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Switching off the reservoir through nonstationary quantum systems

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Abstract

In this paper, we demonstrate that the inevitable action of the environment can be substantially weakened when considering appropriate nonstationary quantum systems. Beyond protecting quantum states against decoherence, an oscillating frequency can be engineered to make the system–reservoir coupling almost negligible. Differently from the program for engineering reservoir and similarly to the schemes for dynamical decoupling of open quantum systems, our technique does not require previous knowledge of the state to be protected. However, differently from the previously-reported schemes for dynamical decoupling, our technique does not rely on the availability of tailored external pulses acting faster than the shortest timescale accessible to the reservoir degree of freedom.

1. Introduction

A great deal of attention has recently been devoted to quantum information theory owing to its strategic position, joining up several areas of theoretical and experimental physics. As eventually all domains of low-energy physics may provide potential platforms for the implementation of quantum logic operations, efforts have been concentrated on overcoming some sensitive problems that constitute a spectacular barrier against their realizations. These problem areas touch on both fundamental physics phenomena—such as decoherence and nonlocality—and outstanding technological issues such as individual addressing of quantum systems, separated by only a few micrometers, with insignificant error [1].

As the debate around nonlocality seems to be subsiding through a set of experimental results—such as (i) technological evidence against the so-called loopholes [2], (ii) the demonstrated violation of Bell's inequality with two-photon fringe visibilities in excess of 97% [3] and (iii) highly successful experimental quantum teleportation [4]—the program for quantum state protection is still at an early stage, despite all the achievements. A promising suggestion on this subject refers to the possibility of manipulating the system–reservoir coupling through an additional interaction between the system and a classical ancilla. This control of decoherence through engineered reservoirs has been theoretically implemented for atomic two-level systems, exploiting a structured reservoir [5] or mimicking a squeezed-bath interaction [6]. In the domain of trapped ions, beyond a theoretical proposition [7], engineered reservoirs have also been experimentally implemented for superposed motional states of a single-trapped atom [8]. Another strategy, also experimentally investigated [9], involves collective decoherence, where a composite system interacting with a common reservoir [10] exhibits a decoherence-free subspace (DFS). Whereas a common reservoir is crucial for shielding quantum coherence in a DFS, the quantum-error correction codes QECC [11] work, instead, on the assumption that the decoherence process acts independently on each of the quantum systems encoding a qubit. The issue of the physical grounds for the assumptions behind a common or distinct reservoir is in detail in [12].

We also mention a recent proposal for the control of coherence of a two-level quantum system [13], based on random dynamical decoupling methods [14]. These methods resemble a previous technique to suppress decoherence that used a tailored external driving force acting as pulses [15] which, as in the present paper, was applied to a cavity-mode superposition state. In [16], in a more general scope, the authors formulated a model for decoupling a generic open quantum system from the environmental influence also bailing out on tailored external pulses to induce motions into the

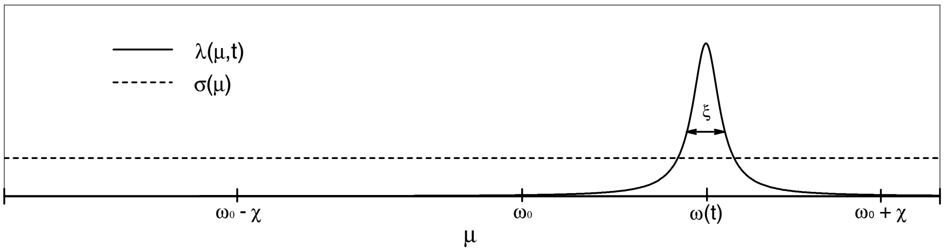

Figure 1. Sketch of the spectral distribution of the reservoir $\sigma(\mu)$, the extrema $\omega_0 \pm \chi$ of the time-dependent frequency of the system, apart from a particular instantaneous frequency $\omega(t)$ around which we have drawn the Lorentzian shape of the system–reservoir coupling with sharpness ξ.

system which are faster than the shortest timescale accessible to the reservoir degree of freedom.

