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Dedicated to the memory of Dr. Andrey Vinogradov on the occasion of the 60th anniversary of his birth.

THE HEISENBERG–LANGEVIN MODEL OF A QUANTUM DAMPED HARMONIC OSCILLATOR WITH TIME-DEPENDENT FREQUENCY AND DAMPING COEFFICIENTS

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Abstract

We develop a consistent model of a quantum damped harmonic oscillator with arbitrary time-dependent frequency and damping coefficients within the framework of the Heisenberg–Langevin equations with two noncommuting delta-correlated noise operators justifying the choice of the "minimal noise" set of damping coefficients and correlation functions.

Keywords: Nonstationary quantum damped oscillator, Heisenberg–Langevin equations, noncommuting noise operators, nonstationary Casimir effect.

1. Introduction

The problem of quantum harmonic oscillator with an arbitrary time-dependent frequency and an external time-dependent force was solved for the first time in the seminal paper by Husimi [1] in 1953. After this, it was considered in numerous publications, references to which can be found, e.g., in reviews [2, 3]. Husimi showed that all dynamical properties of the quantum oscillator are determined by the fundamental set of solutions of the classical equation of motion

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0. \tag{1}$$

In particular, if $\omega(t) = \omega_i$ for $t \to -\infty$ and initially (at $t \to -\infty$) the oscillator was in the vacuum state, then the mean energy at the time moment t equals

$$\mathcal{E}(t) = \frac{1}{4}\left[|\dot{\varepsilon}(t)|^2 + \omega^2(t)|\varepsilon(t)|^2\right], \tag{2}$$

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where the function $\varepsilon(t)$ satisfies Eq. (1) and the initial condition

$$\varepsilon_{t\to-\infty} = \omega_i^{-1/2} e^{-i\omega_i t}.$$

Formula (2) holds for any arbitrary function $\omega(t)$ provided this function is real, i.e., the quantum evolution is unitary.

Our goal is to obtain a generalization of formula (2) to the case of nonunitary evolution due to the presence of damping. Although the problem of quantum damped oscillator has been intensively studied since the first years of quantum mechanics [4, 5], only the case of constant frequency and damping coefficient was considered until now.[1] The case of harmonically varying frequency and constant damping coefficients was considered in [8]. However, there are important cases where all the parameters, including damping coefficients, are time-dependent.

An example is the nonstationary Casimir effect (NSCE) in cavity QED. This refers to the phenomenon of photon creation from vacuum (or another initial state of the field) due to the change of geometry or material properties of the cavity (see [9, 10] for extensive reference lists). Indeed, such a change is accompanied by the shift of eigenfrequencies of the field modes. If the interaction between the modes can be neglected, then it seems reasonable to believe that one can use the model of quantum time-dependent oscillator to evaluate the number of photons created in the selected mode [11, 12]. The most promising scheme of obtaining the required changes of the cavity parameters consists in the periodical creation of a thin highly conducting layer on the surface of a semiconductor slab (attached to one of the cavity walls) by means of short (picosecond) laser pulses [13]. However, dealing with the semiconductor mirror one cannot neglect the fact that the dielectric permeability $\epsilon(x)$ of the conducting medium (semiconductor slab after excitation by the laser pulse) is a complex function

$$\epsilon = \epsilon_1 + i\epsilon_2, \qquad \epsilon_2 = \frac{2\sigma}{f_0},$$

where σ and f_0 are the conductivity (in the CGS units) and frequency in [Hz], respectively. For example, $\epsilon_2 \sim 10^8$ for Cu at 2.5 GHz. This means that the time-dependent shift of the resonance frequency of the cavity is not a real but a complex function. Although the conductivity can be very small for a nonilluminated semiconductor slab at low temperatures, and it can be very high after the illumination (when high concentrations of carriers are achieved), it inevitably passes through intermediate values during the process of excitation and recombination, when ε_2 gradually increases from very low to very high values and after that returns to the initial value. Thus ϵ_2 has the same order of magnitude as ϵ_1 during some intervals of time. As a consequence, the time-dependent instantaneous eigenfrequency of the field mode becomes complex

$$\Omega(t) = \omega(t) - i\gamma(t). \tag{3}$$

Moreover, the time-dependent damping coefficient $\gamma(t)$ can be of the same order of magnitude as the frequency variation $\omega(t) - \omega_i$ [10].

