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Hole burning in Fock space

B. Baseia [a,1], M.H.Y. Moussa [b], V.S. Bagnato [c]

[a] *Instituto de Fisica, Universidade Federal de Goias, Cx. Postal 131, CEP 74001-970 Goiania (GO), Brazil*

[b] *Departamento de Fisica, Universidade Federal de Sao Carlos, Via Washington Luiz, Km 235 CEP 13565-235, Sao Carlos (SP), Brazil*

[c] *Instituto de Fisica, Universidade de Sao Paulo, Cx. Postal 369, CEP 13.560-970, Sao Carlos (SP), Brazil*

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Abstract

As shown in the literature, a field state having holes in its photon-number distribution (PND) corresponds to a nonclassical state. Here we discuss the problem of how to make such holes, by exhibiting a scenario where the involved parameters control their positions and depth in the PND. © 1998 Elsevier Science B.V.

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In quantum optics there are an endless number of states, some of them being clearly important, others seemingly exotic, or artificial, as one finds in the recent literature. Among them, one class of states deserves special attention in the present context: we refer to the interpolating states, which coincide with two distinctive states in two different limits of the parameters involved. To our knowledge, the first interpolating state introduced in the literature was the binomial state (BS), of Stoler et al. [1], which interpolates between the number state $|N\rangle$ and the coherent state $|\alpha\rangle$. Several nonclassical properties exhibited by the BS have been studied, including its application (and influence) as initial state of a field interacting with two-level atoms, through the Jaynes–Cummings model (JMC) [2]. Another example of an interpolating state is the negative-BS, introduced by Agarwal [3], which intermediates between a number state $|N\rangle$ and the (Susskind–Glogower [4]) phase state $|\theta\rangle_{SG}$. More recently, new interpolating states have been proposed: (i) the intermediate number-phase state (INPS) [5], which interpolates between a number state $|N\rangle$ and the (Pegg–Barnett [6]) phase state $|\theta\rangle_{PB}$; (ii) the intermediate number-squeezed state (INSS) [7], which interpolates between the number state $|N\rangle$ and the squeezed-coherent state $|z,\alpha\rangle = \hat{U}(z)|\alpha\rangle$; and (iii) the binomial–binomial state (BBS) [8], which interpolates between two different (complementary) binomial states. In each case, interesting properties shown by these states were investigated, such as sub-Poissonian statistics [9], antibunching [10], squeezing [11], etc.

Besides the interpolating states mentioned before, alternative intermediate states of the light field were also studied in the literature. Here we refer to the superposition states, first introduced by Wodkiewicz et al. [12], involving the superposition on two different number states, $|N_1\rangle$ and $|N_2\rangle$; by Hillery [13], involving two particular coherent states; by Gerry et al. [14],

[1] On leave from Departamento de Fısica, Universidade Federal da Paraíba, João Pessoa (PB), Brazil. E-mail: basilio@fis.ufg.br.

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involving even and odd coherent states; by Xin et al. [15], involving even and odd squeezed states, etc. More recently yet, other superposition states have been investigated: as a general superposition of two arbitrary coherent states [16], $|\alpha_1\rangle$ and $|\alpha_2\rangle$; as a general superposition of an arbitrary number state $|N\rangle$ with an arbitrary coherent state $|\alpha\rangle$ [17], etc. A unified approach for these was presented in Ref. [18].

Besides the mentioned nonclassical properties (sub-Poissonian, antibunching and squeezing), these alternative interpolating states also allow us to study quantum coherence [19], oscillations in the photon-number distribution [20,16], "macroscopic" quantum superposition [21], etc., being also useful as tools for some theoretical demonstrations [22]. We will return to this point after the clarifying discussion in the next paragraph.