In the present work we achieve the goal of [13–16], which goes beyond the quest for quantum state protection through an engineered reservoir, from a different approach: we demonstrate, arguing from quite general and current assumptions, that a nonstationary resonator could almost be completely decoupled from the environment, rendering the damping factor that characterizes the environment negligible. Differently from the dynamical decoupling methods presented in the literature [13–17], where one must interfere in the system in timescales less than the bath correlation time, our technique takes advantage of the preexisting natural frequency of the system, adding to it a small amplitude to achieve such timescales. Note that, differently from our proposal as well as those in [13–16], the schemes of engineered reservoirs require previous knowledge of the state to be protected. Evidently, this requirement forbids the use of engineered reservoirs for the implementation of logic operations, making the schemes of switching off the system–reservoir interaction more attractive. We observe that the control of decoherence through the frequency modulation of the system–heat-bath coupling has been proposed earlier [18], but as in [13, 14], such control is achieved for a two-level system instead of a cavity mode. Again relying on the assumption of an arbitrarily fast control of the Hamiltonian, the topics of optimal [19] and stochastic [20] control of decoherence count on appropriate tailoring of external laser pulses in the former case and of stochastic modulation of a system parameter in the latter case. We finally mention the multilevel encoding of logical states [21], which makes quantum gates immune to mixing and decoherence effects occurring within encoding subspaces.

2. Our model

Assuming a nonstationary mode coupled to the environment, we get the Hamiltonian

$$H(t) = \omega(t)a^\dagger a + \sum_k \omega_k b_k^\dagger b_k + \sum_k \lambda_k(t)\left(ab_k^\dagger + a^\dagger b_k\right), \qquad (1)$$

with $a^\dagger(a)$ and $b_k^\dagger(b)$ standing for the creation (annihilation) operators of the nonstationary field $\omega(t)$ and the kth bath mode ω_k, respectively. Assuming the time-dependent (TD) relation $\omega(t) = \omega_0 - \chi\sin(\zeta t)$, the system–reservoir couplings also turn out to be TD functions $\lambda_k(t)$. The simple TD form of the free Hamiltonian $H_0 = \omega(t)a^\dagger a + \sum_k \omega_k b_k^\dagger b_k$ enables us to describe, through the transformation $U(t) = \exp\left(-\mathrm{i}\int_0^t H_0(\tau)\,\mathrm{d}\tau\right)$, the Hamiltonian in the interaction picture

$$V(t) = a\Lambda^\dagger(t) + a^\dagger\Lambda(t), \qquad (2)$$

where we have defined the TD operator $\Lambda(t) = \sum_k \lambda_k(t) b_k \exp[\mathrm{i}\Delta_k(t)]$ and parameter $\Delta_k(t) = \Omega(t) - \omega_k t$, with $\Omega(t) = \int_0^t \omega(\tau)\,\mathrm{d}\tau$. For the case of the weak system–reservoir coupling the evolution of the density matrix of the nonstationary field, in the interaction picture and to the second order of perturbation, is given by

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\int_0^t \mathrm{d}t'\,\mathrm{Tr}_R[V(t), [V(t'), \rho_R(0)\otimes\rho(t)]], \qquad (3)$$

where we have employed the usual approximation $\rho_R(0)\otimes\rho(t)$. Assuming that the reservoir frequencies are very closely spaced, with spectral density $\sigma(\mu)$, to allow the continuum summation of the coupling strength of the resonator to the reservoir, such that $\sum_k \to (2\pi)^{-1}\int_0^\infty \mathrm{d}\mu\,\sigma(\mu)$, we have to solve integrals appearing in equation (3), related to correlation functions of the form

$$\int_0^t \mathrm{d}t'\langle\Lambda^\dagger(t)\Lambda(t')\rangle = \mathrm{e}^{-\mathrm{i}\frac{\chi}{\zeta}\cos(\zeta t)}\int_0^t \mathrm{d}t'\,\mathrm{e}^{\mathrm{i}\frac{\chi}{\zeta}\cos(\zeta t')} \times \int_0^\infty \frac{\mathrm{d}\mu}{2\pi}\,\mathrm{e}^{-\mathrm{i}(\mu-\omega_0)(t-t')}\sigma(\mu)N(\mu)\lambda(\mu,t)\lambda(\mu,t'), \qquad (4)$$

