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# QUANTUM HARMONIC OSCILLATOR AND NONSTATIONARY CASIMIR EFFECT[1]

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**Abstract**

We consider the relations between the theory of quantum nonstationary damped oscillator and nonstationary Casimir effect in view of the problem of photon creation from vacuum inside the cavity with periodical time-dependent conductivity of a thin semiconductor boundary layer, which simulates periodical displacements of the cavity boundaries. We develop a consistent model of quantum damped harmonic oscillator with arbitrary time-dependent frequency and damping coefficients within the framework of Heisenberg–Langevin equations with two noncommuting delta-correlated noise operators. For the minimum noise set of correlation functions, whose time dependence follows that of the damping coefficient, we obtain the exact solution, which is a generalization of the Husimi solution for undamped nonstationary oscillator. It yields the general formula for the photon-generation rate under the resonance condition in the presence of dissipation. We obtain a simple approximate formula for a time-dependent shift of the complex resonance frequency. It depends only on the total energy of a short laser pulse (which creates an effective time-dependent electron–hole "plasma mirror" on the semiconductor-slab surface) and the recombination time. We show that damping due to a finite conductivity of the material significantly diminishes the photon-generation rate in the selected field mode of the cavity. Nonetheless, we have found optimum values of the parameters (laser pulse power, recombination time, and cavity dimensions), for which the effect of photon generation from vacuum could be observed in the experimental set-up proposed in the University of Padua. We also provide with a list of publications from 2001 to 2005 devoted to the study on quantum-field interactions with moving boundaries (mirrors).

**Keywords:** nonstationary Casimir effect, cavities with time-dependent nonideal boundaries, nonstationary quantum damped oscillator, parametric resonance, Heisenberg–Langevin equations, noncommuting noise operators, semiconductor materials, recombination time, ultrashort laser pulses.

## 1. Introduction: Beauty of Casimir Effect

In 1948, H. B. G. Casimir published the article [1] where he showed that quantum fluctuations of vacuum could result in the attractive force between neutral ideal metallic plates. (He remembered 50 years later [2] that a hint on the importance of zero-point energy in this case was given, although in a dim form, by N. Bohr in their conversation either in 1946 or 1947). Since that time, the stationary

[1] Dedicated to Prof. Vladimir I. Man'ko on the occasion of his 65th birthday.

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Casimir effect attracted attention of many physicists, both theoreticians and experimentalists (see the monography [3] and reviews [4–8]).

Now let us suppose that the plates or boundaries of some cavity can move. What could happen in this case? First of all, the Casimir force can be modified by some factor which depends, besides the instantaneous geometry (the distance between the plates), on the velocity or higher derivatives of the boundary positions. This is one of possible manifestations of the nonstationary Casimir effect (NSCE), which was considered by many authors since 1970s starting with papers by Moore [9] and Fulling and Davies [10], although this name appeared for the first time in [11, 12].

Another consequence of changing the quantum state of the electromagnetic field between plates or inside a cavity due to the motion of boundaries is a possibility to generate quanta of electromagnetic field (photons) from vacuum. This effect was considered for the first time by Moore [9] for a simple one-dimensional model of "scalar electrodynamics" and since that time it was also discussed by many authors. However, during the first two decades NSCE was considered only as a curious consequence of quantum theory, which could help to understand fundamentals of the theory but hardly could be observed in real life. The situation began to change slowly since 1990s, when the idea of a possible significant amplification of the effect in a cavity with oscillating boundaries and under the resonance condition was put forward in publications of the Lebedev Institute group [13–18] (being actively propagandized by Prof. V. I. Man'ko at several conferences [19–25]), as well as some other groups [26–29]. In this connection, it should be noted that a possibility of strong amplification of the classical field inside a laser cavity with resonantly oscillating mirrors was predicted by Askar'yan at the Lebedev Institute in 1962 [30].

As a matter of fact, the study of classical fields in cavities with moving boundaries began as far back as in 1920s by Nicolai [31, 32] and Havelock [33]. However, this field of mathematical physics did not attract much attention before 1960s. The new splash of interest arose only after the experiments performed at that period in two areas — plasma physics [34] (experiments on "field compression" accompanied by the frequency multiplication due to multiple reflections of the initial electromagnetic wave with $\lambda_{\rm in} = 10\,$cm from the opposite sides of a resonator, one of whose walls was a "plasma piston" moving uniformly with the velocity $v \sim 2 \cdot 10^7\,$cm/s) and laser physics (experiments on laser resonators with vibrating [35] or uniformly moving [36, 37] mirrors). These works stimulated the theoretical study of classical field dynamics in the presence of moving boundaries [38–40]; the most important results for different geometries were obtained in [41–49]. We shall discuss in this paper how the excitation of electron–hole plasma in semiconductors by short laser pulses can help in performing the experiment on generating quanta of electromagnetic field (with wavelength $\lambda \sim 10\,$cm) from vacuum in cavities with time-dependent effective mirrors.

There are several different approaches to nonstationary Casimir effect (frequently called also dynamical Casimir effect after the papers [50, 51]), whose descriptions can be found, e.g., in [52–58]. In the case of closed cavities, the most simple approach consists in using the mode decomposition. It is well known that the field inside the cavity with a fixed geometry and ideal boundaries (i.e., in the absence of absorption, damping, leakage, and so on) can be expanded in a series of the complete orthonormal set of stationary eigenmodes with time-dependent coefficients. The evolution of these coefficients is governed by the Hamiltonian, which has the same form as for the set of uncoupled harmonic oscillators. This fact opens the direct way to quantize electromagnetic field inside the cavity. What happens if the cavity shape depends on time (and the boundaries continue to be ideal)? Obviously, at each time instant the field can be expressed as a series with respect to the complete set of instantaneous "eigenmodes." Putting this expansion into the partial differential equations, which govern the field evolution (wave equation in the simplest case) and using the property of orthogonality of instantaneous modes, one can obtain the set of ordinary differential equations for time-dependent coefficients of the decomposition. This approach was proposed by Grinberg [59] in the classical case and independently by Razavy [60,61] in the quantum field case (where coefficients of the expansion are treated as the Heisenberg operators).

In the generic case, one arrives at an infinite set of coupled nonlinear equations. However, for small and slow (compared with the speed of light) displacements of the boundaries, nonlinear terms can be neglected, so that it becomes possible to describe the field behavior with the aid of some effective Hamiltonian, which is infinite-dimension quadratic form of the boson creation/annihilation operators with time-dependent coefficients responsible for the coupling between different modes [62–68]. This Hamiltonian is also complicated, because all modes are coupled and the time-dependent coupling coefficients (which are proportional to the boundary velocity in the simplest case) have a rather complicated dependence on the mode numbers. However, as was shown in [63–68], it can be essentially simplified in the resonance case where the boundary performs harmonic oscillations at the frequency, which is a multiple of the frequency of some field mode. Then the long-time evolution is governed by a reduced effective Hamiltonian, which couples only those modes which are in resonance with the field and with each other (the difference of eigenfrequencies of "survived" modes must be also a multiple of the boundary-oscillation frequency). In many cases, the equations of motion resulting from (or equivalent to) the reduced Hamiltonian can be solved analytically. The most impressive example is the one-dimensional model where the number of resonant modes is infinite (because the unperturbed spectrum of field eigenfrequencies is equidistant) but, nonetheless, closed analytical solutions can be obtained and analyzed due to a particular form of the coupling coefficients [18, 69–71]. Another interesting special case of two resonantly coupled modes was considered in [72, 73].

If the eigenmode spectrum has no equidistant blocks (as happens for real three-dimensional cavities without some accidental symmetry), then it seems reasonable to suppose that only one mode is in resonance with the oscillating boundary. Then the effective Hamiltonian is reduced to that of a one-dimensional harmonic oscillator with a time-dependent frequency. This simple idea was suggested by V. I. Man'ko in [14,19–22] and later by other authors [74]. Actually, this is the hypothesis, which still needs the specification and justification. Some justifications at the physical level were given in [17, 18, 65] for harmonic oscillations of the boundary. In this paper, we assume that the model of a one-dimensional time-dependent oscillator can be used even for arbitrary periodical motion of nonideal boundaries. Our main goals are: (1) to find the conditions of photon generation from vacuum in the cavity, whose parameters periodically change with time and (2) to find the optimum set of parameters, which would permit one to create the maximum number of photons at minimum expenses, within the framework of a concrete experimental set-up developed in the University of Padua [75].

The paper is organized as follows.

In Sec. 2, we briefly discuss the problem of a time-dependent oscillator in unitary quantum mechanics and its consequences with respect to the nonstationary Casimir effect. In Sec. 3, we propose a new model of the quantum damped oscillator with arbitrary time-dependent frequency and damping coefficient, which is based on the concept of "minimum noise operators." Then we give the exact solution of the model, which turns out to be a nice generalization of the remarkable Husimi solution in the unitary case. In Sec. 4, we apply the general solution to the special case of a periodically varying frequency with a small modulation depth and a periodically varying damping coefficient, making some simplification and approximations, which are justified in the NSCE case. In Sec. 5, we calculate the complex shift of the fundamental-mode eigenfrequency in the cylindrical cavity due to the presence of a thin nonuniform (in the normal direction) dielectric slab, whose complex dielectric permeability assumes very big values on the surface but rapidly decreases to a constant value inside the slab (as happens in materials with a high absorption coefficient of laser radiation). We obtain simple approximate analytical expressions, which hold in the case of short laser pulses and short recombination times in the semiconductor. Using these expressions, we calculate the photon-generation rate under the conditions of the Padua experiment and analyze how this rate depends on different parameters. Section 6 contains conclusions. In Appendix we give a brief description of publications on the cavities with moving boundaries for the period from 2001 to 2005, in order to amend the previous review [76], which contained references to publications from 1921 to 2000.

## 2. Quantum Time-Dependent Oscillator and Nonstationary Casimir Effect

The first important contribution to the theory of nonstationary quantum harmonic oscillator with arbitrary time-dependent frequency and external time-dependent force was made in 1953 by Husimi in the seminal paper [77]. In particular, he found a general solution of the time-dependent Schrödinger equation and calculated transition amplitudes and probabilities between the initial and final number states. His results were much more general than those obtained by Feynman [78] and Schwinger [79], who considered only the case of time-dependent force. Perhaps, it is worth mentioning here two other papers of the series on "miscellanea in quantum mechanics" [80, 81]. In the first paper, Husimi found interesting one-parameter families of time-dependent Gaussian states, which are, from the modern point of view, nothing but the squeezed states of the harmonic oscillator and the charged particle in homogeneous stationary magnetic field. The second paper (the less known of the series but not less brilliant) was devoted to nontrivial generalization of the uncertainty relations and their applications to the harmonic oscillator, hydrogen atom, and other potentials. Later similar relations were obtained by Bargmann [82], Faris [83], and others (see review [84]).

After Husimi, the problem of quantum oscillator with time-dependent parameters was studied in numerous publications, which shed additional light on different aspects of the problem. In particular, the relations between the Husimi solution and quantum time-dependent quadratic integrals of motion and linear integrals of motion were established in the series of fundamental papers by Lewis and Riesenfeld [85] and Malkin and Man'ko with coauthors [86–90], respectively. Extensive lists of references can be found in [91, 92].

The most fascinating feature of Husimi solution is that it shows that all dynamical properties of the quantum oscillator are determined by the fundamental set of solutions of the classical equation of motion

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0. \tag{1}$$

In particular, if $\omega(t) = \omega_i$ for $t \to -\infty$ and $\omega(t) = \omega_f$ for $t \to \infty$, then information on the final state of the quantum oscillator, which was initially in the vacuum state, is encripted in complex coefficients $\rho_\pm$ of the asymptotical form of the solution

$$\varepsilon_{t\to\infty} = \omega_f^{-1/2}\left[\rho_- e^{-i\omega_f t} + \rho_+ e^{i\omega_f t}\right], \tag{2}$$

satisfying the initial condition $\varepsilon_{t\to-\infty} = \omega_i^{-1/2} e^{-i\omega_i t}$.