As is well known, the generation of all these states employed extensively in quantum optics, is not a task of immediate implementation. Even the generation of the number state (a "battle-horse" of quantum optics), is of difficult experimental realization: to our knowledge, only number states $|n\rangle$ with small values of n have been (roughly) obtained in laboratory [23]. However, to stimulate the optimism in our community, we must not forget that the first experimental observation of a squeezed state [24] had to wait for fifteen years after its theoretical prediction [25]. Also, in this case the experimental realization was obtained via the four-wave mixing technique, contrary to a previous (but unsuccessful) suggestion using a two-photon laser [26]. Recently, Barnett and Pegg [27] suggested an interesting proposal to measure the phase of an arbitrary (pure or mixed) field, in which an apparently exotic state, called reciprocal-binomial state (RBS), plays a crucial role in the experiment. The authors then claimed that in view of the recent advances of quantum states engineering [28] it would be possible in future to realize the RBS, allowing one to measure the field phase. Of course, many readers would fail to see any relevance of an experiment depending crucially on the existence of an apparently artificial state, such as the RBS – mainly remembering that even the realization of the BS was not achieved yet. Notwithstanding, a feasible proposal to realize the RBS has already been presented [29]. The proposal follows one line of the quantum state engineering technique: accordingly, in principle, any state of the light field in a (good) cavity, with a limited number of photons, can be engineered either through conditional measurements as proposed by Vogel et al. [28], or by direct manipulation of the interaction, as discussed by Law and Eberly [28]. In this context, the work by Barnett and Pegg [27] leads to new optimism in the area, since it shows the potential importance of some states (as the RBS) which at first sight would seem very "artificial", "academic", or even "baroque".

Motivated by the work of Barnett and Pegg [27], the present Letter considers this scenario for the potential application of a state describing a light field, as mentioned above. Here we will restrict ourselves to a convenient superposition state showing its usefulness for an interesting practical purpose, as the creation of holes in Fock space. The present procedure plays the role of a technique for re-engineering an interpolating field state, by burning holes in Fock space through the photon number distribution p_n, representing the state. We will return to this point below.

More recently, it appears that the extension of the concept of "hole" to Fock space could be of some interest, regarding applications in theoretical physics. For example, a recent theorem by Lee [32] in quantum optics claims the necessity of removing the vacuum component $|0\rangle$ of an arbitrary field state $|\Psi\rangle = \sum_{n=0}^{\infty} C_n|n\rangle$. In this case the resulting state $|\Psi_t^{(0)}\rangle$ has the form

$$|\Psi_t^{(0)}\rangle = \eta_0 \sum_{n=0}^{\infty} (c_n - c_0)|n\rangle, \tag{1}$$

where the subscript t stands for "truncated", superscript (0) stands for the removed component $|n=0\rangle$ and η_0 is the normalization constant

$$\eta_0 = \left(\sum_{n=1}^{\infty} |c_n|^2 \right)^{-1/2}. \tag{2}$$

According to the theorem in Ref. [32] the truncated state $|\Psi_t^{(0)}\rangle$ turns out to be as nonclassical as possible in view of a measure of nonclassicality introduced in Ref. [33].

More recently, [22] claims the importance of removing a component $|N\rangle \neq |0\rangle$, of a general state $|\Psi\rangle = \sum_{n=0}^{\infty} c_n|n\rangle$, in order to increase the nonclassical depth of a state, when using the traditional measure of non-classicity (sub-Poissonian, squeezing and antibunching). In this instead of Eq. (1), the state

$$|\Psi_t^{(N)}\rangle = \eta_N \sum_{n=0}^{\infty} (c_n - c_N)|n\rangle , \quad (3)$$

with the normalization constant

$$\eta_N = \left(\sum_{n=0}^{\infty} (1 - \delta_{n,N})|c_n|^2 \right)^{-1/2} , \quad (4)$$

corresponds to a state $|\Psi\rangle$ having its component $|N\rangle$ truncated. As an attempt to this end, the interpolating states scenario was assumed [18], introducing

$$|\Psi(\xi,\phi)\rangle = \eta(\xi,\phi)(\sqrt{\xi}\,e^{i\phi}|\Psi_1\rangle + \sqrt{1-\xi}|\Psi_2\rangle), \quad (5)$$

which can be identified with (3) if

$$\phi = \pi, \quad |\Psi_1\rangle = |N\rangle,$$

$$|\Psi_2\rangle = |\Psi\rangle = \sum_0^{\infty} c_n|n\rangle, \quad c_N = \sqrt{(1-\xi)/\xi}.$$

Although the above strategy is able to remove a component $|N\rangle$ of a state $|\Psi\rangle$, it sounds somewhat precarious, since the experimental generation of a number state $|N\rangle$ is a difficult task until now, in the same way that we are not aware of any practical procedure to remove the vacuum component of a state, as it was also emphasized in Ref. [32].