where the thermal average excitation of the reservoir $N(\mu)$ is defined by $\langle b^\dagger(\mu)b(\mu')\rangle = N(\mu)\delta(\mu-\mu')$, while the system–reservoir coupling is modelled as

$$\lambda(\mu,t) = \lambda_0\frac{\xi^2}{(\omega(t)-\mu)^2+\xi^2}, \qquad (5)$$

with the parameter ξ accounting for the spectral sharpness around the TD frequency of the nonstationary mode. It is quite reasonable, for the case of the weak system–reservoir coupling considered here, to assume a Lorentzian shape for the function $\lambda(\mu,t)$, centred around the frequency $\omega(t)$. Moreover, as expected, an estimate of the time average of the operator $\Lambda(t)$ reveals that the TD system–reservoir coupling falls with $\lambda_0/|\mu-\omega_0|$, so that the larger the detuning, the smaller the coupling. In figure 1 we present a sketch illustrating the spectral distribution of the reservoir, assumed to be of the usual and realistic Markovian white-noise type, the extrema $\omega_0 \pm \chi$ of the time-dependent frequency of the system, apart from a particular instantaneous frequency $\omega(t)$ around which we have drawn the Lorentzian shape of the (weak) system–reservoir coupling $\lambda(\mu, t)$, with sharpness ξ.

Performing the variable transformations $\tau = \zeta(t - t')$ and $\nu = (\omega_0 - \mu)/\chi - \sin(\zeta t)$ in equation (4) and assuming, as usual, that σ and N are functions that vary slowly around the frequency ω_0, we obtain

$$\int_0^t \mathrm{d}t' \langle \Lambda^\dagger(t)\Lambda(t')\rangle = \varkappa\kappa^4\chi N(\omega_0)\int_0^{\zeta t}\mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\varepsilon F(\tau)} \times \int_{-\infty}^{a}\frac{\mathrm{d}\nu}{2\pi}\frac{\mathrm{e}^{\mathrm{i}\nu\varepsilon\tau}}{(\nu^2+\kappa^2)[(\nu+G(\tau))^2+\kappa^2]}, \tag{6}$$

where, apart from the functions $F(\tau) = \cos(\zeta t - \tau) + \cos(\zeta t) - \tau\sin(\zeta t)$, $G(\tau) = \sin(\zeta t) - \sin(\zeta t - \tau)$ and $a = \omega_0/\chi - \sin(\zeta t)$, we have defined the dimensionless parameters $\varkappa = \Gamma_0/\zeta$, $\kappa = \xi/\chi$ and $\varepsilon = \chi/\zeta$, where $\Gamma_0 = \sigma(\omega_0)\lambda_0^2$ is the well-known damping rate of a stationary mode. Under the assumption that $\chi/\omega_0 \ll 1$, the upper limit a can be extended to infinity and the corresponding integral can be evaluated analytically, leading to the correlation function

$$\begin{aligned}\int_0^t \mathrm{d}t' \langle \Lambda^\dagger(t)\Lambda(t')\rangle &= 2N(\omega_0)\varkappa\kappa^4\chi \\ &\times \int_0^{\zeta t}\mathrm{d}\tau \frac{\mathrm{e}^{\mathrm{i}\varepsilon[F(\tau)+\frac{1}{2}\tau G(\tau)]}}{G^3(\tau)(1+\Theta^2)}\mathrm{e}^{-\varepsilon\kappa\tau} \\ &\times \left\{G(\tau)\cos\left(\frac{\varepsilon\tau G(\tau)}{2}\right) + 2\kappa\sin\left(\frac{\varepsilon\tau G(\tau)}{2}\right)\right\} \\ &= N(\omega_0)\gamma(t).\end{aligned} \tag{7}$$

where $\Theta = 2\kappa/G(\tau)$ and $\gamma(t)$ is related to an effective time-dependent damping rate. We remember that the above correlation function is the only one among sixteen others. Those of the forms $\langle\Lambda^\dagger(t)\Lambda(t')\rangle$ and $\langle\Lambda(t)\Lambda^\dagger(t')\rangle$ give results proportional to $N(\omega_0)$ and $N(\omega_0)+1$, respectively, while those of the forms $\langle\Lambda^\dagger(t)\Lambda^\dagger(t')\rangle$ and $\langle\Lambda(t)\Lambda(t')\rangle$ are null. For the sake of completeness, before analysing the influence of the parameters $\varkappa$, κ and ε on the damping rate of a nonstationary mode, we compute its reduced density operator, obtaining the master equation