In the limit $t \to +\infty$, the mean number of quanta in the state, which was the vacuum state at $t \to -\infty$, equals [77, 90, 91]

$$\mathcal{N} = \frac{|\dot{\varepsilon}|^2 + \omega_f^2 |\varepsilon|^2}{4\omega_f} - \frac{1}{2}\,. \tag{3}$$

It can be also expressed in terms of $\rho_\pm$ as follows [93]:

$$\mathcal{N} = |\rho_+|^2 = \frac{R}{T}\,, \tag{4}$$

where the quantities

$$R \equiv \left|\frac{\rho_+}{\rho_-}\right|^2, \qquad T \equiv 1 - R \equiv |\rho_-|^{-2} \tag{5}$$

can be interpreted as the energy reflection and transmission coefficients from an effective "potential barrier" $\omega^2(t)$ [86] and the identity $|\rho_-|^2 - |\rho_+|^2 = 1$ holds due to the unitary evolution.

Now, using formulas (3) or (4) and identifying the time-dependent frequency $\omega(t)$ in Eq. (1) with the field eigenfrequency corresponding to "instantaneous" geometry of the cavity, one can calculate the number of photons created in selected *TE*-mode due to nonstationary Casimir effect for any law of the boundary motion [17, 18, 74, 93]. (The case of other polarizations is more complicated [94, 95] and it is not considered here.)

It is important that the effective reflection coefficient $R_1$ corresponding to a single cycle of the boundary motion cannot exceed a value of the order of $(\Delta L/L)^2$, where $\Delta L$ is the maximum displacement of the boundary and $L$ is the average distance between moving walls. Under any realistic assumptions $(\Delta L/L)^2 \ll 1$, consequently, no photons can be created during the single cycle.

However, if one can arrange many periodical cycles with period $T$, then the total reflection coefficient can be made very close to unity due to interference. In this case, the mean number of photons created from the initial vacuum state after $n$ identical pulses of arbitrary shape (with $\omega_i = \omega_f = \omega_0$) was calculated in [96] (see Sec. 4. for details of calculations):

$$\mathcal{N}_n = |rf|^2 \frac{\sinh^2(n\nu)}{\sinh^2(\nu)}, \quad \cosh\nu \equiv \pm \mathrm{Re}\left(fe^{i\theta}\right). \tag{6}$$

In formula (6) $r$ is the complex amplitude reflection coefficient from a single "potential barrier" (corresponding to one cycle of the motion), $f \equiv |f|e^{i\varphi}$ is the inverse amplitude transmission coefficient from the single barrier ($|f| > 1$), and $\theta = \omega_0 T$ (it is assumed that each cycle contains some interval of time, when $\omega(t) = \omega_0 = \text{const}$). For the initial thermal state with $\langle n\rangle_{\text{th}}$ quanta, the increase in the mean number of quanta is given by the same formula (6) with the additional amplification factor $(1 + 2\langle n\rangle_{\text{th}})$ on the right-hand side of the first equation [97]. The generation of photons is possible, if parameter $\nu$ is real, i.e., $|f|\,|\cos(\varphi+\theta)| > 1$. Adjusting the phases in such a way that $\theta + \varphi = \pi m$ (with $m$ an integer), we obtain $\nu \approx |r|$ (for $|r| \ll 1$). Thus the number of photons which can be created under the ideal resonance condition equals

$$\mathcal{N}_n^{(\text{res})} = \sinh^2(n\nu)\,. \tag{7}$$

The resonance condition can be written as the relation between the periodicity of pulses $T = \theta/\omega_0$, the period of electromagnetic field oscillations $T_0 = 2\pi/\omega_0$, and phase $\varphi$:

$$T = \frac{1}{2} T_0 \left(m - \frac{\varphi}{\pi}\right). \tag{8}$$

Unfortunately, parameter $\nu$ is very small for real oscillating boundaries; it cannot exceed the value $\nu_{\max}{\sim}10^{-8}$ [17, 18], otherwise deformations inside the oscillating wall exceed the limit of material destruction. This means that the necessary number of cycles must be very big, which imposes hard restrictions on the cavity quality factor and admissible detuning from the exact resonance. Moreover, it is still unclear, how to excite the boundary oscillations of required amplitude at frequency of the order of GHz or higher.

These difficulties can be partially avoided in the experimental set-up recently proposed in the University of Padua [75]. The main idea is to use, instead of a real moving metallic surface, some effective electron–hole "plasma mirror" which can be periodically created on the surface of a semiconductor slab by illuminating with a sequence of femtosecond laser pulses. If the interval between the pulses exceeds the recombination time of carriers in the semiconductor, a highly-conducting layer will periodically appear and disappear on the semiconductor-film surface, thus simulating periodical displacements of the boundary. Using the standard GaAs or Si plates of thickness of 600 $\mu$m – 1 mm, one can obtain a relative change of the field mode eigenfrequency inside the cavity by several orders of magnitude bigger than in the case of real metallic vibrating boundary. As a consequence, one can reduce in the same proportion the number of boundary oscillations necessary to produce photons in the cavity, thus relaxing the requirements for the cavity $Q$-factor and admissible detuning from the exact resonance. Another advantage is a possibility to use periodical pulses of arbitrary shape, whose repetition frequency can be significantly less than the frequency of a chosen mode of electromagnetic field [95].

Note that the laser wavelength $\lambda_{\rm las}$ is of the order of 1 $\mu$m, while the wavelength $\lambda_{\rm cav}$ of the fundamental cavity mode, which is supposed to be excited due to NSCE, is $\approx$10 cm. Consequently, if the antenna put somewhere inside the cavity and tuned on the resonance frequency 5 GHz will register a strong signal after the set of laser pulses, one can be sure that quanta of electromagnetic field in the fundamental mode were created due to NSCE and they do not belong to some "tail" of the laser pulse, just due to the difference by five orders of magnitude between $\lambda_{\rm las}$ and $\lambda_{\rm cav}$.

The idea to simulate the so-called Unruh effect and "nonadiabatic" Casimir effect in a medium with a rapidly decreasing-in-time refractive index ("plasma window") was put forward for the first time by Yablonovitch [98]. Following this idea V. I. Man'ko proposed [20] to use semiconductors with time-dependent properties to produce an analogue of nonstationary Casimir effect (similar ideas were discussed, e.g., in [99,100]). A more developed scheme based on the creation of electron–hole "plasma mirror" inside a semiconductor slab illuminated by a femtosecond laser pulse was proposed in [101] (in the single-pulse case). But only recently the possibility of creating the effective "plasma mirror" in the semiconductor slab was confirmed experimentally [102]. (Real plasma mirrors created by powerful ultrashort laser pulses were studied in [103].)

However, dealing with the semiconductor mirror one meets the challenging theoretical problem due to the fact that the dielectric permeability $\epsilon(x)$ of a conducting medium (semiconductor slab after excitation by a laser pulse) is a complex function $\epsilon$,

$$\epsilon = \epsilon_1 + i\epsilon_2, \qquad \epsilon_2 = \frac{2\sigma}{f_0}\,, \tag{9}$$

where $\sigma$ and $f_0$ are the conductivity (in the CGS units) and frequency in Hz, respectively. For example, $\epsilon_2{\sim}10^8$ for Cu at 2.5 GHz. This means that the time-dependent shift of the cavity resonance frequency is not real but a complex function. At the same time, all existing models of photon generation in cavities with time-dependent parameters [76] were based on the assumption that boundaries are made either of the ideal conductor or dielectric material with real $\epsilon$, so that the quantum evolution is unitary. But this is not the case for the Padua experiment. Although the conductivity can be very small for the non-illuminated semiconductor slab at low temperatures, and it can be very high after the illumination (when high carrier concentrations are achieved), it inevitably passes through intermediate values during the process of excitation and recombination, when $\varepsilon_2$ gradually increases from very low to very high values and after that returns to the initial value. Thus $\epsilon_2$ has the same order of magnitude as $\epsilon_1$ during some intervals of time. As a consequence, the time-dependent instantaneous eigenfrequency of the field mode becomes complex

$$\Omega(t) = \omega(t) - i\gamma(t),$$

moreover, the time-dependent damping coefficient $\gamma(t)$ can be of the same order of magnitude as the frequency variation $\omega(t) - \omega_i$ [95]. This means that, in order to make correct predictions of the results of planned-in-Padua experiment, we have to find a generalization of Husimi formula (3) to nonunitary case, i.e., for the quantum oscillator with arbitrary time-dependent frequency $\omega(t)$ and nonzero time-dependent damping coefficient $\gamma(t)$.

## 3. Model of Quantum Damped Oscillator with "Minimum Noise"

Thus we arrive at the problem of "quantization" of the classical harmonic oscillator with time-dependent complex frequency (whose imaginary part is interpreted as the damping coefficient). In principle, it is well known that the most correct and consistent treatment of dissipative systems consists in the "microscopic" approach. Since the dissipation arises due to the coupling of the system under study with many degrees of freedom of some reservoir ("the rest of the world"), one has to consider this system as a small part of some large system, to write down the interaction Hamiltonian, to try to solve the arising dynamical equation, and to trace out the variables of the "rest of the world" at the final stage. The field quantization in lossy dielectrics with time-independent parameters was considered within such framework, e.g., in [104–107], and the case of nonstationary media was discussed in [108] (for other treatments see, e.g., [109–111]). Unfortunately, it is very difficult to follow this way in the case of electromagnetic field in the cavity with moving or nonstationary boundaries. The only known attempt is [112]. Even writing the realistic interaction Hamiltonian is a difficult problem, which was not considered anywhere until now in nonstationary case. Therefore we have to choose a simplified phenomenological approach.

Our first assumption is that we can still reduce the field problem to the dynamics of a single selected mode described in the classical limit as a harmonic oscillator with time-dependent complex frequency

$$\Omega(t) = \omega(t) - i\gamma(t),$$

which can be obtained from the solution to the classical electrodynamical problem of finding eigenfrequencies in a cavity with given instantaneous geometry and material properties. Since this scheme works in the unitary case, we suppose that it can be also used at least for a weak dissipation, which is always implied (obviously, there is no chance to observe NSCE in the case of strong dissipation).

A very nice solution of the problem consistent with principles of quantum mechanics can be obtained, if one follows the quantum noise operator approach proposed as far back as in the beginning of 1960s by several authors [113–116] (for recent applications of the approach see, e.g., [117–119]). It consists in describing the dissipative quantum system within the framework of Heisenberg–Langevin operator equations. In this case, the equations can be written as follows:

$$\frac{d\hat{x}}{dt} = \hat{p} - \gamma(t)\hat{x} + \hat{F}_x(t), \tag{10}$$

$$\frac{d\hat{p}}{dt} = -\gamma(t)\hat{p} - \omega^2(t)\hat{x} + \hat{F}_p(t), \tag{11}$$

where $\hat{x}$ and $\hat{p}$ are the quadrature operators of selected mode. To simplify the formulas, we normalize these operators by the initial frequency in such a way that they are dimensionless and the mean number of photons equals

$$\mathcal{N} = \frac{1}{2}\langle \hat{p}^2 + \hat{x}^2 - 1\rangle.$$

In other words, in the subsequent formulas $\omega$ and $\gamma$ are the frequency and damping coefficient normalized by the initial frequency $\omega_i$.

