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Nonlocality of a single particle: From the Fock space to cavity QED

M.H.Y. Moussa [a,1], B. Baseia [b]

[a] *Departamento de Física, Universidade Federal de São Carlos, Via Washington Luis, km 235, São Carlos 13565-905, SP, Brazil*

[b] *Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, São Carlos 13565-905, SP, Brazil*

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Abstract

We have transposed Peres' argument [Phys. Rev. Lett. 74 (1995) 4571] about the single-particle nonlocality from the Fock space into an experimentally feasible scheme in cavity QED. The present scheme consists of a new proposal to demonstrate the single-particle nonlocality, where the single-particle state is constructed with a coherent factor which is crucial to reveal its nonlocal character. © 1998 Elsevier Science B.V.

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Through a modification of an earlier proposal by Tan, Walls and Collet [1], Hardy [2] has recently suggested an experimental setup which displays "nonlocality of a single photon". Essentially, Hardy's argument is an adaptation for a single particle of his demonstration of nonlocality of a two-particle state without using inequalities [3]. Parallel to the controversy arisen around the interpretation of Hardy's single photon experiment [4], in a recent Letter Peres [5] has discussed the origin of such a nonlocality in the Fock space. In a way somewhat similar to the conclusion by Greenberger et al. [4], Peres claims that the latter is simply due to the creation of new particles by the required detection process itself. The detectors may also supply some of their own particles since the total number of particles is not conserved in the process [5].

In the present work we have transposed Peres' argument from the Fock space to an experimentally feasible scheme in cavity QED [6]. We thus suggest a new proposal demonstrating nonlocality of a single particle which is based on a model recently presented by Gerry [7]. A sketch of the experiment is displayed in Fig. 1. C_A and C_B are two identical and initially empty high-Q cavities which are prepared, by a two-level Rydberg atom (levels $|e\rangle$ and $|g\rangle$) and a Ramsey zone R, in a single-photon field state which couples them. Additional Rydberg atoms 1 and 2 are sent across cavities C_A and C_B, respectively, to read them out. The single-photon field state shared by the cavities is thus transferred to atoms 1 and 2 probing each cavity. Subsequently, atom α ($\alpha = 1,2$) is analyzed by a Ramsey zone R_α and a state-selective field ionization detector D_α revealing, as in Peres' argument, that the Clauser–Horne inequality [8] is violated.

First, the single-photon field state occupying simul-

[1] E-mail: miled@power.ufscar.br.

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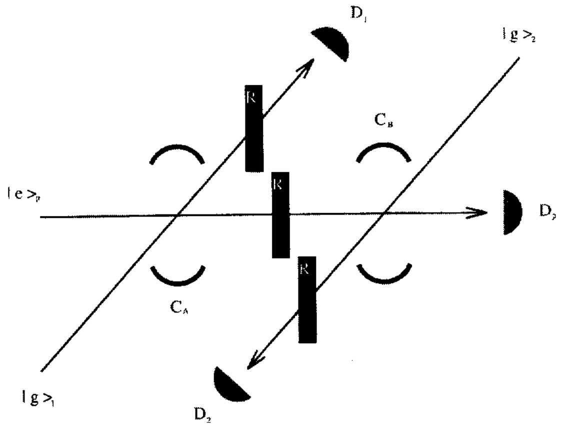

Fig. 1. Sketch of the experimental setup to demonstrate the nonlocality of a single photon.

taneously two cavities, C_A and C_B, is prepared. The preparing atom is initially laser excited to the state $|e\rangle_P$ and thus sent across the cavity pair, as indicated in Fig. 1. The transition frequency of the atomic Rydberg states is made resonant with the cavity field modes, and the preparing atom undergoes, on the $|e\rangle_P \longrightarrow |g\rangle_P$ transition, a $\frac{1}{2}\pi$ pulse in C_A and a π pulse in C_B. However, after crossing C_A the atom enters the Ramsey zone R which is set to produce a ϕ rotation around the z-axis. Therefore, the preparing atom exits the cavities in its ground state while the cavity pair is left in the single-photon field state

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|1\rangle_A|0\rangle_B + e^{i\phi}|0\rangle_A|1\rangle_B\right), \tag{1}$$

where the index A (B) refers to cavity C_A (C_B).

