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One-cavity scheme for atomic-state teleportation through GHZ states

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Abstract

We present a one-cavity scheme for atomic-state teleportation employing the GHZ entangled state as the quantum channel. A scheme employing a single high-Q cavity turns out to be attractive to realize experimentally, while the GHZ state simplifies the procedure for the accomplishment of the required joint measurement. © 1998 Published by Elsevier Science B.V.

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1. Introduction

Some extremely important experiments in fundamental quantum physics have recently been carried out. These experiments, based on cavity QED techniques, permit coherent atom-field interactions to dominate over dissipative processes due to cavity losses and atomic spontaneous emission [1]. The required two-level atoms are considered as circular Rydberg atoms which are suitable for preparing and detecting atom-field long-lived correlations [2]. They present a strong coupling to microwaves and a very long radiative decay time of the order of cavity lifetimes for high-Q superconducting cavities. Moreover, Rydberg atomic states, which can be prepared at a given time with a well-defined velocity, present a high detection efficiency when counted by state-selective field ionization detection [1]. Such properties make the entanglements between circular Rydberg atoms suitable for improved tests that challenge local realistic theories [3,4], and to examine related phenomena such as teleportation [5,6] and quantum computation [7].

To mention some of the most significant experiments, we point out the direct test of field quantization through a quantum Rabi oscillation which has been realized employing circular Rydberg atoms in vacuum and in small coherent fields stored in a high-Q cavity [8]. Rydberg atoms interacting one at a time with a coherent field of a few photons have also been employed for observing progressive decoherence of a "meter" in a quantum measurement [9]. More recently, a quantum memory made of a field in a superposition of zero- and one-photon Fock states was reported [10]. Through this "memory," the quantum information carried by a two-level atom was transferred to a high-Q cavity and, after a delay, to another atom. Finally, we mention the generation of Einstein–Podolsky–Rosen (EPR) pairs of entangled atoms in a cavity QED experiment [4]. The EPR nonlocal corre-

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lations have been measured by a Ramsey interferometric method which can be straightforwardly generalized to an atom triplet or to a large number of particles. It thus opens the way to new tests of nonlocality in mesoscopic quantum systems. Such a "manipulation" of entangled atoms besides being another important aspect of the new EPR experimental proposals [4], also opens the way for the experimental realization of atomic-state teleportation and quantum gates through cavity QED phenomena.

Schemes employing cavity QED phenomena [5,11] have been suggested to realize the teleportation of quantum states following the principle outlined by Bennett et al. [12]. By teleportation the authors refer to the process by which an unknown quantum state $|\Psi\rangle_A$ of a system A is exactly replicated into another system B far away from A. Let us assume that a system A has been given to a sender, Alice, who shares an EPR state, a quantum channel, with a receiver, Bob. This EPR pair, $|\psi\rangle_{BC}$, consists of the above-mentioned particle B, which has been given to Bob, and a third particle C, which has also been given to Alice. By performing a joint measurement on particles A and C (whose possible eigenstates constitute a Bell basis $|\Psi^{\pm}\rangle_{AC}, |\Phi^{\pm}\rangle_{AC}$ [13]), Alice couples the particle to be teleported with the EPR state. As a result of this measurement, system B is automatically projected into a pure state that differs from $|\Psi\rangle_A$ just by an irrelevant phase factor or a rotation around the x, y, or z axis (depending on the eigenstates of the joint measurement). Through a classical channel Alice communicates the outcome of her measurement to Bob, who finally performs a unitary transformation on his previously entangled systems that brings it to the original state of Alice's system A. Obeying the no-cloning theorem [14] Alice's original state is destroyed in the process.

In Davidovich et al.'s scheme [5] a nonlocal superposition of microwave field states simultaneously occupying two cavities is assumed to set up the required quantum channel. Two cavities are also employed in the Cirac and Parkins' scheme [11], one to prepare an atomic-entangled state (the quantum channel) and another for the accomplishment of the required joint measurement. Similarly to the proposal by Sleator and Weinfurter [7], in this Letter, we present a one-cavity scheme which simplifies the experimental procedure for atomic-state teleportation. In fact, all the above-mentioned experiments carried out through cavity QED phenomena have been realized through a single high-Q cavity. The consideration of additional cavities in the experimental set up, which has to be cooled to about 0.6 K by a cryostat making the mean blackbody photon number negligible (it was measured to be only 0.05 at 0.8 K [8]), leads to additional technical difficulties. Moreover, despite the cavity damping mechanism, the vacuum Rabi pulse in a cavity cannot transfer more than 94% of the atoms due to coupling dispersions related to the atomic position spread in the cavity mode [10].