The choice of equal coefficients at the damping terms $-\gamma\hat{x}$ and $-\gamma\hat{p}$ in Eqs. (10) and (11) is motivated by the following reason. In principle, one could introduce two coefficients $\gamma_x$ and $\gamma_p$, whose sum equals exactly $2\gamma$. For example, in the case of "material" oscillator (a point mass attached to a spring), the choice $\gamma_x = 0$ seems to be the most natural. But in the case of "optical" oscillator, the quadrature components have no direct physical meaning and namely symmetrical choice of damping coefficients is the most natural [120]. Two constant damping coefficients were introduced in [121] where the effect of damping and fluctuations was described within the framework of Fokker–Planck equation for the Wigner function $W(x,p,t)$ of the form

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial x}\left([\gamma_x x - p]W\right) + \frac{\partial}{\partial p}\left([\gamma_p p + \omega^2(t)x]W\right) + D_{xx}\frac{\partial^2 W}{\partial x^2} + D_{pp}\frac{\partial^2 W}{\partial p^2} + 2D_{px}\frac{\partial^2 W}{\partial p \partial x}. \tag{12}$$

However, the approximate solution of Eq. (12) found in [121] for the harmonically changing-in-time frequency

$$\omega(t) = \omega_0[1 + 2\epsilon\sin(2\omega_0 t)]$$

(i.e., under the resonance condition) in the case of small damping and small modulation depth ($\gamma_x \ll 1$, $\gamma_p \ll 1$, $|\epsilon| \ll 1$) contained only the sum of damping coefficients $\gamma_x + \gamma_p = 2\gamma$. This observation allows one to suppose that, in the limit of small damping (and small frequency variation), the concrete choice of coefficients $\gamma_x$ and $\gamma_p$ is not significant for arbitrary functions $\omega(t)$ and $\gamma(t)$, and one can choose $\gamma_x$ and $\gamma_p$ in the form which is most convenient for calculations. (This is not true, if damping is not small [122,123].) Another justification of symmetrical choice of damping coefficients is our intention to use the minimum possible number of parameters. This is especially important for the application to NSCE where $\gamma(t)$ can be quite arbitrary function of time (because admitting that $\gamma_x$ can be different from $\gamma_p$, one can take, in principle, any function $\gamma_x(t)$ satisfying the condition $2\gamma(t) - \gamma_x(t) \geq 0$). We shall show soon how the choice of equal damping coefficients permits one to reduce Eqs. (10) and (11) to Husimi problem (1) in the most direct way.

The noncommuting noise operators $\hat{F}_x(t)$ and $\hat{F}_p(t)$ in Eqs. (10) and (11) are assumed to be delta-correlated:

$$\langle \hat{F}_j(t)\hat{F}_k(t')\rangle = \delta(t-t')\chi_{jk}(t), \qquad j,k = x,p, \tag{13}$$

where functions $\chi_{jk}(t)$ will be determined from the condition of "minimum noise" (see below). Equation (13) means that we use the Markov approximation (again due to two obvious reasons — to obtain the solution in the most simple way and to avoid introducing extra parameters, such as memory times). A crucial point is the presence of two noise operators, because the usual classical Langevin equations contain only one stochastic force. But here the quantum nature manifests itself, because one cannot preserve the canonical commutation relations, using only one noise operator [92, 115, 116, 123]. The noise operators commute with $\hat{x}$ and $\hat{p}$.

Although Eqs. (10) and (11) with condition (13) were studied in numerous papers, only the case of constant coefficients was considered until now. Here we obtain the exact solution for arbitrary time-dependent coefficients. Making the substitution

$$\hat{x}(t) = \hat{\xi}(t)e^{-\Gamma(t)}, \qquad \hat{p}(t) = \hat{\eta}(t)e^{-\Gamma(t)}, \tag{14}$$

with

$$\Gamma(t) = \int_{-\infty}^{t} \gamma(\tau)d\tau, \tag{15}$$

we can eliminate "friction" terms in Eqs. (10) and (11) reducing them to the set of equations for a time-dependent forced oscillator without damping (namely at this point the choice of "symmetric" damping shows its great advantage):

$$\frac{d\hat{\xi}}{dt} = \hat{\eta} + \hat{F}_x(t)e^{\Gamma(t)}, \tag{16}$$

$$\frac{d\hat{\eta}}{dt} = -\omega^2(t)\hat{\xi} + \hat{F}_p(t)e^{\Gamma(t)}. \tag{17}$$

Excluding the variable $\hat{\eta}$ we arrive at the equation, whose homogeneous part coincides exactly with Eq. (1). Thus the solution of inhomogeneous equations (16) and (17) can be expressed in terms of the fundamental set of solutions of Eq. (1) and their convolutions with the noise operators. Hereafter, we use the symbol $\varepsilon(t)$ for the solution of Eq. (1), which satisfies the initial condition $\varepsilon(t) = \exp(-it)$ for $t \to -\infty$. This is equivalent to fixing the value of Wronskian:

$$\varepsilon\dot{\varepsilon}^* - \dot{\varepsilon}\varepsilon^* = 2i \tag{18}$$

(note that we use here the function which is complex conjugated to that used in [73, 86, 88–93]). Now we can express the Heisenberg operators $\hat{x}(t)$ and $\hat{p}(t)$ in terms of the initial operators $\hat{x}(-\infty) \equiv \hat{x}_0$, $\hat{p}(-\infty) \equiv \hat{p}_0$, the function $\varepsilon(t)$, and integrals of the noise operators as follows:

$$\hat{x}(t) = e^{-\Gamma(t)}\Big\{\hat{x}_0 \mathrm{Re}\,[\varepsilon(t)] - \hat{p}_0\ \mathrm{Im}\,[\varepsilon(t)]\Big\} + \hat{X}(t), \tag{19}$$

$$\hat{p}(t) = e^{-\Gamma(t)}\Big\{\hat{x}_0\ \mathrm{Re}\,[\dot{\varepsilon}(t)] - \hat{p}_0\ \mathrm{Im}\,[\dot{\varepsilon}(t)]\Big\} + \hat{P}(t), \tag{20}$$

where

$$\hat{X}(t) = e^{-\Gamma(t)}\mathrm{Im}\left(\varepsilon^*(t)\int_{-\infty}^{t} e^{\Gamma(\tau)}\left[\hat{F}_p(\tau)\varepsilon(\tau) - \hat{F}_x(\tau)\dot{\varepsilon}(\tau)\right]d\tau\right), \tag{21}$$

$$\hat{P}(t) = e^{-\Gamma(t)}\mathrm{Im}\left(\dot{\varepsilon}^*(t)\int_{-\infty}^{t} e^{\Gamma(\tau)}\left[\hat{F}_p(\tau)\varepsilon(\tau) - \hat{F}_x(\tau)\dot{\varepsilon}(\tau)\right]d\tau\right). \tag{22}$$

The consequence of Eqs. (19) and (20) together with Wronskian (18) is the formula

$$[\hat{x}(t), \hat{p}(t)] = i\hbar e^{-2\Gamma(t)} + \left[\hat{X}(t), \hat{P}(t)\right]. \tag{23}$$

The commutator $\left[\hat{X}(t), \hat{P}(t)\right]$ can be calculated with the aid of Eqs. (13), (21), and (22). Using again several times the important Wronskian from (18), we find

$$\left[\hat{X}(t), \hat{P}(t)\right] = e^{-2\Gamma(t)} \int_{-\infty}^{t} \left[\chi_{xp}(\tau) - \chi_{px}(\tau)\right] e^{2\Gamma(\tau)} d\tau. \tag{24}$$

If we assume now that

$$\chi_{xp}(t) - \chi_{px}(t) = 2i\dot{\Gamma}(t) \equiv 2i\gamma(t), \tag{25}$$

then

$$\left[\hat{X}(t), \hat{P}(t)\right] = i\left[1 - e^{-2\Gamma(t)}\right], \tag{26}$$

and the commutator $[\hat{x}(t), \hat{p}(t)] = i$ is preserved exactly for any function $\gamma(t)$ (since we use dimensionless variables, the Planck's constant is equal formally to 1). To make the simplest choice of coefficients $\chi_{jk}(t)$, let us calculate the mean value $\langle \hat{X}^2(t)\rangle$

$$\begin{aligned}\langle \hat{X}^2(t)\rangle \;=\; & \frac{1}{2}e^{-2\Gamma(t)} \int_{-\infty}^{t} d\tau\, e^{2\Gamma(\tau)} \Bigg( \left[\chi_{xp}(\tau) + \chi_{px}(\tau)\right] \mathrm{Re}\left[\varepsilon^2(t)\varepsilon^*(\tau)\dot{\varepsilon}^*(\tau) - |\varepsilon(t)|^2\varepsilon^*(\tau)\dot{\varepsilon}(\tau)\right] \\ & + \chi_{xx}(\tau)\left\{|\varepsilon(t)\dot{\varepsilon}(\tau)|^2 - \mathrm{Re}\left(\left[\varepsilon(t)\dot{\varepsilon}^*(\tau)\right]^2\right)\right\} + \chi_{pp}(\tau)\left\{|\varepsilon(t)\varepsilon(\tau)|^2 - \mathrm{Re}\left(\left[\varepsilon(t)\varepsilon^*(\tau)\right]^2\right)\right\}\Bigg).\end{aligned} \tag{27}$$

The mean value $\langle \hat{P}^2(t)\rangle$ has the same form, with only difference that function $\varepsilon(t)$ should be replaced by its derivative $\dot{\varepsilon}(t)$, without changes in the functions which depend on the integration variable $\tau$. In the special case $\omega(t) = \text{const} = 1$, we have $\varepsilon(t) = \exp(-it)$, and (27) is simplified as follows:

$$\begin{aligned}\langle \hat{X}^2(t)\rangle \;=\; & \frac{1}{2}e^{-2\Gamma(t)} \int_{-\infty}^{t} d\tau e^{2\Gamma(\tau)}\Big\{ \left[\chi_{xx}(\tau) - \chi_{pp}(\tau)\right]\cos[2(t-\tau)] + \left[\chi_{xp}(\tau) + \chi_{px}(\tau)\right]\sin[2(t-\tau)] \\ & + \chi_{pp}(\tau) + \chi_{xx}(\tau)\Big\}.\end{aligned} \tag{28}$$

Choosing $\chi_{pp}(\tau) = \chi_{xx}(\tau)$ and $\chi_{px}(\tau) = -\chi_{xp}(\tau)$, we can remove oscillating terms in the integrand of (28). Thus we postulate the following set of coefficients $\chi_{jk}(t)$ (remind the normalization by the initial frequency):

$$\chi_{pp}(t) = \chi_{xx}(t) = \gamma(t)G, \qquad G = 1 + 2\langle n\rangle_{\text{th}}, \tag{29}$$

$$\chi_{xp}(t) = -\chi_{px}(t) = i\gamma(t), \tag{30}$$

where $\langle n\rangle_{\text{th}}$ is the equilibrium mean number of photons in the selected mode at given temperature. This choice ensures that the asymptotical mean number of quanta $\frac{1}{2}\langle \hat{X}^2(\infty) + \hat{P}^2(\infty) - 1\rangle$ coincides with $\langle n\rangle_{\text{th}}$ (if $\omega(t) = \text{const}$). We call the set of coefficients (29) and (30) as the "minimum noise" set, because we did not introduce any additional parameter besides the time-dependent damping coefficient and temperature.