Once the preparing atom is detected in D, interrupting the source which has ejected it, atoms 1 and 2, initially prepared in their ground states $|g\rangle_1$ and $|g\rangle_2$, are injected in cavities A and B, respectively. Both atoms undergo, on the $|e\rangle \longrightarrow |g\rangle$ transition, a π pulse in their respective cavities, so that from the complete state vector of the system, $|\Psi\rangle_{AB}|g\rangle_1|g\rangle_2$, we obtain

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|e\rangle_1|g\rangle_2 + e^{i\phi}|g\rangle_1|e\rangle_2\right). \tag{2}$$

The atoms have thus carried the photon out from the cavity pair. It is interesting to note that the phase factor has also been transposed from $|\Psi\rangle_{AB}$ to $|\Psi\rangle_{12}$

Now, following the Peres' route to nonlocality, we have the choice of two different experiments realized on atoms 1 and 2. The first experiment is to test the projection operator $\hat{\mathcal{P}}_\alpha$ on the state $|e\rangle_\alpha$. We do not need the Ramsey zones R_α for these two measurements whose expected values for the state $|\Psi\rangle_{12}$ are

$$\langle\hat{\mathcal{P}}_\alpha\rangle = \tfrac{1}{2}, \tag{3a}$$

$$\langle\hat{\mathcal{P}}_1\hat{\mathcal{P}}_2\rangle = 0. \tag{3b}$$

Concerning the second experiment, we have to measure the projection operator $\hat{\mathcal{P}}'_\alpha$ on the coherent superposition of the atomic excited and ground states $a|e\rangle_\alpha + (-1)^\alpha b|g\rangle_\alpha$, with $|a|^2 + |b|^2 = 1$. For this purpose we need the Ramsey zones R_α in order to adjust the measurement of this special superposition. The Ramsey zones R_α play the role of atomic states "polarizers" and "analyzers" which, combined with detectors D_α, allow us to analyze an arbitrary superposition of $|e\rangle_\alpha$ and $|g\rangle_\alpha$. As considered by Freyberger [9], such a measurement works as follows: The Ramsey zone R_α is appropriately adjusted in a way that the two-level atom α which crosses it in the superposition $a|e\rangle_\alpha + (-1)^\alpha b|g\rangle_\alpha$ undergoes a unitary transformation to the state $|e\rangle_\alpha$. As a consequence, atom α leaves the Ramsey zone in the state $|g\rangle_\alpha$ if it crosses it in the state orthogonal to $a|e\rangle_\alpha + (-1)^\alpha b|g\rangle_\alpha$, namely $(-1)^\alpha b^*|e\rangle_\alpha - a^*|g\rangle_\alpha$. After crossing R_α, atom α is thus counted with high efficiency by the ionization detection chamber D_α. However, the two-level atom α will be in a superposition of these orthogonal states, so that in some cases the detector will deliver a click (measuring $|e\rangle_\alpha$) and in some not (measuring $|g\rangle_\alpha$). Once we register a click, atom α has been projected in the required superposition $a|e\rangle_\alpha + (-1)^\alpha b|g\rangle_\alpha$. So, the analyzer α is able to measure observables of the form $(a|e\rangle_\alpha + (-1)^\alpha b|g\rangle_\alpha)(a^*\,{}_\alpha\langle e| + (-1)^\alpha b^*\,{}_\alpha\langle g|)$, which represent projection operators with measurable eigenvalues 1 (click) and 0 (no click). The quantum theory prediction concerning these two last experiments gives us for the state $|\Psi\rangle_{12}$

$$\langle\hat{\mathcal{P}}'_\alpha\rangle = \tfrac{1}{2}, \tag{4a}$$

$$\langle\hat{\mathcal{P}}_1\hat{\mathcal{P}}'_2\rangle = \langle\hat{\mathcal{P}}'_1\hat{\mathcal{P}}_2\rangle = \tfrac{1}{2}|b|^2, \tag{4b}$$

$$\langle\hat{\mathcal{P}}'_1\hat{\mathcal{P}}'_2\rangle = |a|^2|b|^2\,(1 - \cos\phi). \tag{4c}$$

By substituting the results (3) and (4) into the Clauser–Horne inequality [8],

$$0 \leqslant \langle \hat{\mathcal{P}}_1' + \hat{\mathcal{P}}_2' - \hat{\mathcal{P}}_1'\hat{\mathcal{P}}_2' - \hat{\mathcal{P}}_1'\hat{\mathcal{P}}_2 - \hat{\mathcal{P}}_1\hat{\mathcal{P}}_2' + \hat{\mathcal{P}}_1\hat{\mathcal{P}}_2\rangle \leqslant 1, \quad (5)$$

we obtain the domain in which the adjustable phase shift ϕ has to be considered in order to violate the inequality (5), namely

$$\phi_0 \leqslant \phi \leqslant 2\pi - \phi_0. \quad (6)$$

where $\phi_0 = \cos^{-1}(1 - 1/|b|^2)$. The phase factor $e^{i\phi}$ is thus crucial for the state $|\Psi\rangle_{12}$ to reveal the single-photon nonlocality and the choice of the coefficient b determines the range of ϕ leading to the nonlocal behavior. Accordingly, the range of ϕ differs from zero when $b \in (1/\sqrt{2}, 1)$. For the sake of generality, even choosing the first experiment to test the arbitrary superposition $c|e\rangle_\alpha + (-1)^\alpha d|g\rangle_\alpha$, it is still possible to demonstrate that there will be a range to nonlocality concerning the values of ϕ. Our choice to test $\hat{\mathcal{P}}_\alpha$ on the state $|e\rangle$ is just to follow Peres' procedures.