[1] The arbitrary functions $\omega(t)$ and $\gamma(t)$ were considered within the framework of the so-called Caldirola–Kanai model of a quantum damped oscillator (see [6] and review [7] for other references). However, this model has defects, because it implies the unitary evolution. In particular, it does not describe the relaxation to the equilibrium state.

2. Choice of the "Minimal Set" of Parameters

We follow the quantum-noise-operator approach proposed by several authors [14–17] as far back as the beginning of the 1960s (see also [18] for an extensive review). It consists in the description of dissipative quantum systems within the framework of the Heisenberg–Langevin operator equations. In the case of damped oscillator, these equations can be written as follows:

$$\frac{d\hat{x}}{dt} = \hat{p} - \gamma_x(t)\hat{x} + \hat{F}_x(t), \tag{4}$$

$$\frac{d\hat{p}}{dt} = -\gamma_p(t)\hat{p} - \omega^2(t)\hat{x} + \hat{F}_p(t). \tag{5}$$

Here $\hat{x}$ and $\hat{p}$ are the quadrature operators of the selected mode. To simplify the formulas, we normalize these operators by the initial frequency in such a way that they are dimensionless and the mean number of photons equals

$$\mathcal{N} = \frac{1}{2}\langle \hat{p}^2 + \hat{x}^2 - 1\rangle.$$

In other words, in the subsequent formulas ω and γ are the frequency and damping coefficient normalized by the initial frequency ω_i. The noise operators $\hat{F}_x(t)$ and $\hat{F}_p(t)$ are assumed to commute with $\hat{x}$ and $\hat{p}$.

The system of linear equations (4) and (5) can be solved explicitly for arbitrary time-dependent functions $\gamma_{x,p}(t)$, $\omega(t)$, and $\hat{F}_{x,p}(t)$

$$\hat{x}(t) = e^{-\Gamma(t)}\left\{\hat{x}_0 \mathrm{Re}\left[\xi(t)\right] - \hat{p}_0 \mathrm{Im}\left[\xi(t)\right]\right\} + \hat{X}(t), \tag{6}$$

$$\hat{p}(t) = e^{-\Gamma(t)}\left\{\hat{x}_0 \mathrm{Re}\left[\eta(t)\right] - \hat{p}_0 \mathrm{Im}\left[\eta(t)\right]\right\} + \hat{P}(t), \tag{7}$$

where $\hat{x}_0$ and $\hat{p}_0$ are the initial values of operators at $t = -\infty$ (taken as the initial instant),

$$\Gamma(t) = \int_{-\infty}^{t} \gamma(\tau)\, d\tau, \qquad \gamma(t) = \frac{1}{2}\left[\gamma_x(t) + \gamma_p(t)\right], \tag{8}$$

$$\hat{X}(t) = e^{-\Gamma(t)} \int_{-\infty}^{t} d\tau\, e^{\Gamma(\tau)} \mathrm{Im}\left\{\xi^*(t)\left[\hat{F}_p(\tau)\xi(\tau) - \hat{F}_x(\tau)\eta(\tau)\right]\right\}, \tag{9}$$

$$\hat{P}(t) = e^{-\Gamma(t)} \int_{-\infty}^{t} d\tau\, e^{\Gamma(\tau)} \mathrm{Im}\left\{\eta^*(t)\left[\hat{F}_p(\tau)\xi(\tau) - \hat{F}_x(\tau)\eta(\tau)\right]\right\}, \tag{10}$$

and $\xi(t)$ is the special solution to Eq. (1) with $\omega^2(t)$ replaced by the effective frequency

$$\omega_{\mathrm{eff}}^2(t) = \omega^2(t) + \dot{\delta}(t) - \delta^2(t), \qquad \delta(t) = \frac{1}{2}\left[\gamma_x(t) - \gamma_p(t)\right]. \tag{11}$$

This special solution is selected by the initial condition $\xi(t) = \exp(-it)$ for $t \to -\infty$, which is equivalent to fixing the value of the Wronskian:

$$\xi\dot{\xi}^* - \dot{\xi}\xi^* = 2i. \tag{12}$$

The function $\eta(t)$ is defined as

$$\eta(t) = \dot{\xi}(t) + \delta(t)\xi(t). \tag{13}$$

It seems natural to identify the functions $\omega(t)$ and $\gamma(t)$ in Eqs. (5), (8), and (11) with the real and imaginary parts of the instantaneous cavity eigenfrequency Ω in Eq. (3).

Now we shall try to find the best set of other coefficients.