Supposing that Eqs. (29) and (30) hold for an arbitrary function $\omega(t)$ (which returns to the initial value at $t \to \infty$), we arrive at the following generalization of Husimi formula (3) in the presence of dissipation with "minimum noise":

$$\mathcal{N}(t) = Ge^{-2\Gamma(t)}\left\{\frac{1}{2}E(t) + \int_{-\infty}^{t} d\tau e^{2\Gamma(\tau)}\gamma(\tau)\left(E(t)E(\tau) - \mathrm{Re}\left[\tilde{E}^*(t)\tilde{E}(\tau)\right]\right)\right\} - \frac{1}{2}, \tag{31}$$

where

$$E(\tau) = \frac{1}{2}\left[|\varepsilon(\tau)|^2 + |\dot{\varepsilon}(\tau)|^2\right], \qquad \tilde{E}(\tau) = \frac{1}{2}\left[\varepsilon^2(\tau) + \dot{\varepsilon}^2(\tau)\right]. \tag{32}$$

This is the first main result of the paper. It is exact and holds for arbitrary functions $\omega(t)$ and $\gamma(t)$. Its beauty consists in the fact that function $\varepsilon(t)$ is the same as in non-dissipative case. To find the influence of time-dependent damping, one has to calculate only the integrals of functions $E(\tau)$ and $\tilde{E}(\tau)$ (which are expressed through the function $\varepsilon(\tau)$ and its derivative) multiplied by the derivative of $\exp[2\Gamma(t)]$.

## 4. Periodical Variations of the Frequency Shift and Damping Coefficient

To apply the results of the preceding section to the NSCE problem, we must know, first of all, the functions $\omega(t)$ and $\gamma(t)$ corresponding to real experimental conditions. To find them, one has to make the following steps:

(i) To calculate the space–time dependence of the carrier concentration in a semiconductor slab illuminated by the laser-radiation pulse of given shape and to determine the complex dielectric function $\epsilon(\mathbf{r}, t)$.

(ii) To calculate the complex time-dependent frequency of a selected mode $\Omega(t) = \omega(t) - i\gamma(t)$ solving the Maxwell equations in the empty part of the cavity and inside the slab, in view of the continuity conditions at the slab surface.

After that, one has to solve the classical oscillator equation (1) with time-dependent frequency $\omega(t)$ and to calculate the integrals containing $\gamma(t)$ in (31). In the most general case, obviously, all these intermediate problems can be solved only numerically. However, under certain conditions, which are fulfilled in the NSCE case with a reasonable accuracy, one can obtain sufficiently good approximation for each step.

We suppose that the functions $\omega(t)$ and $\gamma(t)$ have the form of periodical pulses separated by time intervals when $\omega = 1$ (remind the frequency normalization by the initial value) and $\gamma = 0$ (this means that we neglect the field damping due to the finite quality factor of the cavity, supposing that it is under our control and can be made sufficiently big). Note that a relative change of the frequency $\omega(t)$ during pulses is very small, being of the order of $|\Delta L|/L$ in the best case (really, it is less, as was shown in [95]). This observation has very important consequences, i.e., functions $E(\tau)$ and $\tilde{E}(\tau)$ defined by Eq. (32) can be considered as approximate integrals of motion (they are exact integrals of motion if $\omega \equiv 1$) during each pulse. Of course, their values change with time but these variations are so slow, that we can assume that $E(\tau)$ and $\tilde{E}(\tau)$ are only slightly different for various pulses, while being constant during each pulse. During the $k$th pulse we can replace $E(\tau)$ and $\tilde{E}(\tau)$ in the integrand of (31) by their values $E_k$ and $\tilde{E}_k$ taken, say, at the pulse end. Then the integral over the pulse period is calculated exactly (since $\gamma(\tau) = d\Gamma/dt$) and it becomes the sum, so that after $n$ pulses we have

$$\mathcal{N}_n = Ge^{-2\Lambda n}\left\{\frac{1}{2}E_n + \left(1 - e^{-2\Lambda}\right)\sum_{k=1}^{n} e^{2\Lambda k}\left[E_n E_k - \mathrm{Re}\left(\tilde{E}_n^* \tilde{E}_k\right)\right]\right\} - \frac{1}{2}, \tag{33}$$

where

$$\Lambda = \int_{t_i}^{t_f} \gamma(\tau)\, d\tau \tag{34}$$

with $t_i$ and $t_f$ being the initial and final time moments of any pulse.

Writing the function $\varepsilon(t)$ in the interval between the $k$th and $(k+1)$th pulses as

$$\varepsilon_k(t) = a_k e^{-it} + b_k e^{it}, \qquad a_0 = 1, \quad b_0 = 0, \tag{35}$$

where $a_k$ and $b_k$ are constant coefficients, we obtain the following expressions for $E_k$ and $\tilde{E}_k$:

$$E_k = |a_k|^2 + |b_k|^2, \qquad \tilde{E}_k = 2a_k b_k. \tag{36}$$

Evidently, every two sets of the nearest constant coefficients, $(a_{k-1}, b_{k-1})$ and $(a_k, b_k)$, are related by means of a linear transformation

$$\begin{pmatrix} a_k \\ b_k \end{pmatrix} = M_k \begin{pmatrix} a_{k-1} \\ b_{k-1} \end{pmatrix}. \tag{37}$$

However, the form of Eq. (35) implies that the $k$th barrier begins at the moment $t = 0$. If it begins at the moment $t_{k-1}$ (so that $t_0 = 0$), then one should make the time shift $t \to t - t_{k-1}$ in Eq. (35), so that (37) will take the form

$$\begin{pmatrix} a_k e^{it_{k-1}} \\ b_k e^{-it_{k-1}} \end{pmatrix} = M_k \begin{pmatrix} a_{k-1} e^{it_{k-1}} \\ b_{k-1} e^{-it_{k-1}} \end{pmatrix} \qquad \Longrightarrow \qquad \begin{pmatrix} a_k \\ b_k \end{pmatrix} = \Phi^\dagger_{k-1} M_k \Phi_{k-1} \begin{pmatrix} a_{k-1} \\ b_{k-1} \end{pmatrix}, \tag{38}$$

where

$$\Phi_k \equiv \begin{Vmatrix} \exp(it_k) & 0 \\ 0 & \exp(-it_k) \end{Vmatrix} \tag{39}$$

is the unitary matrix describing the time-shift effect. For $n$ consecutive barriers shifted in time with respect to the initial instant $t = 0$ by $t_k$, $k = 1, \ldots, n-1$, we have the relation

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \mathcal{M}_n \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}, \qquad \mathcal{M}_n = \Phi^\dagger_{n-1} M_n \Phi_{n-1} \Phi^\dagger_{n-2} M_{n-1} \cdots \Phi^\dagger_1 M_2 \Phi_1 M_1, \tag{40}$$

where $\mathcal{M}_n$ is the total transfer matrix, whereas $M_n$, $M_{n-1}$, …, $M_1$ are the matrices describing the action of single barriers.

In the case of periodic barriers, all single barrier matrices $M_k$ coincide with $M_1$ and $\Phi_k \equiv \Phi_1^k$, so that

$$\mathcal{M}_n = \Phi^{\dagger n} (\Phi M_1)^n, \qquad \Phi \equiv \begin{Vmatrix} \exp(iT) & 0 \\ 0 & \exp(-iT) \end{Vmatrix}, \tag{41}$$

where $T$ is the period of temporal changes. It is well known that the matrix $M_1$ is unimodular and, consequently, the matrices $\Phi M_1$ and $\mathcal{M}_n$ possess the same property. Then we can use the well-known formula for the powers of any two-dimensional unimodular matrix $S$ (see, e.g., [124])

$$S^n = U_{n-1}(z) S - U_{n-2}(z) E, \qquad z \equiv \frac{1}{2} \operatorname{Tr} S, \tag{42}$$

where $E$ means the unit matrix and $U_n(z)$ is the Tchebyshev polynomial of a second kind.

Four complex elements of the $2\times2$ matrix $M_1$ for the single barrier can be expressed through two complex amplitude reflection coefficients and two complex amplitude transmission coefficients, which connect the plane waves coming from the left and from the right. Namely, if the barrier begins at $t = 0$ and terminates at $t = t_*$, then one can write two independent solutions of Eq. (1) outside the barrier as follows (remind that we assume the constant initial and final values of frequency to be equal to $\omega_i = 1$):

$$\varepsilon^{(-)}(t) = \begin{cases} e^{it} + r_- e^{-it}, & t < 0 \\ s_- e^{it}, & t > t_* \end{cases}, \qquad \varepsilon^{(+)}(t) = \begin{cases} s_+ e^{-it}, & t < 0 \\ e^{-it} + r_+ e^{it}, & t > t_* \end{cases}. \tag{43}$$

The reflection coefficients $r_\pm$ and transmission coefficients $s_\pm$ are not independent, because Eq. (1) is invariant with respect to complex conjugation ($\omega(t)$ is real). The following relations hold [125] (the simplest way to obtain them is to calculate Wronskians for suitable pairs of independent solutions):

$$s_- = s_+ \equiv s, \qquad r_- s^* + r_+^* s = 0, \qquad |r_-|^2 + |s_-|^2 = |r_+|^2 + |s_+|^2 = 1. \tag{44}$$

Comparing Eqs. (35) and (37) with (43) and taking into account identities (44), one can express the elements of matrix $M_1$ as follows:

$$M_1 = \left\| \begin{matrix} f & g^* \\ g & f^* \end{matrix} \right\|, \qquad f \equiv s^{-1}, \quad g \equiv \frac{r_+}{s}, \qquad \det M_1 = |f|^2 - |g|^2 \equiv 1. \tag{45}$$

Accordingly, let us write the total transfer matrix after $n$ barriers as follows:

$$\mathcal{M}_n = \left\| \begin{matrix} F_n & G_n^* \\ G_n & F_n^* \end{matrix} \right\|. \tag{46}$$

It is convenient to use the following parametrization for the trace of matrix $\Phi M_1$:

$$\frac{1}{2}\,\mathrm{Tr}\,(\Phi M_1) = \mathrm{Re}\,(f e^{iT}) \equiv \cosh(\nu). \tag{47}$$

Then using Eqs. (41) and (42) together with the hyperbolic representation of Tchebyshev polynomial

$$U_n(\cosh\nu) \equiv \frac{\sinh[(n+1)\nu]}{\sinh(\nu)}, \tag{48}$$

we obtain the following explicit expressions for coefficients $F_n$ and $G_n$:

$$F_n = f\,\frac{\sinh(n\nu)}{\sinh(\nu)} e^{-iT(n-1)} - \frac{\sinh[(n-1)\nu]}{\sinh(\nu)} e^{-iTn}, \qquad G_n = g\,\frac{\sinh(n\nu)}{\sinh(\nu)} e^{iT(n-1)}. \tag{49}$$

One can check the identity $|F_k|^2 - |G_k|^2 \equiv 1$.

We shall suppose that the period between pulses (barriers) $T$ is adjusted to the phase of the inverse transmission coefficient $f$ in such a way that $fe^{iT}$ is the real number. This means that we confine ourselves here to the exact resonance case. The case where some detuning from the exact resonance was permitted (but without damping) was studied in [96] (one should have in mind that the phases of coefficients $f$ and $g$ in that paper were different from those used here). The most general case (with arbitrary detuning and damping) will be considered elsewhere.