In the present Letter we will look for a different and feasible way to make a hole in Fock space. To this end we consider the interpolating superposition of two (available) coherent states

$$|\Psi(\xi,\phi)\rangle = \eta(\sqrt{\xi}|\alpha_1\rangle + e^{i\phi}\sqrt{1-\xi}|\alpha_2\rangle) , \quad (6)$$

where

$$\eta = [1 + 2\sqrt{\xi(1-\xi)}\,\mathrm{Re}\,(e^{i\phi}\langle\alpha_1|\alpha_2\rangle)]^{-1/2} . \quad (7)$$

Note that for $\xi \to 1(0)$, $|\Psi(\xi,\phi)\rangle \to |\alpha_1\rangle$ $(|\alpha_2\rangle)$. Hence the state in (6) interpolates between the two arbitrary coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$. If we assume that $\alpha_j = r\exp(i\theta_j)$, $j = 1, 2$, then we have

$$|\Psi(\xi,\phi)\rangle = \eta\, e^{-r^2/2} \sum \frac{r^n}{\sqrt{n!}} \times e^{in\theta_1}(\sqrt{\xi} + e^{i(\phi+n\Delta\theta)}\sqrt{1-\xi})|n\rangle, \quad (8)$$

where $\Delta\theta = \theta_2 - \theta_1$ and the representation $|\alpha_i\rangle = e^{-r^2/2}\sum(\alpha_i^n/\sqrt{n!})|n\rangle$ was employed. Next, setting $\xi = 1/2$ and

$$\phi + n\Delta\theta = (2m+1)\pi , \quad m = 0, 1, 2, \ldots , \quad (9)$$

we burn a hole at the component $|n\rangle = |N\rangle$ if we choose in Eq. (5)

$$\phi = (1 - N/N_0)\pi , \quad (10a)$$

$$\Delta\theta = \pi/N_0 , \quad (10b)$$

where N_0 is an integer. In fact, we obtain from the substitution of Eqs. (10) in Eq. (9),

$$n = N + 2mN_0 , \quad (11)$$

hence holes at $n = N$, $n = N + 2N_0$, $n = N + 4N_0$, etc., are created in Fock space. These holes will appear in the photon-number distribution PND, $p_n = |\langle n|\Psi(\xi,\phi)\rangle|^2$.

Now, if we want to make (effectively) only a single hole at $n = N$, then we must choose N_0 sufficiently large, in such a way that the remaining holes at $n = N + 2N_0$, $n = N + 4N_0$, $n = N + 6N_0$, etc., are far away from the peak of the PND, so their contributions become negligible. This condition is accomplished if $2N_0 \gg \bar{n}$, where $\bar{n} \simeq |\alpha|^2 = r^2$ is the average number excitation in the field state $|\Psi(\xi,\phi)\rangle$, namely, $\bar{n}\langle\Psi(\xi,\phi)|\hat{n}|\Psi(\xi,\phi)\rangle$. For example, if we have $r = |\alpha| = 2$, then $\bar{n} \simeq 4$. In this case, if we want to make a hole at $n = 1$ we set $N = 1$ and, say, $N_0 = 10$ in Eqs. (10) to get $\phi = (1 - 1/10)\pi = 9\pi/10$ and $\Delta\theta = \pi/10$. Hence from Eq. (11) one obtains holes at $n = 1, 21, 41$, etc. However, since $N_2 = 21 \gg \bar{n} \simeq 4$, all the holes, with exception of the first one at $n = 1$, are negligible, because they effectively lie outside the PND.

On the other hand, if we want to make a hole at $n = 2$, then we might set $N = 2$ and $N_0 \geqslant 10$ (say, $N_0 = 10$). In this case one would get, from Eqs. (10), $\phi = (1 - 2/10)\pi = 4\pi/5$ and $\Delta\theta = \pi/10$; so, from Eq. (11) holes would be produced at $n = 2, 22, 42$, etc. As before, the holes at $n = 22, 42, \ldots$, are negligible. If the excitation $\bar{n}$ of the field increases then the previous strategy will work for larger values of N_0, concerning the production of single hole.