Concerning the physical meaning of the relevant phase factor $e^{i\phi}$ we refer to Eq. (4c), from which we observe that the detection of the component $|e\rangle_1|e\rangle_2$ is phase-sensitive. The required coincident measurements depend on the interference effect coming from the coherence factor $e^{i\phi}$ which determines the possibility for the violation of the inequality (5).

Through the Peres' choice to test the projection operator $\hat{\mathcal{P}}_\alpha'$ on the state $1/2(|e\rangle_\alpha + (-1)^\alpha\sqrt{3}|g\rangle_\alpha)$, i.e., $|a|^2 = \frac{1}{4}$ and $|b|^2 = \frac{3}{4}$, results $2\pi - \phi_0 \geqslant \phi \geqslant \phi_0 \approx 110°$. When adjusting $\phi = \pi$, the state (1) turns out to be exactly the pure one-particle and maximally entangled state considered by Peres; hence, the actual value of expression (5) is -0.125.

It is worth stressing that in the four different experiments described above, we have mapped the one-particle (vacuum) state $|1\rangle$ ($|0\rangle$) of Peres' work in our atomic excited (ground) state $|e\rangle$ ($|g\rangle$). In fact, the entangled state $|\Psi\rangle_{12}$ is generated by having the atoms carry the single photon out from the cavity pair. So, in the present scheme the atomic entangled state $|\Psi\rangle_{12}$ represents the single-photon state in Peres' argument, once atoms 1 and 2 (through their atomic levels $|e\rangle$ and $|g\rangle$) are just playing the role of cavities A and B which store the photon in the entanglement $|\Psi\rangle_{AB}$. As a matter of fact, the four different experiments whose expected values are given in Eqs. (3) and (4) can be exactly interpreted as the expected values for measuring the single photon (the click in detectors D_α). Thus, the mapping process we have done maintains exactly Peres' conclusion, that the origin of the nonlocality of a single particle (our single photon) is "due to the creation of a two-particle (two-photon) component ($|e\rangle_1|e\rangle_2$) by the detection process itself". Of course, the creation of the component $|e\rangle_1|e\rangle_2$ is here due to the Ramsey zones R_1 and R_2, what provides the result (4c) and consequently the violation of the Clauser–Horne inequality.

We finally mention that, by considering the entanglement $|\Psi\rangle_{12}$ (also obtained from the nonlocal field state $|\Psi\rangle_{AB}$), Gerry has shown that Bell's inequalities are violated. Gerry's conclusion is that "it is apparent that the origin of the nonlocality (in $|\Psi\rangle_{12}$) can be attributed only to the nonlocality of the single-photon emitted by the preparing atom" (generating $|\Psi\rangle_{AB}$). In view of what we have discussed above, Gerry's conclusion does not apply exactly to the *nonlocality of a single photon in cavity QED*. Instead, it applies to such a subject just when added by Peres' conclusion; otherwise the violation of Bell's inequality when considering $|\Psi\rangle_{12}$ refers just to the usual two-particle EPR nonlocality [10]. As in the violation of the Clauser–Horne inequality described above, the violation of Bell's inequality presented by Gerry could be (somewhat) interpreted as being due to the *nonlocality of a single photon in cavity QED* only when considering that it is due to the creation of a component $|e\rangle_1|e\rangle_2$ by Gerry's Ramsey zones R_1 and R_2. In fact, for the violation of Bell's inequalities using entanglements of two-level atoms the measurement of the Pauli operators along any direction σ_r is carried out by applying a controlled pulse to the atom which transform the states $|e\rangle$ and $|g\rangle$ into the eigenstates of σ_r. A subsequent measurement of the state of the atom would give the desired result of the observable corresponding to σ_r [11,7]. Furthermore, it is evident from Gerry's paper that its Bell's inequality violation does not depend on the phase factor $e^{i\phi}$ which, as we have seen from Eq. (6), is crucial to reveal the single-photon nonlocality. Contrarily, even if Gerry had considered the atomic entangled state $(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2)/\sqrt{2}$ (obtained from the single-photon field state $(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B)/\sqrt{2}$) instead of $(|e\rangle_1|g\rangle_2 - |g\rangle_1|e\rangle_2)/\sqrt{2}$, the violation of Bell's inequality would just depend on a proper choice of the directions σ_r. Since the Clauser–Horne inequality is a variant of the Bell's, it is also expected through the latter that a range to the single-photon nonlocality must be found concerning the values of ϕ.

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