Assuming for definiteness that $fe^{iT}$ is positive, we have

$$fe^{iT} = |f| = \cosh(\nu), \qquad |g| = \sinh(\nu), \qquad |r| = \tanh(\nu).$$

Then coefficients $a_k$ and $b_k$ from (35) (they coincide with $F_k$ and $G_k$, in view of Eqs. (40), (46), and the initial conditions $a_0 = 1$ and $b_0 = 0$) take the form

$$a_k = \cosh(k\nu)e^{-ikT}, \qquad b_k = \sinh(k\nu)e^{iT(k-1)+i\phi}, \tag{50}$$

where $\phi$ is the phase of the complex number $g$. Combining (36) and (50) we obtain

$$E_k = \cosh(2k\nu), \qquad \tilde{E}_k = \sinh(2k\nu)e^{i(\phi-T)}. \tag{51}$$

Since the phase of the complex number $\tilde{E}_k$ does not depend on index $k$, it does not influence the sum in Eq. (33) which is reduced to

$$\sum_{k=1}^{n} e^{2\Lambda k}\cosh[2\nu(n-k)],$$

so that it can be easily calculated, giving rise to the following final result:

$$\mathcal{N}_n = \frac{G}{2}\cosh(2n\nu)e^{-2\Lambda n} - \frac{1}{2} + \frac{G}{4}\left(1-e^{-2\Lambda}\right)\left\{e^{(\Lambda+\nu)(1-n)}\frac{\sinh[(\Lambda+\nu)n]}{\sinh(\Lambda+\nu)} + e^{(\nu-\Lambda)(n-1)}\frac{\sinh[(\nu-\Lambda)n]}{\sinh(\nu-\Lambda)}\right\}. \tag{52}$$

If $\Lambda = 0$, then (52) goes to (7) (for $G = 1$). We see that the condition of photon generation is $\nu > \Lambda$. In the asymptotical regime $n(\nu - \Lambda) \gg 1$, formula (52) can be simplified as follows (we assume that $\nu \ll 1$, which corresponds to real conditions for NSCE):

$$\mathcal{N}_n \approx \frac{G\nu}{4(\nu-\Lambda)}\exp[2n(\nu-\Lambda)]. \tag{53}$$

This formula holds provided that the difference $\nu - \Lambda$ is not close to zero. If $\nu = \Lambda$, then formula (52) gives the asymptotical dependence $\mathcal{N}_n \sim n$, and for $\nu < \Lambda$ the increase in the mean number of photons (above the initial thermal value) tends to a small number $G\nu^2/[2(\Lambda^2 - \nu^2)]$.

## 5. Photon Rate Generation in the $TE$ Mode of a Cavity With a Thin Time-Dependent Semiconductor Slab and Optimum Conditions for NSCE

The absolute value of the "amplitude reflection coefficient" for a time-dependent pulse, which starts at the instant $t_i$ and ends at $t_f$, can be calculated with the aid of approximate formula [126]

$$|r| \approx \frac{1}{2}\left|\int_{t_i}^{t_f}\frac{\dot{\omega}(t)}{\omega(t)}\exp\left[-2i\int_{t_0}^{t}\omega(\tau)d\tau\right]dt\right|,$$

which holds for small variations of effective frequency $\omega(t)$. Writing

$$\omega(t) = \omega_0[1+\chi(t)], \qquad |\chi| \ll 1, \tag{54}$$

we obtain

$$|r| \approx \left|\int_{t_i}^{t_f}\frac{d\chi}{dt}e^{-2i\omega_0 t}\frac{dt}{2}\right| = \left|\int_{t_i}^{t_f}\omega_0\chi(t)e^{-2i\omega_0 t}dt\right| \tag{55}$$

for the absolute value of effective amplitude reflection coefficient from the single barrier and

$$\varphi \approx \omega_0 \int_{t_i}^{t_f} \chi(t)\, dt \tag{56}$$

for the phase of the single-barrier inverse transmission coefficient. Consequently, if the time-dependent relative shift of the real part of the cavity resonance frequency $\chi(t)$ is known, one can calculate the effective reflection coefficient and phase of the transmission coefficient with the aid of Eqs. (55) and (56). After that, the number of created photons can be calculated from Eq. (52), where the value of parameter $\Lambda$ can be extracted from the imaginary part of the cavity resonance frequency $\gamma(t)$ by means of integral (34).

For harmonic oscillations at double frequency, $\chi(t) = \chi_0 \sin(2\omega_0 t)$ so that taking $t_i = 0$ and $t_f = \pi/\omega_0$ we have $\varphi \equiv 0$. Then the resonance condition (8) with $m = 1$ coincides formally with the parametric resonance condition $T = T_0/2$ used in [17, 18]. For other shapes of function $\chi(t)$, some fine tuning of the period between pulses $T$ must be made, taking into account a concrete value of the phase shift $\varphi$. Moreover, for anharmonic modulation of the resonance frequency, the modulation period can be greater than the period of field oscillations.

Let us consider a cylindrical cavity with an arbitrary cross section and the axis parallel to the $x$ direction. We assume that the main part of the cavity is empty:

$$\varepsilon(x) \equiv 1 \qquad \text{for} \qquad -L < x < 0$$

but there is a thin slab of a semiconductor material with $\varepsilon(x) \neq 1$ in the domain $0 < x < D \ll L$. We use the Maxwell equations in the form, which does not contain derivatives of function $\varepsilon(\mathbf{r}, t)$ with respect to the time variable:

$$\operatorname{rot} \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \qquad \operatorname{rot} \frac{\mathbf{D}}{\varepsilon} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{57}$$

(the medium is assumed to be non-magnetic). Excluding the magnetic field $\mathbf{B}$, we obtain the equation

$$\operatorname{rot}\operatorname{rot} \frac{\mathbf{D}}{\varepsilon} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{D}}{\partial t^2}. \tag{58}$$

For monochromatic field $\mathbf{D}(\mathbf{r}, t) = \mathbf{D}(\mathbf{r}) \exp(-i\Omega t)$, it takes the form

$$\operatorname{grad}\operatorname{div} \frac{\mathbf{D}}{\varepsilon} - \Delta \frac{\mathbf{D}}{\varepsilon} = \frac{\Omega^2}{c^2}\mathbf{D}. \tag{59}$$

In the case of $TE$ modes, vectors $\mathbf{E}$ and $\mathbf{D}$ are perpendicular to the $x$ axis (and parallel to plane surfaces of the cylinder and slab). Since we assume that the dielectric permeability depends only on the longitudinal space variable $x$, the first term in Eq. (59) is zero:

$$\operatorname{div} \frac{\mathbf{D}}{\varepsilon} = \frac{1}{\varepsilon(x)} \operatorname{div} \mathbf{D} \equiv 0,$$

because differentiation is made in this case with respect to transversal coordinates, which function $\varepsilon(x)$ does not depend on. Consequently, the electric field $\mathbf{E} = \mathbf{D}/\varepsilon(x)$ satisfies the usual three-dimensional Helmholtz equation

$$\Delta \mathbf{E} + \left(\frac{\Omega}{c}\right)^2 \varepsilon(x)\mathbf{E} = 0. \tag{60}$$

It is known that Eq. (60) allows one to factorize any scalar component of the electric field $E(x,\mathbf{r}_\perp)$ (where $\mathbf{r}_\perp$ stands for the coordinates in the plane perpendicular to the $x$ axis and the choice of scalar components depends on a concrete form of the cavity):

$$E(x,\mathbf{r}_\perp) = \psi(x)\Phi(\mathbf{r}_\perp),$$

where the function $\Phi(\mathbf{r}_\perp)$ obeys the two-dimensional Helmholtz equation

$$\Delta_\perp \Phi + k_\perp^2 \Phi = 0. \tag{61}$$

Thus the equation for $\psi(x)$ becomes

$$\psi'' + \left[\left(\frac{\Omega}{c}\right)^2 \varepsilon(x) - k_\perp^2\right]\psi = 0. \tag{62}$$

Its solution in the domain $-L < x < 0$ satisfying the boundary condition $\psi(-L) = 0$ is obvious:

$$\psi(x) = F_1 \sin[k(x+L)], \tag{63}$$

where the constant coefficient $k$ is related to the field eigenfrequency $\Omega$ and the corresponding wavelength in vacuum $\lambda$ as follows:

$$\Omega = c\left(k^2 + k_\perp^2\right)^{1/2}, \qquad \lambda = 2\pi\left(k^2 + k_\perp^2\right)^{-1/2}. \tag{64}$$

Since the electric field and its derivative in the $x$ direction must be continuous at the surface $x = 0$, the admissible values of parameter $k$ can be determined from the continuity condition for the logarithmic derivative of $\psi(x)$ at $x = 0$:

$$\tan(kL) = \frac{\psi_+(0;k)}{\psi_+'(0;k)}, \tag{65}$$

where $\psi_+(x;k)$ is the solution of Eq. (62) in the domain $0 < x < D$ satisfying the boundary condition $\psi_+(D) = 0$. In the generic case, (65) is a complicated transcendental equation, which can be solved only numerically. But for a thin layer, $D \ll \lambda \sim L$, the value of $k$ must be close to $\pi/L$ (we consider the lowest mode of the cavity). Thus we can write

$$k = \frac{(1+\xi)\pi}{L}, \qquad |\xi| \ll 1, \tag{66}$$

and replace $\tan(\pi\xi)$ on the left-hand side of (65) simply by $\pi\xi$. Moreover, in the first approximation we can identify $k$ with $\pi/L$ on the right-hand side. It is convenient to introduce the dimensionless variable $\tilde{x} = x/D$ (so that $0 \le \tilde{x} \le 1$). Then the small coefficient $\xi$ can be calculated as follows:

$$\xi = \frac{\eta\Delta\tilde{\psi}_+(0)}{\tilde{\psi}_+'(0)}, \tag{67}$$

where the function $\tilde{\psi}_+(\tilde{x})$ satisfies the equation with a fixed value of the parameter $k = \pi/L$

$$\frac{d^2\tilde{\psi}_+}{d\tilde{x}^2} + \pi^2\Delta^2\left[\varepsilon(\tilde{x}) - 1 + \eta^2\right]\tilde{\psi}_+(\tilde{x}) = 0. \tag{68}$$

New dimensionless parameters are defined as follows:

$$\eta = \frac{\lambda}{2L} < 1, \qquad \Delta = \frac{2D}{\lambda} \ll 1. \tag{69}$$

It is convenient to introduce the function $R(\tilde{x}) = \tilde{\psi}_+(\tilde{x})/\tilde{\psi}'_+(\tilde{x})$. It satisfies the first-order nonlinear generalized Riccati equation, which is the immediate consequence of Eq. (68):

$$\frac{dR}{d\tilde{x}} = 1 + \pi^2\Delta^2\left[\varepsilon(\tilde{x}) - 1 + \eta^2\right]R^2. \tag{70}$$

Solving (70) in the interval $0 < \tilde{x} < 1$ with the condition at the right boundary $R(1) = 0$, we can rewrite (67) as follows:

$$\xi = \eta\Delta R(0).$$

For an arbitrary function $\varepsilon(\tilde{x})$, Eq. (70) should be solved numerically. However, we can obtain a simple approximate analytical solutions in the case under consideration by taking into account several additional circumstances.

First of all, Eq. (64) gives rise to the following expression for complex relative shift of the resonance frequency:

$$\chi_\Omega \equiv \frac{\Omega - \omega_0}{\omega_0} = \eta^2\left(\xi - \xi_0\right) = \eta^3\Delta\left[R(0) - R_0(0)\right],$$

where $\xi_0$ or $R_0(0)$ correspond to a non-excited semiconductor slab (we take into account that $|\xi| \ll 1$ and $|\xi_0| \ll 1$). In the case of $\xi_0$, we have $\varepsilon(\tilde{x}) = \varepsilon_1 = \text{const}$, so that Eq. (70) has the exact solution

$$R_0(\tilde{x}) = \frac{\tan\left[\pi\Delta a\left(\tilde{x} - 1\right)\right]}{\pi\Delta a}, \qquad R_0(0) = -\frac{\tan(\pi\Delta a)}{\pi\Delta a}, \qquad a = \sqrt{\varepsilon_1 - 1 + \eta^2}.$$

Since $\varepsilon_1 \sim 10$ for semiconductors, the product $\pi\Delta a$ does not exceed a value of the order of 0.1, if $\Delta \leq 0.01$. But in such a case, $R_0(0) \approx -1$, with an accuracy of the order of 0.01 or even better. When the semiconductor slab is illuminated by the laser pulse, $\varepsilon(\tilde{x}) = \varepsilon_1 + i\varepsilon_2(\tilde{x})$, where the imaginary part $\varepsilon_2(\tilde{x})$ can attain very big values, so that $\pi^2\Delta^2\varepsilon_2(\tilde{x}) \gg 1$ in some domain $0 < \tilde{x} < \delta$ near the slab surface (otherwise, we cannot simulate the displacement of an effective boundary). And here it is important to note that $\delta \ll 1$, because carriers are created in a thin layer of the depth $\alpha^{-1}$, where $\alpha$ is the absorption coefficient of laser radiation. Obviously, to create an effective "plasma mirror" one needs the material with $\delta \sim \alpha^{-1} \ll D$. It is important that the generation of new carriers under the laser-pulse action influences on the imaginary part $\varepsilon_2$ of $\varepsilon(\tilde{x})$ but it practically does not change the real part $\varepsilon_1$ at the fundamental-mode frequency ($\sim 5$ GHz), because this frequency is much less than the optical frequencies and the concentration of new carriers is still much less than the concentration of "optical electrons" in the material (i.e., the corresponding plasma frequency is much less than the frequency of optical transitions).