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} &= [N(\omega_0)+1]\{2\,\mathrm{Re}[\gamma(t)]a\rho(t)a^\dagger \\ &- \gamma^*(t)\rho(t)a^\dagger a - \gamma(t)a^\dagger a\rho(t)\} \\ &+ N(\omega_0)\{2\,\mathrm{Re}[\gamma(t)]a^\dagger\rho(t)a - \gamma(t)\rho(t)aa^\dagger \\ &- \gamma^*(t)aa^\dagger\rho(t)\}.\end{aligned} \tag{8}$$

Assuming a reservoir at absolute zero, where $N(\omega_0) = 0$, we obtain the simplified form

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} &= [\gamma(t)+\gamma^*(t)]a\rho(t)a^\dagger - \gamma^*(t)\rho(t)a^\dagger a \\ &- \gamma(t)a^\dagger a\rho(t),\end{aligned} \tag{9}$$

whose c-number version, for the normal ordered characteristic function $\chi(\eta,\eta^*,t) = \mathrm{Tr}[\rho(t)\exp(\eta a^\dagger)\exp(-\eta^* a)]$, is given by

$$\begin{aligned}\frac{\partial\chi(\eta,\eta^*,t)}{\partial t} &= -\gamma^*(t)\eta\frac{\partial\chi(\eta,\eta^*,t)}{\partial\eta} \\ &- \gamma(t)\eta^*\frac{\partial\chi(\eta,\eta^*,t)}{\partial\eta^*}.\end{aligned} \tag{10}$$

Assuming a solution of the form $\chi(\eta,\eta^*,t) = \chi(\eta(t),\eta^*(t))$, we obtain $\eta(t) = \eta_0\,\mathrm{e}^{-\Gamma(t)/2}$, where $\eta_0 \equiv \eta(t=0)$ and $\Gamma(t) = \int_0^t \gamma(\tau)\,\mathrm{d}\tau$ is the effective damping rate. Assuming, in addition, that $\chi(\eta,\eta^*,t) = \chi(\eta,\eta^*,t=0)|_{\eta\to\eta(t)}$, we obtain from the Glauber–Sudarshan P-representation and the initial superposition state $|\Psi(t=0)\rangle = \sum_\ell c_\ell|\alpha_{0\ell}\rangle$, the reduced density operator of the nonstationary mode

$$\rho(t) = \mathcal{N}^2\sum_{\ell\ell'} C_{\ell\ell'}(t)|\alpha_\ell(t)\rangle\langle\alpha_{\ell'}(t)|, \tag{11}$$

where $\alpha_\ell(t) = \alpha_{0\ell}\,\mathrm{e}^{-\gamma(t)}$ and

$$\begin{aligned}C_{\ell\ell'}(t) &= \exp\left\{\left[-\tfrac{1}{2}(|\alpha_{0\ell}|^2 + |\alpha_{0\ell'}|^2) + \alpha_{0\ell'}^*\alpha_{0\ell}\right]\right. \\ &\left.\times\left[1 - \mathrm{e}^{-2\,\mathrm{Re}[\Gamma(t)]}\right]\right\}c_{\ell'}^* c_\ell.\end{aligned} \tag{12}$$

We note that, as expected, the decay rate turns out to be a real function even when $\Gamma(t)$ is complex. For the particular case where the nonstationary mode is prepared in the superposition state $|\Psi(0)\rangle = \mathcal{N}(|\alpha_0\rangle + |-\alpha_0\rangle)$, the function multiplying the nondiagonal elements of the density matrix reads

$$C_{12}(t) = \exp[-2|\alpha_0|^2(1 - \mathrm{e}^{-2\,\mathrm{Re}[\Gamma(t)]})]. \tag{13}$$