The first step is to calculate the commutator $[\hat{x}(t), \hat{p}(t)]$. An immediate consequence of Eqs. (6) and (7) is the formula

$$[\hat{x}(t), \hat{p}(t)] = i\hbar e^{-2\Gamma(t)} + \left[\hat{X}(t), \hat{P}(t)\right]. \tag{14}$$

In proving (14) the Wronskian formula (12) and its consequence

$$\mathrm{Im}\,[\xi(t)\eta^*(t)] \equiv 1 \tag{15}$$

play the crucial role.

We suppose, as frequently assumed in the theory of Heisenberg–Langevin equation, that the noise operators are delta-correlated (this assumption is equivalent to the Markov approximation)

$$\langle \hat{F}_j(t)\hat{F}_k(t')\rangle = \delta(t-t')\chi_{jk}(t), \qquad j,k = x,p, \tag{16}$$

Using Eqs. (9), (10), (15), and (16), we find

$$\left\langle \left[\hat{X}(t), \hat{P}(t)\right]\right\rangle = e^{-2\Gamma(t)} \int_{-\infty}^{t} \left[\chi_{xp}(\tau) - \chi_{px}(\tau)\right] e^{2\Gamma(\tau)} d\tau. \tag{17}$$

If we assume now that

$$\chi_{xp}(t) - \chi_{px}(t) = 2i\hbar\dot{\Gamma}(t) \equiv 2i\hbar\gamma(t), \tag{18}$$

then

$$\left\langle \left[\hat{X}(t), \hat{P}(t)\right]\right\rangle = i\hbar\left[1 - e^{-2\Gamma(t)}\right], \tag{19}$$

and the commutator $[\hat{x}(t), \hat{p}(t)] = i\hbar$ is preserved exactly (after averaging over noise operators) for any function $\gamma(t)$.

In contrast to the classical Langevin equations, which contain a single stochastic force, in the quantum case one has to use two noise operators, otherwise the canonical commutation relations cannot be saved in the presence of damping.

For the mean values of the operators $\hat{x}^2(t)$ and $\hat{p}^2(t)$, we find the following expressions (from now on we shall use dimensionless variables, making the Planck's constant formally equal to 1; besides, we shall replace sometimes the symbol of function $f(t)$ by a short form f_t):

$$\begin{aligned}\langle \hat{x}^2(t)\rangle \;=\;& e^{-2\Gamma(t)}\left\{\langle \hat{x}^2\rangle_0 \left[\mathrm{Re}\,(\xi_t)\right]^2 + \langle \hat{p}^2\rangle_0 \left[\mathrm{Im}\,(\xi_t)\right]^2 - \langle \hat{x}\hat{p} + \hat{p}\hat{x}\rangle_0 \mathrm{Re}\,(\xi_t)\,\mathrm{Im}\,(\xi_t)\right\}\\ &+ e^{-2\Gamma(t)}\int_{-\infty}^{t} d\tau\, e^{2\Gamma(\tau)}\Big\{\chi_{xx}(\tau)\left[\mathrm{Im}\,(\xi_t\eta_\tau^*)\right]^2 + \chi_{pp}(\tau)\left[\mathrm{Im}\,(\xi_t\xi_\tau^*)\right]^2\\ &- \left[\chi_{xp}(\tau) + \chi_{px}(\tau)\right]\mathrm{Im}\,(\xi_t\eta_\tau^*)\,\mathrm{Im}\,(\xi_t\xi_\tau^*)\Big\},\end{aligned} \tag{20}$$

$$\begin{aligned}\langle \hat{p}^2(t)\rangle \;=\;& e^{-2\Gamma(t)}\left\{\langle \hat{x}^2\rangle_0 \left[\mathrm{Re}\,(\eta_t)\right]^2 + \langle \hat{p}^2\rangle_0 \left[\mathrm{Im}\,(\eta_t)\right]^2 - \langle \hat{x}\hat{p} + \hat{p}\hat{x}\rangle_0 \mathrm{Re}\,(\eta_t)\,\mathrm{Im}\,(\eta_t)\right\}\\ &+ e^{-2\Gamma(t)}\int_{-\infty}^{t} d\tau\, e^{2\Gamma(\tau)}\Big\{\chi_{xx}(\tau)\left[\mathrm{Im}\,(\eta_t\eta_\tau^*)\right]^2 + \chi_{pp}(\tau)\left[\mathrm{Im}\,(\eta_t\xi_\tau^*)\right]^2\\ &- \left[\chi_{xp}(\tau) + \chi_{px}(\tau)\right]\mathrm{Im}\,(\eta_t\eta_\tau^*)\,\mathrm{Im}\,(\eta_t\xi_\tau^*)\Big\}.\end{aligned} \tag{21}$$