In the domain $0 < \tilde{x} < \delta$, we can neglect the first term (unity) on the right-hand side of Eq. (70), as well as the real part of the coefficient at $R^2$. The simplified equation can be integrated immediately resulting in the formula:

$$\frac{1}{R(0)} - \frac{1}{R(\delta)} = i(\pi\Delta)^2\int_0^\delta \varepsilon_2(\tilde{x})\,d\tilde{x}. \tag{71}$$

On the other hand, in the domain $\delta < \tilde{x} < 1$ the nonlinear term in (70) becomes insignificant, so that we can write $R(1) - R(\delta) = 1 - \delta$. Since $R(1) = 0$ and $\delta \ll 1$, we can take $R(\delta) = -1$. Moreover, since the function $\varepsilon_2(\tilde{x})$ quickly goes to zero outside the interval $(0, \delta)$, we can extend the upper limit of integration in (71) to infinity. Thus we have $R(0) = (iA - 1)^{-1}$ and finally

$$\chi_\Omega = \eta^3 \Delta \frac{A^2 - iA}{A^2 + 1}, \qquad \chi = \eta^3 \Delta \frac{A^2}{A^2 + 1}, \qquad \gamma = \eta^3 \Delta \frac{A}{A^2 + 1}, \tag{72}$$

where

$$A = (\pi\Delta)^2 \int_0^\infty \varepsilon_2(\tilde{x}) d\tilde{x}. \tag{73}$$

Consequently, to find the functions $\chi(t)$ and $\gamma(t)$, we need only the integral of the instantaneous carrier concentration over the slab at every instant of time $t$. Obviously, approximate formulas in (72) are not very good when $A \ll 1$ but contributions of corresponding pieces of functions $\chi(t)$ and $\gamma(t)$ to integrals (34), (55), and (56) are so small that they can be neglected. Consequently, the approximation (72) is quite adequate for the NSCE applications. As follows from (72), $\chi > \gamma$ if $A > 1$.

The imaginary part of dielectric permeability $\epsilon_2$ is proportional to the conductivity

$$\sigma(x, t) = n(x, t)|eb|, \tag{74}$$

where $e$ is the electron charge and $b$ is the total mobility of carriers (for each electron–hole pair). Strictly speaking, the carrier mobility can also depend on $x$ and $t$ but we assume that it is constant. Moreover, for evaluations, we use the static values of the mobility, neglecting its possible dependence on the frequency (taking into account once again that the frequency of the selected cavity mode is much less than the characteristic frequencies of atoms in the semiconductor slab). The carrier concentration inside the semiconductor slab is governed by the equations, which take into account, besides the photon absorption, different recombination processes. The typical equation has the form [127–130]

$$\frac{\partial n}{\partial t} = \nabla \cdot (Y \nabla n) + \frac{\alpha\kappa}{E_g} I(t) e^{-\alpha x} - \beta_3 n^3 - \beta_2 n^2 - \beta_1 n, \tag{75}$$

where $Y$ is the coefficient of ambipolar diffusion, $\alpha$ is the absorption coefficient of laser radiation inside the layer, $E_g$ is the semiconductor energy gap (which is close to the laser photon energy), $I(t)$ is the time-dependent intensity of the laser pulse, which enters the slab (it can be less than the intensity of the pulse outside the slab, because the reflection coefficient from the semiconductor surface can be rather big, due to a big value of the dielectric constant $\varepsilon_1 \sim 10$; however, the reflection can be diminished if some quarter wavelength film is put on the surface), $\kappa \le 1$ is the efficiency of the photon–electron conversion, $\beta_3$ is the Auger recombination coefficient, $\beta_2$ is the radiative recombination coefficient, and $\beta_1$ is the trap-assisted recombination coefficient. For powerful pulses, one should add the equation for the space–time evolution of temperature, because many coefficients in Eq. (75) are temperature-sensitive (moreover, this equation contains, in the generic case, the term proportional to the temperature gradient [129]). However, the experimental data show that the increase in temperature is rather small under the conditions of the Padua experiment [131], therefore we consider the simplest model based on Eq. (75). In the most general case, one should use the function $I(t - x/v)$ instead of $I(t)$, where $v$ is the group velocity. But for materials with high absorption coefficients the coordinate dependence can be neglected. For example, if $\alpha = 10^6\,\text{cm}^{-1}$ and $v \sim 10^{10}\,\text{cm/s}$, then $x/v \sim (\alpha v)^{-1} \sim 10^{-16}\,\text{s}$, and this shift is insignificant for pulses, whose characteristic time scale is of the order of picoseconds.

It is clear that in the most general case, Eq. (75) can be solved only numerically [128–130]. However, for modeling the nonstationary Casimir effect one needs very small recombination times, of the order of

$T_r \sim 20 - 30\,\text{ps}$ (see estimations at the end of this section). This can be achieved in doped materials with a high impurity concentration. Doping increases the coefficient $\beta_1$ but does not change significantly $\beta_2$ and $\beta_3$. Thus we can assume that $\beta_1 \approx T_r^{-1} \sim 5 \cdot 10^{10}\,\text{s}^{-1}$. On the other hand, typical values of coefficients $\beta_2$ and $\beta_3$ are [128–130] $\beta_2 \sim 5 \cdot 10^{-14}\,\text{cm}^3/\text{s}$ and $\beta_3 \sim 4 \cdot 10^{-31}\,\text{cm}^6/\text{s}$ (strictly speaking, the data presented are for Si but we suppose that the orders of magnitude in GaAs are the same). Then even for a very high carrier concentration $n \sim 10^{18}\,\text{cm}^{-3}$ we have $\beta_2 n \sim 5 \cdot 10^4\,\text{s}^{-1}$ and $\beta_3 n^2 \sim 4 \cdot 10^5\,\text{s}^{-1}$. These values are six and five orders of magnitude smaller than $\beta_1$. Consequently, in the special case of heavy doped materials, which corresponds to the conditions of the proposed experiment, the terms with $n^2$ and $n^3$ in Eq. (75) can be neglected without any doubts, so that we have, in fact, the linear equation, which can be solved analytically. Moreover, according to Eqs. (72) and (73), we need only the time-dependent integral value

$$\mathcal{N}(t) = \int_0^\infty n(x,t)\,dx$$

(we consider the case of a high absorption coefficient, where the integration over the slab can be extended to infinity with very small error). Integrating Eq. (75) (without nonlinear terms) over $dx$ from 0 to $\infty$ and taking into account the boundary condition

$$Y \frac{\partial n}{\partial x}\bigg|_{x=0} = 0 \tag{76}$$

(this means that we neglect the surface recombination), we obtain the equation

$$\frac{\partial \mathcal{N}}{\partial t} = \frac{\kappa}{E_g} I(t) - \beta_1 \mathcal{N}, \tag{77}$$

whose obvious solution is

$$\mathcal{N}(t) = \frac{\kappa}{E_g} \int_0^t e^{-\beta_1 (t-\tau)} I(\tau)\,d\tau. \tag{78}$$

For short laser pulses, whose duration is much less than the recombination time (this is the real experimental situation), the integral in (78) is reduced to

$$\mathcal{N}(t) = \frac{\kappa W}{E_g S}\, e^{-\beta_1 t}, \qquad \frac{W}{S} = \int_{-\infty}^{\infty} I(\tau)\,d\tau, \tag{79}$$

where $W$ is the total pulse energy and $S$ is the area of the semiconductor-slab surface (it is assumed that the energy is distributed uniformly over the surface area). Combining Eqs. (9), (73), (74), and (79), we can write the time-dependent function $A(t)$ in Eq. (72) as follows:

$$A(t) = A_0 e^{-\beta_1 t}, \qquad A_0 = \frac{4\pi^2 |eb| \kappa W \Delta}{c E_g S} = \frac{4\pi^2 e^2 \tau_c \kappa W \Delta}{m c E_g S}, \tag{80}$$

where $\tau_c$ is the mean time between collisions and $m$ is the effective mass of the carriers, so that $b = e\tau_c/m$. The parameter $A_0$ coincides, up to a numerical coefficient, with the parameter $\mathcal{F}$ used in [95]. As a reference point we find, taking $\kappa = 1$, $\lambda = 12\,\text{cm}$ (or $f_0 = 2.5\,\text{GHz}$), $D = 0.6\,\text{mm}$, $S = 71{\times}22\,\text{mm}^2$ (according to [75]), and $E_g = 1.4\,\text{eV}$ (as for GaAs), that $A_0 = 1$ for $W \sim 2 \cdot 10^{-5}\,\text{J}$ and $b = 3 \cdot 10^7$ [CGS units] $= 10\,\text{m}^2/\text{V}\cdot\text{s}$ (which corresponds, e.g., to $\tau_c = 10^{-11}\,\text{s}$ and $m = 0.16$ of free electron

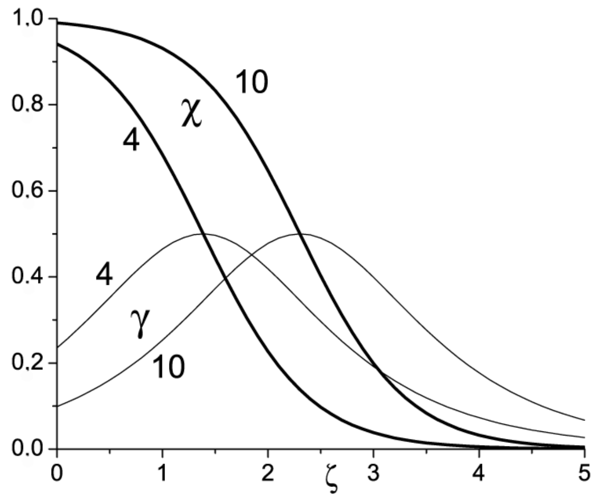


**Fig. 1.** Relative shift of the cavity eigenfrequency $\chi$ and normalized damping coefficient $\gamma$ versus dimensionless time variable $\zeta = \beta_1 t = \tau/Z$. Numbers 4 and 10 at each line correspond to fixed values of parameter $A_0$.

mass). Note that the parameter $A_0$ does not depend on the absorption coefficient (obviously, this happens due to the condition $\alpha D \gg 1$).