2.1. The mechanism behind the attenuation of the system–reservoir coupling

We now analyse the influence of the parameters $\varkappa$, κ and ε on the effective damping rate $\Gamma(t)$ which, in its turn, determines the decoherence time of the superposition $|\Psi(0)\rangle$, as given by equations (12) and (13). Starting with the parameter $\varkappa = \Gamma_0/\zeta$, a measure of the rate of variation of the frequency ζ, compared to the damping constant Γ_0, it is evident from equation (7), as expected, that the damping function $\Gamma(t)$ decreases in proportion to $\varkappa$. Otherwise, in the adiabatic regime where ζ approaches Γ_0 we also expect $\Gamma(t)$ to be close to the damping constant Γ_0. Regarding parameter $\kappa = \xi/\chi$, which accounts for the range of oscillation of $\omega(t)$ compared to the Lorentzian sharpness ξ, we expect the damping function to decrease as the range of oscillation χ increases, as long as the variation rate ζ is adjusted to be significantly higher than Γ_0. When both parameters κ and $\varkappa$ are adjusted accordingly, to be significantly smaller than unity, the system–reservoir coupling is weakened as well as the damping function $\Gamma(t)$, consequently increasing the decoherence times of superposition states. Differently from $\varkappa$, our expectation concerning κ is blurred in equation (7): just as is confirmed by the factor κ^4, it is refuted by the decay function $\mathrm{e}^{-\varepsilon\kappa\tau}$ in the integral. Finally, the parameter $\varepsilon = \chi/\zeta$ may also be defined as $\varepsilon = \varkappa/\kappa$, as long as the damping rate Γ_0 approximates the sharpness ξ, weighing the contribution of parameters $\varkappa$ and κ. For the same reason as κ, the role played by ε in the behaviour of $\Gamma(t)$ is also blurred in equation (7).

We stress that the above mathematical analysis could be expected from physical considerations. In fact, although we have assumed a sudden system–reservoir coupling, the characteristic time interval for an appreciable action of the reservoir over a stationary system is around Γ_0^{-1}. However, when the frequency of the system changes continuously, its

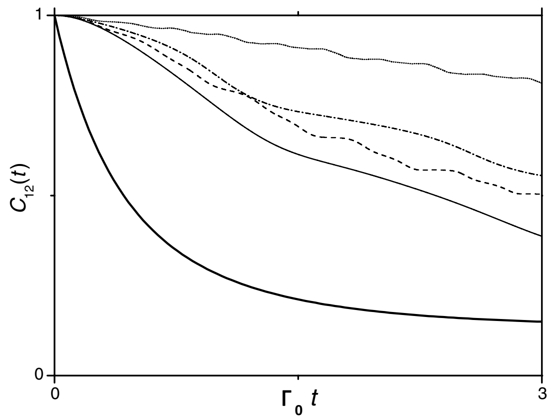

Figure 2. Plot of the function $C_{12}(t)$ against the scaled time $\Gamma_0 t$ for the initial state $|\Psi(0)\rangle = \mathcal{N}(|\alpha_0\rangle + |-\alpha_0\rangle)$ with $\alpha_0 = 1$. The thick solid line corresponds to the case of a stationary mode where $\omega(t) = \omega_0$; the solid and dashed lines correspond to $\varkappa = 1/2$ and $1/10$, for fixed $\kappa = 1/2$; finally, the dashed-dotted and dotted lines correspond to $\varkappa = 1/2$ and $1/10$, for fixed $\kappa = 1/10$.

rate of variation (proportional to ζ) plays a crucial role in the effective coupling between the system and the reservoir. Remembering that this coupling occurs around $\omega(t)$, in a region defined by the Lorentzian sharpness ξ, a significant rate of variation, with ζ larger than Γ_0, makes difficult an effective action of the reservoir over the system since their interaction time is reduced proportionally to $\varkappa$. Otherwise, when ζ is smaller than Γ_0, an effective action of the reservoir takes place, inducing the relaxation of the system before a significant change of its frequency. In its turn, the role of the amplitude of the oscillation χ is to trigger the action of the rate of variation ζ. In fact, when the amplitude χ is smaller than the Lorentzian sharpness ξ, the nonstationary system does not leave the region (in frequency space) of its effective coupling with the reservoir, thus decaying as a stationary system, whatever the value of ζ. However, when χ is larger than ξ, the effective system–reservoir coupling moves to a different region of the spectrum, triggering the action of the rate of variation ζ as described above.