Formulas (20) and (21) are exact, and they hold formally for arbitrary functions $\gamma_{x,p}(t)$, $\omega(t)$, and $\hat{F}_{x,p}(t)$. However, admissible physical sets of damping and force correlation coefficients must obey certain restrictions, which follow from the condition of nonnegative definiteness of the statistical operator, or its consequence — the requirement of nonviolation of the uncertainty relations. This subject was studied in detail in [3, 19–23] (similar or equivalent results can be found also in [24–31]). It was shown in these studies that the nonnegative definiteness of the statistical operator can be preserved during the evolution for arbitrary physical initial states provided the "noise matrix"

$$\mathcal{K} = \left\| \begin{array}{cc} \chi_{xx} & \chi_{xp} \\ \chi_{px} & \chi_{pp} \end{array} \right\| \tag{22}$$

is hermitian and non-negatively definite. The hermiticity of (22) together with Eq. (18) gives rise to the relations

$$\chi_{xp} = \chi_s + i\gamma, \qquad \chi_{px} = \chi_s - i\gamma, \tag{23}$$

where χ_s is a real parameter. Then the non-negativity of matrix (22) results in the important restriction

$$\det \mathcal{K} = \chi_{xx}\chi_{pp} - \chi_s^2 \geq \gamma^2, \tag{24}$$

which tells us once again that one cannot use only one noise operator (taking $\chi_{xx} = 0$ or $\chi_{pp} = 0$).

Let us consider the case of time-independent frequency $\omega = \omega_i = 1$ and time-independent damping and noise coefficients. Moreover, suppose that $\gamma \ll 1$ (small damping). Then one can neglect the correction $\delta^2 \sim \gamma^2$ in ω_{eff} (11) and use the solution $\xi(t) = \exp(-it)$. The integrals in Eqs. (20) and (21) can be calculated exactly in this special case. We assume that $\gamma_{x,p} \sim \gamma$ and $\chi_{jk} \sim \gamma$ (in accordance with the fluctuation–dissipation theorem). Then neglecting terms of the order of γ^2, we obtain for $t \to \infty$ the following expressions for the steady-state statistical moments of the second order:

$$\langle \hat{x}^2 \rangle_\infty = \frac{1}{4\gamma} \left[\chi_{xx} + \chi_{pp} + 2\gamma_p \chi_s \right], \qquad \langle \hat{p}^2 \rangle_\infty = \frac{1}{4\gamma} \left[\chi_{xx} + \chi_{pp} - 2\gamma_x \chi_s \right], \tag{25}$$

$$\langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle_\infty = \frac{1}{2\gamma} \left[\gamma_x \chi_{pp} - \gamma_p \chi_{xx} \right]. \tag{26}$$

We see that the steady-state moments of the second order coincide with the thermodynamic equilibrium values

$$\langle \hat{x}^2 \rangle_{\text{eq}} = \langle \hat{p}^2 \rangle_{\text{eq}} = \frac{1}{2} + \langle n \rangle_{\text{th}}, \qquad \langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle_{\text{eq}} = 0$$

(where $\langle n \rangle_{\text{th}}$ is the mean number of quanta in the thermal state; remember that we assume here $\hbar = \omega = 1$), with accuracy of the order of γ^2 (i.e., without linear corrections with respect to the damping coefficients) provided the noise correlation coefficients are chosen as

$$\chi_s = 0, \qquad \chi_{xx} = \gamma_x G, \qquad \chi_{pp} = \gamma_p G, \qquad G = 1 + 2\langle n \rangle_{\text{th}} = \coth\left(\frac{\hbar\omega_i}{2k_B T} \right), \tag{27}$$

where T is the temperature of the reservoir. In such a case, the condition (24) takes the form

$$G^2 \gamma_x \gamma_p \geq \frac{(\gamma_x + \gamma_p)^2}{4}. \tag{28}$$

At zero temperature of the reservoir ($G = 1$) inequality (28) can be satisfied with a unique choice of the damping coefficients

$$\gamma_x = \gamma_p = \gamma. \tag{29}$$

For $G > 1$, strictly speaking, the choice (29) is not the only possible one but it seems to be the most natural (if one assumes that the ratio of damping coefficients γ_p/γ_x should not depend on the reservoir temperature). The consequences of this choice (called "the minimal noise set") were studied in detail in [10,32]. In particular, the following generalization of Husimi's formula (2) in the presence of dissipation was obtained (for the initial thermal state):