Combining Eqs. (72) and (80), we see that the dimensionless frequency shift $\chi(\tau)$ and damping coefficient $\gamma(\tau)$ depend on a dimensionless time $\tau = \omega_0 t$ as follows:

$$\chi(\tau) = \frac{\eta^3 \Delta A_0^2 \exp(-2\tau/Z)}{1 + A_0^2 \exp(-2\tau/Z)}, \qquad \gamma(\tau) = \frac{\eta^3 \Delta A_0 \exp(-\tau/Z)}{1 + A_0^2 \exp(-2\tau/Z)}, \tag{81}$$

where

$$Z = \frac{\omega_0}{\beta_1} = \frac{2\pi T_r}{T_0}\,, \tag{82}$$

$T_r$ is the recombination time, and $T_0$ is the field-oscillation period in the selected mode. The evolution of functions $\chi(\tau)$ and $\gamma(\tau)$ for different values of parameter $A_0$ is shown in Fig. 1. We see that the influence of damping cannot be neglected for two reasons. First, the maximum value of $\gamma$ is only twice smaller than the maximum value of $\chi$. Second, $\chi(\tau)$ asymptotically goes to zero as $\exp(-2\tau/Z)$, whereas $\gamma$ decays much more slowly, as $\exp(-\tau/Z)$. Therefore, the effect of damping can significantly diminish the photon-generation rate, which is determined, according to Eq. (53), by the difference

$$\nu - \Lambda = \eta^3 \Delta\, F(A_0, Z). \tag{83}$$

The integral (34) for parameter $\Lambda$ can be calculated exactly, if the function $\gamma(\tau)$ is given by (81):

$$\Lambda = Z\eta^3 \Delta \tan^{-1}(A_0) = \begin{cases} Z\eta^3\Delta\pi/4 & \text{for } A_0 = 1, \\ Z\eta^3\Delta\pi/2 & \text{for } A_0 \gg 1. \end{cases} \tag{84}$$

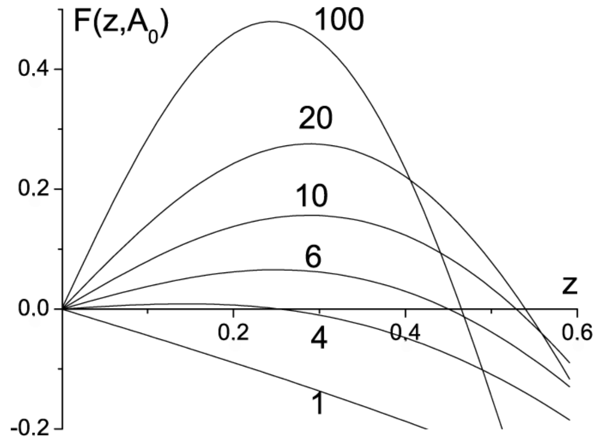


**Fig. 2.** Dependence of the amplification coefficient $F$ on parameter $Z$ for fixed values of parameter $A_0$ (given at each line).

Exact expression can be also obtained for integral (56), which determines the pulse periodicity:

$$\varphi = \frac{1}{2} Z\eta^3 \Delta \ln\left(1 + A_0^2\right) = \begin{cases} Z\eta^3\Delta(\ln 2)/2 & \text{for } A_0 = 1, \\ Z\eta^3\Delta \ln(A_0) & \text{for } A_0 \gg 1. \end{cases} \tag{85}$$

Integral (55) for the effective reflection coefficient takes the form

$$\nu = |r| = \frac{Z}{2}\eta^3\Delta \left| \int_0^\infty \frac{\exp(-iZ\tau)\, d\tau}{1 + a\exp(\tau)} \right|, \qquad a \equiv \frac{1}{A_0^2}. \tag{86}$$

The integral in (86) can be calculated exactly if $A_0 = 1$ [132]:

$$|r| = \frac{1}{4} Z\eta^3\Delta \left| \psi\left(1 + \frac{iZ}{2}\right) - \psi\left(\frac{1}{2} + \frac{iZ}{2}\right) \right|, \tag{87}$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$ is the logarithmic derivative of Gamma function. For $A_0 \neq 1$, this integral can be calculated only numerically. However, simple evaluations can be made in certain limit cases.

It is easy to show that the amplification coefficient $F$ is negative (i.e., the photon creation is impossible) if $A_0 \ll 1$. Indeed, in this case, $a \gg 1$ so that the denominator in the integrand can be replaced by $a\exp(\tau)$. After this, integral (86) can be calculated exactly and one obtains the expression

$$F = Z\left(\frac{A_0^2}{2\sqrt{Z^2+1}} - A_0\right),$$

which is obviously negative for $A_0 \ll 1$. If $Z \ll 1$, then replacing $\exp(-iZ\tau)$ by 1 we obtain for $\nu$ the same expression as for $\varphi$. In this case, we have the formula

$$F \approx Z\left[\frac{1}{2}\ln\left(1 + A_0^2\right) - \tan^{-1}(A_0)\right].$$

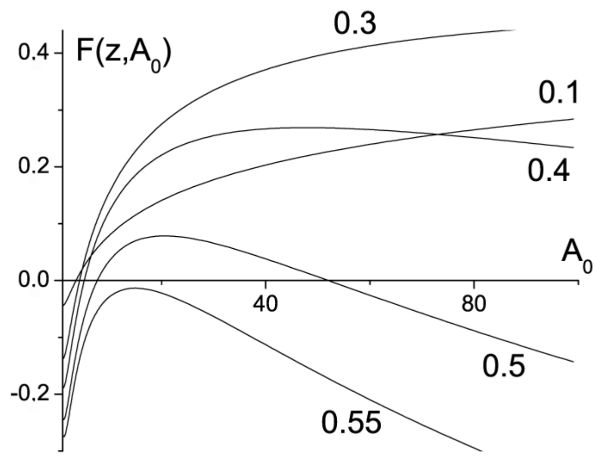


**Fig. 3.** Dependence of the amplification coefficient $F$ on parameter $A_0$ for fixed values of parameter $Z$ (given at each line).

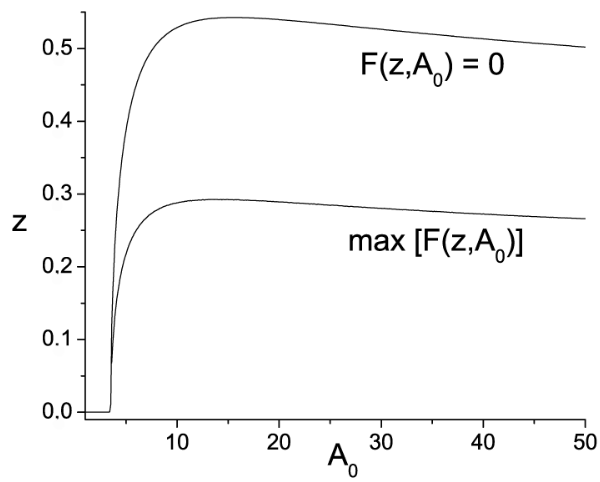


**Fig. 4.** Boundary between domains of positive and negative values of the amplification coefficient $F$ (the upper line) and the "optimum trajectory" in the plane of parameters $A_0$ and $Z$ (where the function $F(Z, A_0)$ takes maximum values).

This expression is obviously negative for $A_0 \ll 1$. Moreover, it is negative even for $A_0 = 1$ (this is confirmed by exact formula (87), because $\psi(1) - \psi(1/2) = 2\ln 2$ [132]) but it is obviously positive if $\ln(1 + A_0^2) > \pi$. Consequently, the photon generation is possible for small recombination times and sufficiently high energy of the laser pulse. In Figs. 2 and 3, we collect the results of numerical integration of (86), showing the dependence of the amplification coefficient $F$ on parameters $Z$ and $A_0$.

In Fig. 4, we show the boundary between the domains of positive and negative values of the amplifi-

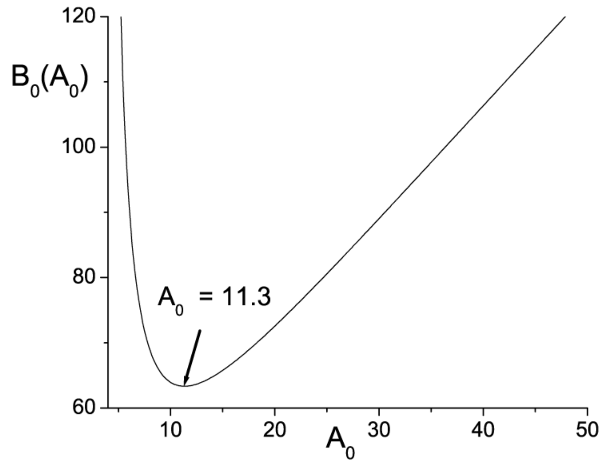


**Fig. 5.** Function $B_0(A_0, Z) = A_0/F(Z, A_0)$ proportional to the total energy of all laser pulses, which is necessary to generate the fixed (big) number of photons, versus parameter $A_0$ (for optimum values of parameter $Z$).

cation coefficient $F$ in the plane of parameters $A_0$ and $Z$ (the generation is possible for the parameters belonging to the domain below the upper line), as well as the "optimal trajectory" corresponding to the sets of parameters, for which the function $F(Z, A_0)$ takes maximum values. We see that the generation is impossible if $Z > 0.54$ or $A_0 < 4$. For moderate values of parameter $A_0$, the maximum values of $F$ are achieved for $Z \approx 0.3$.

Integral (86) can be calculated analytically for $A_0 = \infty$ (and fixed $Z$). In this case, the denominator in the integrand can be replaced by unity and the convergence of the integral can be ensured by adding a small term $-\varepsilon\tau$ to the argument of exponential function. Then we obtain $F_\infty(Z) = (1 - Z\pi)/2$. This means that the boundary line $F(Z, A_0) = 0$ tends asymptotically to the horizontal line $Z_c = 1/\pi \approx 0.32$, but this happens for very big values of $A_0$.

Analyzing Figs 2–4, we conclude that the optimum value of $Z$ is close to 0.3. The corresponding recombination time is close to $T_r^{\rm opt} = T_0/(2\pi^2)$. For $T_0 = 400$ ps (or $f_0 = 2.5$ GHz), we obtain $T_r^{opt} \approx 20$ ps, and there is no case where it can be more than 35 ps. As was reported in [133], it is possible to reduce the recombination time in GaAs down to 3 ps, using the implantation of $Au^+$ ions. Even more short recombination times, less than 1 ps, were reported (for other materials) in [134]. Consequently, the value $T_r^{\rm opt} \approx 20$ ps is quite realistic from the viewpoint of available technology.

The optimum value of parameter $A_0$ can be found in the following way.

We have already shown that this parameter is proportional to the laser-pulse energy. On the other hand, if we fix the number of photons which can be created after $n$ pulses, then formula (53) shows that $n \approx {\rm const}/F$. Consequently, the function $B_0(A_0, Z) = A_0/F(Z, A_0)$ is proportional to the total energy of all necessary laser pulses. Choosing for each value of $A_0$ the optimum value of parameter $Z$ (taken from the lower line in Fig. 4), we obtain plot given in Fig. 5.