For the implementation of the present one-cavity teleportation scheme (OCTS) we have modified the protocol outlined by Bennett et al. [12], but keeping its principle, the possibility of entanglement between separated quantum systems and the projection postulate. Basically, as in Ref. [7], the single cavity is employed both to produce the quantum channel and for the accomplishment of the joint measurement. However, and here is the essential modification we have introduced in the original Bennett scheme, the quantum channel will consist of three quantum systems, two identical two-level atoms and the cavity itself. An entangled state of the Greenberger–Horne–Zeilinger (GHZ) [15] type, which can be achieved through Hagley et al.'s scheme [4], is thus necessary to implement the proposed experiment. As a consequence of the present modified protocol for teleportation it will be shown that we overcome the necessity for the accomplishment of a joint measurement to distinguish the Bell states $|\Psi^{\pm}\rangle$ and $|\Phi^{\pm}\rangle$. Only the entanglement pairs $|\Psi^{\pm}\rangle$ or $|\Phi^{\pm}\rangle$ occur in this OCTS.

A sketch of the OCTS is displayed in Figs. 1a,b,c. The set up consists of an initially empty microwave cavity C, a set of identical two-level atoms, state-selective field ionization detectors D and Ramsey zones R, whose roles in the experiment are discussed below. The cavity field can be described through the basis of zero- and one-photon Fock states ($|0\rangle$ and $|1\rangle$, respectively) since there is never more than one photon in the cavity. Two different kinds of atom–field interactions are used in the present scheme: resonant and dispersive interactions. The former is used to transfer quantum states between atom and cavity while the latter is used to produce conditional shifts in the atomic states, controlled by the photon number in the cavity field. Atomic Rydberg states

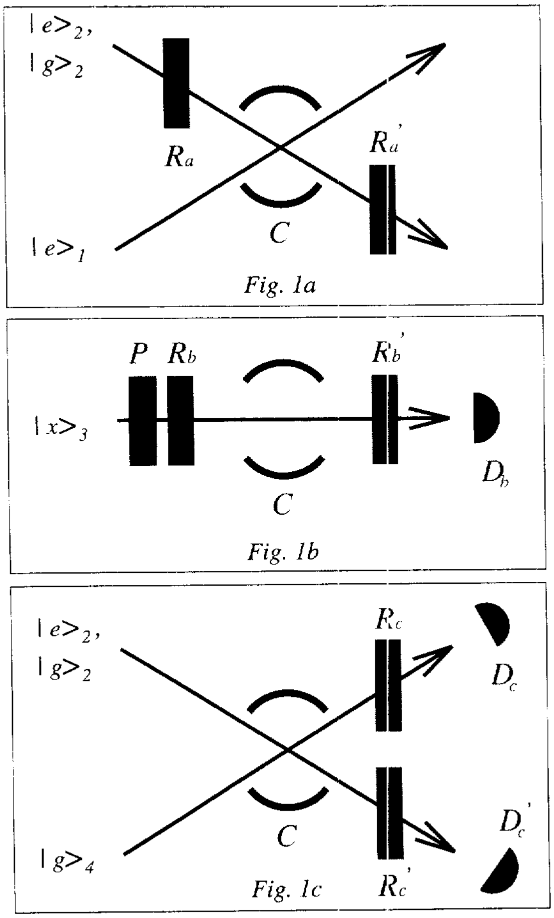

Fig. 1.

with adjacent principal quantum numbers are considered and the transition from the excited ($|e\rangle$) to the ground state ($|g\rangle$) is tuned to resonance with the cavity mode frequency. However, when considering a Ramsey type arrangement (RTA), the atoms are tuned to have a dispersive interaction with the cavity field. In a RTA the atoms are made to cross two separated microwave fields (R and R', respectively) with the cavity placed between them. After passing across a RTA ($R - C - R'$), an atom undergoes a transformation such that the probability for an $|e\rangle \to |g\rangle$ transition is characteristic for the photon number in the cavity. Such a transition probability depends also on a given setting of the Ramsey zones besides depending on the dispersive atom–cavity interaction. For the present purpose, we assume that the microwave zones are set so that an atom exactly undergoes a $\pi/2$ pulse, on the $|e\rangle \to |g\rangle$ transition, in each zone. The cavity detuning is set so that the atom undergoes a phase shift per photon exactly equal to π. In this way, as obtained in Ref. [16], the $|e\rangle \to |g\rangle$ transfer probability is one when the cavity field is $|0\rangle$, and therefore, zero when the cavity field is $|1\rangle$. Owing to the dispersive atom–cavity interaction the photon number in the cavity remains unchanged and the atom–cavity system undergoes the transformations,

$$|g\rangle \begin{cases} |0\rangle \to |e\rangle|0\rangle \\ |1\rangle \to |g\rangle|1\rangle \end{cases}, \quad |e\rangle \begin{cases} |0\rangle \to -|g\rangle|0\rangle \\ |1\rangle \to -|e\rangle|1\rangle. \end{cases} \tag{1}$$