$$\mathcal{N}(t) = Ge^{-2\Gamma(t)}\left\{\frac{1}{2}E(t) + \int_{-\infty}^{t} d\tau\, e^{2\Gamma(\tau)}\gamma(\tau)\left(E(t)E(\tau) - \mathrm{Re}\left[\tilde{E}^*(t)\tilde{E}(\tau)\right]\right)\right\} - \frac{1}{2}, \tag{30}$$

$$E(\tau) = \frac{1}{2}\left[|\varepsilon(\tau)|^2 + |\dot{\varepsilon}(\tau)|^2\right], \qquad \tilde{E}(\tau) = \frac{1}{2}\left[\varepsilon^2(\tau) + \dot{\varepsilon}^2(\tau)\right]. \tag{31}$$

Formula (30) is exact, and it holds for arbitrary functions $\omega(t)$ and $\gamma(t)$. Its beauty consists in the fact that function $\varepsilon(t)$ is the same as in the nondissipative case.

As a generalization, we can introduce the "asymmetry parameter" y defined according to the relations

$$y = \frac{\gamma_p - \gamma_x}{\gamma_p + \gamma_x}, \qquad \gamma_p = \gamma(1+y), \quad \gamma_x = \gamma(1-y). \tag{32}$$

Then condition (28) gives rise to the following limitations on admissible values of this parameter:

$$y^2 \le 1 - G^{-2} = \cosh^{-2}\left(\frac{\hbar\omega_i}{2k_BT}\right). \tag{33}$$

The generalization of (30) to the case $y \neq 0$ has the form

$$\begin{aligned}\mathcal{N}(t) \;=\; & Ge^{-2\Gamma(t)}\Bigg\{\frac{1}{2}E_\eta(t) + \int_{-\infty}^{t} d\tau\, e^{2\Gamma(\tau)}\gamma(\tau)\Bigg(E_\eta(t)\left[E_\eta(\tau) + yD_\eta(\tau)\right] \\ & -\mathrm{Re}\left(\tilde{E}^*_\eta(t)\left[\tilde{E}_\eta(\tau) + y\tilde{D}_\eta(\tau)\right]\right)\Bigg)\Bigg\} - \frac{1}{2},\end{aligned} \tag{34}$$

$$E_\eta(\tau) = \frac{1}{2}\left[|\xi_\tau|^2 + |\eta_\tau|^2\right], \quad \tilde{E}_\eta(\tau) = \frac{1}{2}\left[\xi_\tau^2 + \eta_\tau^2\right], \quad D_\eta(\tau) = \frac{1}{2}\left[|\xi_\tau|^2 - |\eta_\tau|^2\right], \quad \tilde{D}_\eta(\tau) = \frac{1}{2}\left[\xi_\tau^2 - \eta_\tau^2\right]. \tag{35}$$

Actually, in the case of small damping, one can replace η by $\dot{\xi}$, with an error of the order of γ. Moreover, the terms containing functions $D_\eta(\tau)$ and $\tilde{D}_\eta(\tau)$ in Eq. (34) become very small after the integration, because these functions exhibit fast oscillations. Therefore, up to corrections of the order of γ, formula (34) can be replaced by an expression that has apparently the same form as (30), with the only difference being that the function $\varepsilon(t)$ satisfying Eq. (1) with frequency $\omega^2(t)$ must be replaced by the function $\xi(t)$ that satisfies the same Eq. (1) but with the effective frequency

$$\omega^2_{\text{eff}}(t) = \omega^2(t) - y\dot{\gamma}(t)$$

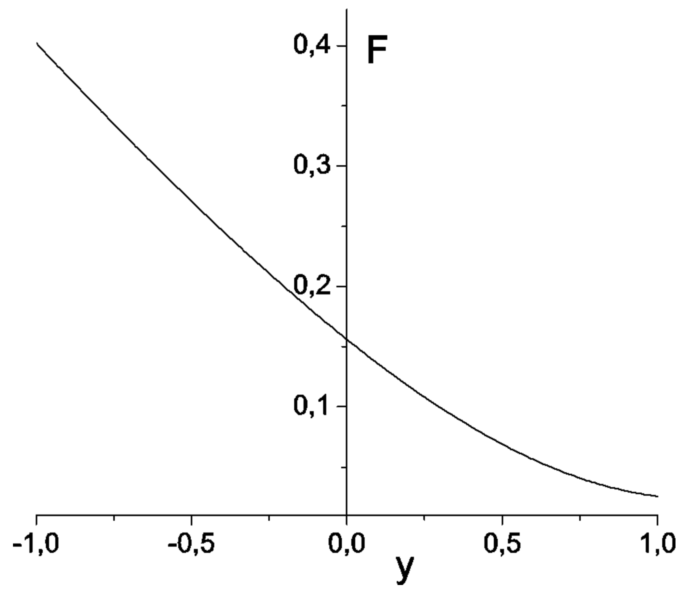

Fig. 1. Normalized amplification factor $F = \nu - \Lambda$ versus the asymmetry parameter y for $\zeta = 1$, $A_0 = 10$, and $\beta = 10/3$.