It shows that the best choice (corresponding to the minimum total energy) is

$$A_{0*} = 11.3, \qquad Z_* = 0.29, \qquad F_* = 0.18, \qquad \nu_* = 0.61, \qquad \Lambda_* = 0.43. \tag{88}$$

We see that the ratio $\Lambda_*/\nu_* \approx 2/3$ is rather big, so that neglecting the damping effect would result in a

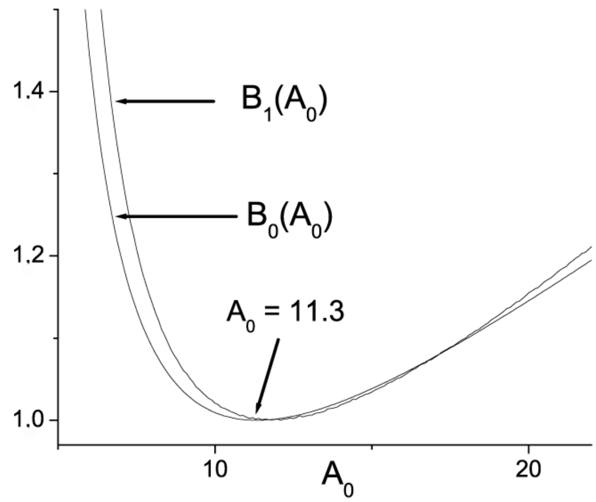


**Fig. 6.** Comparison of functions $B_0(A_0, Z) = A_0/F(Z, A_0)$ and $B_1(A_0, Z) = A_0/[ZF(Z, A_0)]$ versus parameter $A_0$ (for optimum values of parameter $Z$). Here both functions are normalized by their minimum values.

significant error. For the numerical values of parameters considered above, we obtain the optimum energy of a single pulse $W \approx 10^{-4}$ J. For the geometry chosen for the Padua experiment [75], ($D = 0.6$ mm, $L = 110$ mm, and $\lambda = 12$ cm) we have $\Delta = 0.01$ and $\eta = 0.55$, so that $\eta^3 = 0.16$. Consequently, according to formula (53), under the optimum conditions one needs $n_4 \approx 16000$ pulses to create $\mathcal{N}_4 = 10^4$ photons in the fundamental mode of the cavity. $\mathcal{N}_3 = 1000$ photons (expected lower detection level [131]) can be created after $n_3 \approx 12000$ pulses. Taking the pulse periodicity $T \approx T_0/2 = 200$ ps, the total duration of the process of generation of $10^4$ photons must be $T_{\text{tot}} \approx 3\,\mu$s, which means that the cavity quality factor must be higher than $Q_{\min} = \pi T_{\text{tot}}/T_0 \approx \pi n/2 \sim 3 \cdot 10^4$, whereas the real cavity quality factor in the Padua experiment is $Q_{\text{real}} \sim 10^7$ [131]. The total energy of all pulses must be about 1.6 J for chosen values of the parameters. It can be diminished, if the carrier mobility can be increased (maximum carrier mobility in GaAs reported in the available literature is about $40\,\text{m}^2/\text{V}\cdot$ s). If one neglected the damping effect (putting formally $\Lambda = 0$), then the necessary number of pulses would be $n_4^{\text{ideal}} \approx 5500$ and $n_3^{\text{ideal}} \approx 4300$.

The evaluations made above are based on the assumption that the carrier mobility $b$ (or the mean time between collisions $\tau_c$) does not depend on the recombination time or, equivalently, on the parameter $Z$. Since the decrease of the recombination time is achieved by doping the semiconductor material, time $\tau_c$ can also diminish due to doping. In the extreme case, one can suppose that $\tau_c \sim T_r \sim Z$. Then the pulse energy is proportional to the ratio $A_0/Z$, and we have to analyze the function $B_1(A_0, Z) = A_0/[ZF(Z, A_0)]$ (with the value of $Z$ taken again from the optimum line in Fig. 4). The functions $B_0$ and $B_1$ normalized by the corresponding minimum values are compared in Fig. 6. The minimum of $B_1$ is achieved for $A_{0**} = 12.1$. We see that the difference between the normalized functions $B_0$ and $B_1$ is very small for $A_0 > 10$. Consequently, the possible dependence of the carrier mobility on the recombination time does not influence the choice of the optimum value of parameter $A_0$. Moreover, since the minima of the both functions are rather flat, one can assume that this optimum value is always close to $A_0 = 10$.

The resonance wavelength $\lambda$ corresponding to the $TE_{101}$ mode with the lowest eigenfrequency of a rectangular cavity is related to the cavity length $L$ and the biggest transverse dimension $B$ as $\lambda = 2LB/\sqrt{L^2+B^2}$. Consequently, $B = \lambda/(2\sqrt{1-\eta^2})$. For fixed values of the parameter $A_0$ and the smallest transverse dimension, the pulse energy is proportional to the surface area, i.e., $B$, whereas the necessary number of pulses depends on $L$ as $\eta^{-3} \sim L^3$. Consequently, the total energy is proportional to the product $BL^3$. Minimizing this product for a fixed value of $\lambda$ (which is equivalent to the maximum of function $\eta^3\sqrt{1-\eta^2}$), we find the optimum value $\eta_{\rm opt} = \sqrt{3}/2 = 0.866$, which corresponds to $L = \lambda/\sqrt{3} \approx 7\,$cm and $B = \lambda = 12\,$cm (for $\lambda = 12\,$cm). This choice could diminish the total energy by 2.5 times and the number of pulses by almost 4 times in comparison with the case of $L = 11\,$cm ($\eta = 0.55$), at the expense of approximately twice increase in the each pulse energy. However, perhaps, it would be difficult to ensure the uniform illuminating of such long plate.

It is worth emphasizing that all estimations have been made under the exact resonance condution, when the period of pulses is adjusted to the phase $\varphi$ of the effective "transmission coefficient" through the "time barrier" in accordance with Eq. (8). Formula (85) shows that $\varphi$ has the same order of magnitude as effective "reflection coefficient" $|r| \approx \nu$, i.e., $\varphi \sim 10^{-3}$ under the conditions discussed. Although this quantity is small, it must be known precisely and the frequency of pulses must be shifted from the usual parametric resonance value $2/T_0 = 5\,$GHz by a quantity $\delta f_{\rm res} = 2\varphi/(\pi T_0)$, which is of the order of several MHz. The relative width of the resonance is of the order of $\nu$ [96], so without making the shift, one cannot be sure that the number of photons in the selected mode grows with time exponentially, instead of performing oscillations with a small amplitude nearby the initial value.

## 6. Conclusions

We have shown interesting intersections between such distant, at first sight, areas as nonstationary Casimir effect, quantum nonstationary damped oscillator, the physics of semiconductors and the physics of ultrashort laser pulses. Let us formulate and discuss the main results of the paper.

We have constructed a consistent model of damped nonstationary quantum oscillator with arbitrary time-dependent frequency and damping coefficients based on a generalization of the Senitzky–Schwinger–Haus–Lax noise operator approach. We have derived the set of "minimum noise" operators, which enabled us to obtain a simple generalization of Husimi solution to the case of quantum damped oscillator — formula (31). The new solution is exact and it can be applied to a great variety of problems in quantum mechanics of nonstationary dissipative systems. For example, one can generalize the formulas for time-dependent transition probabilities between different states obtained in [77,86–90] for the unitary evolution to the nonunitary case. It is interesting also to analyze the equations for the density matrix or Wigner function, which are equivalent to Heisenberg–Langevin equations (10) and (11) with arbitrary functions $\omega(t)$ and $\gamma(t)$, following the general scheme given in [92,135]. Using the approach of [91,136], one can study the influence of damping on the degree of squeezing, nonclassical statistics, and so on. One could think also on possible multidimensional generalizations.

Another important result is approximate formula (52) for the number of photons, which can be produced after $n$ periods of small perturbations of the oscillator frequency in the resonance case. It depends on two parameters — the absolute value $\nu$ of the amplitude "reflection coefficient" from the effective single pulse barrier in the time domain, which is given by the Fourier transform of time-dependent frequency $\omega(t)$ calculated at double resonant frequency and "accumulated damping factor" $\Lambda$ given by integral (34) of the periodical damping coefficient (absolute value of the imaginary part of the complex eigenfrequency) over the period of this function (pulse duration). The asymptotical rate of the photon generation is twice the difference $\nu - \Lambda$.

We have obtained simple approximate analytical expressions for the real and imaginary parts of the complex shift of the cavity resonance frequency $\Omega(t) = \omega(t) - i\gamma(t)$ in the case of a thin semiconductor slab irradiated by short laser pulses. They hold under the following conditions (which are satisfied in the experiment planned in the University of Padua): small absorption length (much less than the slab thickness) and small recombination time of photo-induced carriers. Using these expressions, we have calculated coefficients $\nu$ and $\Lambda$ and found their dependence on several dimensionless parameters, which determine the process dynamics — normalized geometrical factors $\Delta$ and $\eta$ (69), normalized recombination time $Z$ (82), and normalized pulse energy $A_0$ (80). We have found the boundary of the photon-generation domain in the plane of parameters $Z - A_0$, as well as the dependence $A_0(Z)$ which gives the maximum value of the photon generation rate $\nu - \Lambda$. Besides, using different criteria we have found optimum values for the pulse energy, recombination time, and geometrical factors. In particular, we have shown that the recombination time should not exceed 35 ps, and the optimum choice is 20 ps.

It is worth repeating that our evaluations are made under the assumption that the interaction between the selected mode and other cavity modes can be neglected in the resonance case in a three-dimensional cavity with non-equidistant spectrum for arbitrary periodical change of the frequency. A strict mathematical proof of this assumption is highly desirable. By analogy with the standard parametric resonance case (where the function $\omega(t)$ performs small harmonical oscillations), we can suppose that our results are valid under the condition $\nu n^2 \ll 1$; otherwise, the influence of neglected nonlinear terms in the equations of motion and interaction with other modes could be important. This inequality explains, in particular, why the semiconductor-slab thickness must be small (so that one cannot take it of the order of, say, the half of the cavity length or more). For the Padua-experiment geometry, we have $\nu \sim 10^{-3}$ so that the number of pulses should be less than $10^6$; and this upper limit is much bigger than the values which people hope to achieve in reality.

We have considered only the fundamental $TE$ mode inside the cavity, in accordance with a concrete scheme chosen for the Padua experiment [75]. There are indications that the generation rate could be enhanced in the $TM$ modes [68, 94, 95]. We shall report on the calculations for this geometry elsewhere.

## Acknowledgments

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## Appendix. Publications on Quantum Fields in the Presence of Moving Boundaries (from 2001 to the First Half of 2005)

Various aspects of dynamical Casimir effect in three-dimensional systems (rectangular and cylindrical cavities or waveguides), such as effects of additional symmetry in cubic cavities, the comparison of $TE$ and $TM$ polarizations, the Hertz potential applications, the temperature influence and some others, were studied in [72,73,94–96,137–140]. Spherical cavities with moving boundaries were considered in [141–143]. The cavities with time-dependent conductivity or dielectric permeability of thin slabs inside them were considered in [68, 95, 144]. Different phenomena related to quantum and classical fields propagating in time-dependent media were discussed in [145–147]. The attempts to describe the effects of dissipation were made in [112, 148, 149]. The importance of taking into account the influence of decoherence in the planned experiments on NSCE was emphasized in [150].

The one-dimensional models and their consequences or applications were the subject of studies [151–167]. The comparison of numerical and analytical calculations in the one-dimensional case was made in [168–170]. The radiation due to a single mirror motion in $1 + 1$ space–time dimensions was the subject of studies [171–186], whereas moving mirrors in three (or more) space dimensions were studied in [187–189]. The connection between nonstationary Casimir effect and decoherence of quantum states of mirrors was established in papers [190–192] (within the framework of a one-dimensional model). The decoherence of quantum states of field modes due to their indirect interaction via the moving mirror was studied in [193].

Different interactions between atoms and field modes in cavities with nonstationary boundaries were considered in [194–197]. New schemes of detection or simulation of the Unruh effect by passing accelerated atoms through resonant cavities were discussed in [198,199] and the review of other proposals can be found in [200]. Different boundary conditions, including the case of deformed mirror surfaces, were considered in [201, 202]. The dynamical Lamb effect was introduced in [203, 204] and the dynamical Casimir–Polder force was considered in [205]. Applications of the dynamical Casimir effect to the cosmological and astrophysical problems were discussed in [206–209].

Many authors studied the quantum motion of mirrors interacting with quantized field modes in the adiabatical regime (i.e., when the influence of NSCE is negligible) [210–222]. (For the pioneer studies see, e.g., [223, 224].) The entanglement between quantum states of the mirror and field was considered in [225–231], whereas the entanglement and exchange of quantum states between the field modes (photons) in cavities with moving boundaries (or beam splitters) were studied in [204, 232–235]. Quantum interferometers with moving mirrors were considered in [236–241] ("dynamical interferometers" with time-dependent reflection and transmission coefficients or space/time field modulation were studied in [242, 243]). A numerical simulation of the reflection of femtosecond laser pulses from the oscillating boundary of overdense plasma was performed in [244]. A proposal to use the dynamical Casimir effect for the spacecraft propulsion was made in [245].

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