Let us begin by generating the GHZ type quantum channel. As sketched in Fig. 1a, atom 1, initially prepared in the excited state $|e\rangle_1$, is sent across cavity C, initially in the vacuum state $|0\rangle$. This atom is made resonant with the cavity and undergoes, on the $|e\rangle \to |g\rangle$ transition, a $\pi/2$ pulse, so that $|e\rangle|0\rangle \to (|e\rangle|0\rangle - |g\rangle|1\rangle)/\sqrt{2}$. Subsequently, atom 2 (it does not matter in which state it has initially been prepared, $|e\rangle_2$ or $|g\rangle_2$, as indicated in Fig. 1a) is sent through the RTA $R_a - C - R'_a$. By assuming that atom 2 is initially prepared in the ground state $|g\rangle_2$, the cavity plus the atomic system end up in the entanglement

$$|\Psi\rangle_{12C} = \frac{1}{\sqrt{2}}(|e\rangle_1|e\rangle_2|0\rangle - |g\rangle_1|g\rangle_2|1\rangle), \tag{2}$$

which constitutes the quantum channel. In fact, since an additional cavity will not be required for the accomplishment of the joint measurement, our single cavity necessarily has to be correlated with the running atomic system which will carry out the teleported state. As indicated in Fig. 1b, such a state, supposedly unknown to the observer, is initially prepared through atom 3 by the microwave zone P. With atom 3 in the arbitrary $|e\rangle$, $|g\rangle$ superposition $|\chi\rangle_3 = c_e|e\rangle_3 + c_g|g\rangle_3$, the state of the whole system can be expanded as

$$\begin{aligned} |\Psi\rangle_{12C}|\chi\rangle_3 = \tfrac{1}{2}[&|\Psi^{(+)}\rangle_{3C}(c_g|g\rangle_1|e\rangle_2 - c_e|e\rangle_1|g\rangle_2) \\ &- |\Psi^{(-)}\rangle_{3C}(c_g|g\rangle_1|e\rangle_2 + c_e|e\rangle_1|g\rangle_2) \\ &- |\Phi^{(+)}\rangle_{3C}(c_g|e\rangle_1|g\rangle_2 - c_e|g\rangle_1|e\rangle_2) \\ &+ |\Phi^{(-)}\rangle_{3C}(c_g|e\rangle_1|g\rangle_2 + c_e|g\rangle_1|e\rangle_2)], \end{aligned} \tag{3}$$

where we have introduced the Bell operator basis of a system composed of an atom (labeled below by α)

plus cavity,

$$|\Psi^{(\pm)}\rangle_{\alpha C} = \frac{1}{\sqrt{2}}(|e\rangle_\alpha|0\rangle_C \pm |g\rangle_\alpha|1\rangle_C), \tag{4a}$$

$$|\Phi^{(\pm)}\rangle_{\alpha C} = \frac{1}{\sqrt{2}}(|e\rangle_\alpha|1\rangle_C \pm |g\rangle_\alpha|0\rangle_C). \tag{4b}$$

So, differently from Bennett et al.'s scheme, by performing a joint measurement (on the system whose state is to be teleported plus one of the systems composing the quantum channel) we do not get the teleportation process accomplished. Instead, we just break the correlation $|\Psi\rangle_{12C}$ leaving the cavity decoupled from atoms 1 and 2. However, when atom 3 is made to cross the RTA $R_b - C - R'_b$, as indicated in Fig. 1b, we find that the state vector of the entire system (Eq. (3)) evolves as

$$\begin{aligned}|\Psi\rangle_{12C}|\chi\rangle_3 \rightarrow \tfrac{1}{2}\{[|\Phi^{(+)}\rangle_{2C}(c_e|e\rangle_1 + c_g|g\rangle_1)\\ - |\Phi^{(-)}\rangle_{2C}(c_e|e\rangle_1 - c_g|g\rangle_1)]|g\rangle_3\\ - [|\Phi^{(+)}\rangle_{2C}(c_g|e\rangle_1 + c_e|g\rangle_1)\\ - |\Phi^{(-)}\rangle_{2C}(c_g|e\rangle_1 - c_e|g\rangle_1)]|e\rangle_3\}.\end{aligned} \tag{5}$$

It is worth noting that if atom 2 had initially been prepared in the excited state $|e\rangle_2$, we would have obtained Eq. (5) in terms of the Bell states $|\Psi^{(\pm)}\rangle$ instead of $|\Phi^{(\pm)}\rangle$. We observe from the above equation that we have simplified the joint measurement required for the accomplishment of the teleportation of the original state $|\chi\rangle$ from atom 3 to atom 1. In Bennett et al.'s scheme one has to discern between the four Bell states in Eqs. (4), first measuring Ψ or Φ, and afterwards their phases. Here, after state-selective detection of atom 3 by D_b (Fig. 1b), we just have to distinguish between the phases in $\Phi^\pm$ (or $\Psi^\pm$ when considering $|e\rangle_2$).