When N_0 is small ($N_0 \simeq 1$) many holes are burned in the PND. For example, an interesting application results for $N_0 = 1$: in this case, if we want the first hole at $n = 0$ ($n = 1$) then we set $N = 0$ ($N = 1$) and $N_0 = 1$ in Eqs. (10) to get $\phi = \pi$ ($\phi = 0$) and $\Delta\theta = \pi$. This yields a regular sequence of holes at $n = 1, 3, 5, \ldots$ ($n = 0, 2, 4, \ldots$) corresponding to the PND of the *odd* (*even*) coherent state [14].

In the foregoing procedure the hole burning is complete. What about making holes with controlled depth? This goal could be attained by choosing the interpolating parameter $\xi \neq 0.5$, in Eq. (5). On the other hand, instead of a hole, could one also burn peaks in the PND? In this case the condition (9) should be modified to

$$\phi + n\Delta\theta = 2m\pi, \quad m = 0, 1, 2, \ldots \tag{12}$$

Now, the convenient choices for ϕ and $\Delta\theta$ will differ from those leading to holes. Here we set, instead of the choices in Eqs. (10),

$$\phi = -(N/N_0)\pi, \tag{13a}$$

$$\Delta\theta = \pi/N_0 \tag{13b}$$

to get, from the substitution of (13) in (12),

$$n = N + 2mN_0, \tag{14}$$

which coincides with Eq. (11). However, note that while Eq. (14) comes from "constructive interference" (see Eq. (12)) Eq. (11) comes from "destructive interference" (see Eq. (9)). Considerations concerning the generation of a single peak or many peaks are strictly parallel to those for generation of single holes and many holes, respectively.

The engineering of quantum states here concerns the generation of the superposition state given in Eq. (5). Re-engineering the state consists in its manipulation, by creating holes at desired components $|N_i\rangle$ of Fock space, through the control of the involved parameters ξ, ϕ and $\Delta\theta$. As we have seen above, while the parameter ξ controls the depth of holes, the parameters ϕ and $\Delta\theta$ control their positions and separation in Fock space, respectively. Now, if the control of such engineering and re-engineering is achieved, then one could think of transposing the successful application concerning the hole burning in frequency space [30,31], the realm of quantum optics. To this end, however, one should make a step beyond the present strategy, by controlling not only the positions and depth of holes, but also the other structure of the PND. We think that for this purpose one would need a superposition of more coherent state components, the present procedure being a first step in this direction.

Now, a remark concerning the generation of states having holes in its PND, is necessary. Recent works, based on quantum states engineering [28], show recurrence formulas for constructing a desired field state having no holes in its PND [29], or having holes for even components ($n = 0, 2, 4, \ldots$) and odd components ($n = 1, 3, 5, \ldots$) [35]. From these recurrence formulas one obtains the required expression ce/cg for the state $|\Psi\rangle = ce|e\rangle + cg|g\rangle$ of the ith atom crossing a (high-Q) cavity. By combining the procedures of Refs. [29,35] one would be able to obtain a recurrence formula for the present context, which would depend on each specified structure of holes in the PND.

As a final remark we mention that the present results stand as a realization of a general demonstration by Mandel and Wolf [34], showing that for an arbitrary field state, $\hat{\rho} = \int p(\alpha)|\alpha\rangle\langle\alpha|\,d^2\alpha$, we have

$$p_n = \int P(\alpha)|\langle n|\alpha\rangle|^2\,d^2\alpha. \tag{15}$$

Since $|\langle n|\alpha\rangle|^2 > 0$ for $|\alpha| > 0$ then $p_n \neq 0$ when $P(\alpha)$ is a true probability density. Hence, $p_n = 0$ will correspond to a state having no classical analog, being purely quantum mechanical. As a consequence, in this context, making holes in Fock space corresponds to the generation of some nonclassical effects in the light field, an additional point deserving attention in the present perspective.

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