To clarify even better the role of the parameters $\varkappa$ and κ in the damping rate, in figure 2 we plot the function $C_{12}(t)$ against the scaled time $\Gamma_0 t$, considering the initial superposition $|\Psi(0)\rangle = \mathcal{N}(|\alpha_0\rangle + |-\alpha_0\rangle)$ with $\alpha_0 = 1$. The thick solid line corresponds to the case of a stationary mode where $\omega(t) = \omega_0$, prompting the expected result $\Gamma(t) = \Gamma_0 t/2$. Setting $\kappa = 1/2$, the solid and dashed lines correspond to $\varkappa = 1/2$ and $1/10$, respectively. As expected, the damping function decreases as the rate of variation of the frequency increases. In fact, a higher rate of variation works to hinder the system–reservoir coupling, lengthening the response time of the system. With $\kappa = 1/10$, the dashed-dotted and dotted lines correspond to $\varkappa = 1/2$ and $1/10$, showing that the amplitude of oscillation χ is more effective in diminishing the damping rate than the rate of variation ζ. This unexpected result reveals interesting aspects of the physics of nonstationary cavity modes: first, as demonstrated below, (i) the time-dependence of $\omega(t)$—the values of the frequencies χ and ζ—required to practically switch off the system–reservoir coupling can be engineered through atom–field interaction; furthermore, (ii) in the adiabatic regime, where $\zeta/\omega_0, \chi/\omega_0 \ll 1$, the atom–field interaction is still modelled by the Jaynes–Cummings interaction despite the nonstationary mode [22]. Consequently, all the protocols developed for the implementation of processes in stationary modes—for example, quantum state or Hamiltonian engineering and logical devices—become directly applicable to the nonstationary mode considered here.

Figures 3(a)–(e) display the damping process in the evolution of the amplitude of the coherent state $\alpha(t) = \alpha_0 \exp[-i\Omega(t) - \Gamma(t)]$ composing the superposition $|\Psi(t)\rangle$. All these figures refer to the same time interval as that used in figure 2, thus leading to the same number of cycles coming from the rotation in phase space, due to the factor $e^{-i\Omega(t)}$. In all figures the ratio $\omega_0/\Gamma_0 = 10$ is set to a fictitious scale to make clear the spiralling of $\alpha(t)$. In figure 3(a), related to the thick solid line of figure 2, we observe the loss of excitation carrying the initial coherent state to the vacuum state. In this figure we also plot, in a thick dotted line, the evolution of the amplitude $-\alpha(t)$ of the other component of the superposition state. Figures 3(b)–(e) correspond respectively to the solid, dashed, dashed-dotted and dotted lines of figure 2, showing a gradual suppression of the loss of excitation which, differently from figure 3(a), does not occur at a uniform rate, due to the oscillatory character—coming from equation (7)—of their corresponding curves in figure 2. Figure 3(d) clearly reveals this nonuniform character of the excitation loss through the distinct gaps between the cycles described by the amplitude $\alpha(t)$ on its (obstructed) way to the vacuum.

To stress that the mechanism of suppression of decoherence—illustrated in figures 2 and 3 for the particular superposition $|\Psi(0)\rangle = \mathcal{N}(|\alpha_0\rangle + |-\alpha_0\rangle)$—applies for any state; in figure 4 we plot the real part of the effective damping rate $\mathrm{Re}[\Gamma(t)]$ against $\Gamma_0 t$. In fact, back to the master equation (9), we observe that as $\gamma(t)$ decreases, the effective damping rate $\Gamma(t)$ also decreases, weakening both the relaxation and decoherence processes. In the extreme case where $\gamma(t)$ becomes null, we create the conditions for a free evolution. As in figure 2, the thick solid line corresponds to the case of a stationary mode, resulting that $\Gamma(t) = \Gamma_0 t/2$. Confirming that the damping function decreases as the rate of variation ζ increases, the solid and dashed lines correspond to $\varkappa = 1/2$ and $1/10$, with fixed $\kappa = 1/2$. Finally, showing that the damping function decreases as the amplitude of oscillation χ increases, the dashed-dotted and dotted lines correspond to $\varkappa = 1/2$ and $1/10$, with fixed $\kappa = 1/10$. Therefore, there is a complete correspondence between the decoherence process of the particular state $|\Psi(0)\rangle = \mathcal{N}(|\alpha_0\rangle + |-\alpha_0\rangle)$ and the general behaviour of the effective damping rate $\Gamma(t)$, showing that our mechanism to suppress decoherence applies for any state.