As indicated in Fig. 1c, an additional atom is employed for the realization of the joint measurement. After atom 1 had reached Bob's hand, atom 4, initially prepared in the ground state $|g\rangle_4$, is sent across the cavity C. This atom is tuned to interact resonantly with the cavity and undergoes a $\pi/2$ pulse so that the Bell states in Eq. (5) evolve as

$$|g\rangle_4|\Phi^{(\pm)}\rangle_{2C} \rightarrow \frac{1}{\sqrt{2}}(|e\rangle_2|e\rangle_4 \pm g\rangle_2|g\rangle_4)|0\rangle, \tag{6}$$

Atom 4 thus leaves cavity C entangled with atom 2 while the cavity, which is left in the vacuum state, gets disentangled of the atomic system. Next, both atoms 2 and 4 undergo a $\pi/2$ pulse in Ramsey zones R_c and R'_c, respectively. The atomic-entangled state in Eq. (6) thus evolves to

$$\begin{aligned}\frac{1}{\sqrt{2}}(|e\rangle_2|e\rangle_4 \pm |g\rangle_2|g\rangle_4) \rightarrow \frac{1}{2\sqrt{2}}[(1 \pm 1)(|g\rangle_2|g\rangle_4\\ + |e\rangle_2|e\rangle_4) + (1 \mp 1)(|g\rangle_2|e\rangle_4 + |e\rangle_2|g\rangle_4)].\end{aligned} \tag{7}$$

Detecting atoms 2 and 4 (through D'_c and D_c, respectively) in common states $|g\rangle_2|g\rangle_4$ or $|e\rangle_2|e\rangle_4$ does indicate the phase (+), while detection in different states $|e\rangle_2|g\rangle_4$ or $|g\rangle_2|e\rangle_4$ indicates the phase (−).

By having Alice tell Bob the outcome of the measurement performed on atom 3 and the joint measurement on $|\Phi^\pm\rangle_{2C}$, Bob finally has to apply an appropriate rotation on atom 1. Through this rotation, accomplished by an additional Ramsey zone, atom 1 finally reaches the original state of atom 3.

Back to the preparation of an entangled state of the GHZ type as in Eq. (3), a crucial step for the experimental realization of the OCTS, we note that the idea, as considered by Hagley et al. [4], is to apply a set of well-controlled interactions to the atomic particles in order to bring them *with the cavity* into a "tailored" entangled state. As is well known, there are some sensitive points which are typical in such experiments employing cavity QED phenomena. The dissipative process due to cavity losses and atomic spontaneous emission, the dispersion in the atomic velocity, and the efficiency of atomic detection constitute basic difficulties which have to be overcome. However, as mentioned above, due to the strong coupling between the considered circular Rydberg states and microwave fields, and due to their long radiative decay times, the atom–field interactions are supposed to dominate the dissipative processes. For high-Q superconducting cavities the cavity damping times, in the 10^{-2} s range [6–9], are about three orders of magnitude longer than typical atom–cavity interaction times. For Rydberg atoms in circular states $l = n - 1$, atomic excited-state lifetimes are also of the order of 10^{-2} s [6–9]. The above-mentioned experiments involving the interaction of circular Rydberg atoms with microwave fields have also reached the parameters necessary in order for the velocity dispersion of the required atom–field entangled states not to appreciably depart from the expected one. Such parameters correspond to a coupling strength between atoms and quantized cavity fields of around 2×10^{-5} s^{-1} and atomic velocities of around 10^3 m/s [3]. As regards the efficiency of atomic detection, this can be estimated for an average success of the one-cavity teleportation process. The dispersion in the atomic velocities can also be assimilated into an effective efficiency of detection [5].

As a final remark we note that the manipulation of quantum systems interacting in a well-controlled environment as required for the realization of OCTS presents strong connections to the theory of quantum information [10]. The realization of the OCTS could thus indicate the way for implementing simple quantum networks [7] and even to improve the emergent area of quantum state engineering of the radiation field [17].

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