glogower phase state [12], also obtained for $\eta \to 1$ [13] [3].

It should be stressed that the present scheme is related with that by Vogel et al. [5], except for the important step represented by the final atom which interacts dispersively with the cavity field. In fact, both proposals are based on conditional measurements since all the $N$ atoms must be detected in the lower state.

As a final remark we mention that in the experimental scheme presented here coherent atom–field interaction can be made to dominate over dissipative processes due to cavity losses and atomic spontaneous emission. As mentioned above characteristic times for these processes are about $10^{-2}$ s, three orders of magnitude longer than typical atom–cavity interaction times [8,14]. Also, current experiments involving the interaction of circular Rydberg atoms

[2] Alternative proposals have been presented in the literature for the generation of the binomial state [1], and the negative binomial state [3].

[3] In Ref. [3] a modification was introduced in the definition of the negative binomial state so that in the limiting cases it corresponds to fields in coherent and thermal states.

with microwave fields can reach the parameters necessary to avoid that the velocity dispersion separates the required atom–field entangled states appreciably from the expected one. Such parameters correspond to a coupling strength between atoms and quantized fields around $10^{+5}$ $s^{-1}$, and atomic velocities around $10^3$ m/s [8,14]. As a consequence of dispersion in the velocity the atoms would arrive at random times and would meet the cavity field with random phases [15] (only diagonal elements of the density matrix would survive). This is a very serious restriction to be accomplished in the experiment. Finally, the efficiency of atomic detections [7] can also be estimated for an average success of the cavity field engineering presented here.

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