[the term $y^2\gamma^2(t)$ can be neglected]. In the cases where $\gamma(t)$ has the same order of magnitude as the variation of $\omega(t)$, the final result can be quite different from the case $y = 0$.

For example, in the case of NSCE, the functions $\omega(t) = 1 + \chi(t)$ and $\gamma(t)$ have the form of periodical "pulses" with $\chi(t) = \gamma(t) = 0$ in the time intervals between pulses and $\chi \sim \gamma \ll 1$ during the pulses [10, 32]. Then the mean number of quanta generated in the cavity mode after n pulses in the resonance case is given by the formula (we give here its asymptotic form, which holds if $n(\nu - \Lambda) \gg 1$ and the difference $\nu - \Lambda$ is not too small) [10]

$$\mathcal{N}_n \approx \frac{G\nu}{4(\nu - \Lambda)} \exp[2n(\nu - \Lambda)], \tag{36}$$

where

$$\nu = \left| \int_{t_i}^{t_f} \chi(t) e^{-2it} \, dt \right|, \qquad \Lambda = \int_{t_i}^{t_f} \gamma(\tau) \, d\tau \tag{37}$$

(t_i and t_f are the initial and final time moments of the pulse). For nonzero asymmetry coefficient it is sufficient to replace $\chi(t)$ in Eq. (37) by

$$\chi_y(t) = \chi(t) - \frac{y\dot{\gamma}(t)}{2},$$

or (integrating by parts) by

$$\tilde{\chi}_y(t) = \chi(t) - iy\gamma(t).$$

As was shown in [10], reasonable approximations for the functions $\chi(t)$ and $\gamma(t)$ are

$$\chi(t) = \frac{\zeta A_0^2 \exp(-2\beta t)}{A_0^2 \exp(-2\beta t) + 1}, \qquad \gamma(t) = \frac{\zeta A_0 \exp(-\beta t)}{A_0^2 \exp(-2\beta t) + 1}, \tag{38}$$

where $\zeta \ll 1$ is proportional to the thickness of the semiconductor slab, A_0 is proportional to the intensity of the laser pulse used to create a highly conducting surface layer, and β is the dimensionless recombination time of carriers generated in this layer due to the photo-absorption of the laser radiation. Numerical evaluations of the integrals in Eq. (37) for the parameters $A_0 = 10$ and $\beta = 10/3$ (which are optimal in the case $y = 0$ studied in [10]) show (see Fig. 1) that the difference $\nu - \Lambda$ varies by almost an order of magnitude when the parameter y runs from -1 to 1 providing the worst (minimum) result for $y = 1$ (which corresponds to the classical Langevin equation with $\gamma_x = \chi_{xx} = 0$). Therefore, although we believe that the choice $y = 0$ is the most adequate, further studies based on some microscopic model of damping are necessary.

3. Conclusions

We have constructed a consistent phenomenological model of a damped nonstationary quantum oscillator with arbitrary time-dependent frequency and damping coefficients, based on the generalization of the Senitzky–Schwinger–Haus–Lax noise operator approach. We have derived the set of "minimal noise" equations, which enabled us to obtain a simple generalization of the Husimi solution to the case of a quantum damped oscillator — formula (30). For zero temperature of the reservoir, this set is practically unique if one uses the natural requirements of preservation of the positivity of the density matrix and the maximum closeness of the asymptotic stationary state to the thermodynamical-equilibrium state in the case of constant damping coefficients. For nonzero temperature, other sets of coefficients (depending on the "asymmetry parameter") are possible in principle, although the "minimal set" seems to be the best one from the point of view of simplicity and mathematical beauty. Therefore, the derivation of the coefficients of the Heisenberg–Langevin equations from some deeper "microscopic" model (such as considered, e.g., in [33–36] but for media with arbitrary time-dependent properties) seems to be an important and urgent problem.

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