Next, considering some sensitive features in the present scheme to control decoherence, we first address the time-dependent system–reservoir coupling $\lambda_k(t)$, which can be justified through the treatment of two coupled harmonic oscillators, one of them with time-dependent frequency. We

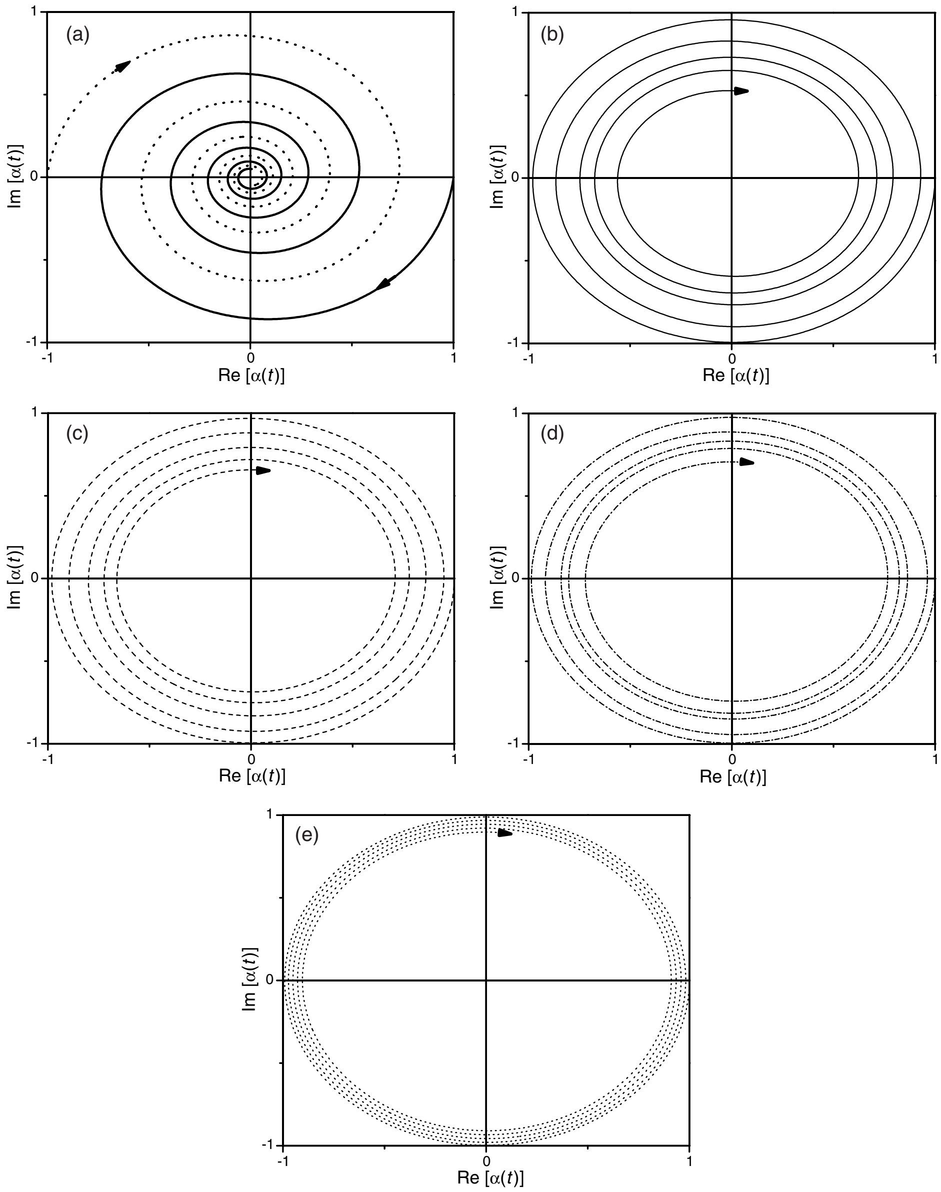

Figure 3. Plot of $\mathrm{Im}(\alpha(t))$ against $\mathrm{Re}(\alpha(t))$ to illustrate the damping process in the evolution of the amplitude of the coherent state $\alpha(t) = \alpha_0 \exp[-\mathrm{i}\Omega(t) - \Gamma(t)]$ composing the superposition $|\Psi(t)\rangle$, with $\alpha_0 = 1$. The patterns of the lines in figures 3(a)–(e) correspond to those in figure 2, with the associated values of parameters κ and $\varkappa$.

start with the usual coupling $\mathcal{C}X_1X_2$, where $X_1(t) = \mathcal{C}_1(t)\big(a_1 + a_1^\dagger\big)$ and $X_2 = \mathcal{C}_2\big(a_2 + a_2^\dagger\big)$. Within the interaction picture and the rotating-wave approximation, we end up with a time-dependent interaction of the form $\mathcal{C}(t)\big(a_1^\dagger a_2 + a_1 a_2^\dagger\big)$, similar to what had been considered in [13, 23–25]. Moreover, as we are assuming a sudden coupling between the system and the reservoir at each time instant, we believe that this coupling 'follows' the modulation of the cavity mode frequency. Our belief is also based on the fact the system–bath coupling in quantum optics systems (as in cavity QED) is sufficiently small, so that we can model it by a Lorentzian function in a good approximation. Considering again two interacting harmonic oscillators [12], now with stationary frequencies ω_1 and ω_2 and coupling strength λ—ω_1 playing the role of the system and ω_2 of one of the bath modes—it can be demonstrated that their effective coupling is given by $\lambda^2/\sqrt{\lambda^2 + \delta^2}$, with $\delta = |\omega_1 - \omega_2|$, justifying a sharp Lorentzian function in the case of weak coupling, where $\lambda \ll \delta$. However, for the case of strong coupling, where $\lambda \gg \delta$ (assuming, as usual, a cutoff frequency for the reservoir) we obtain, instead, a constant effective coupling λ.

3. Engineering the nonstationary mode

The most sensitive point, however, is the engineering of the nonstationary mode whose state is to be protected. There is a great deal of the literature exploring nonstationary modes,

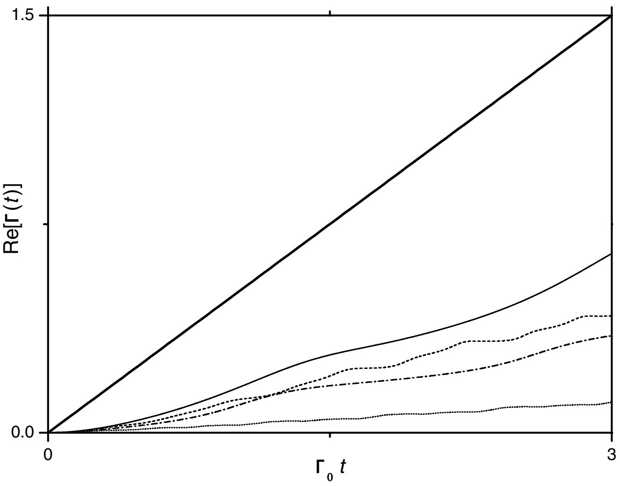

Figure 4. Plot of the real part of the effective damping rate $\mathrm{Re}[\Gamma(t)]$ against $\Gamma_0 t$, illustrating that the mechanism of suppression of decoherence applies for any initial state. As in figure 3, the patterns of the lines correspond to those in figure 2, with the associated values of parameters κ and $\varkappa$.

especially in respect of Casimir effect [26]. We present below a scheme to engineer a nonstationary mode $\omega(t) = \omega_0 + \chi\sin(\zeta t)$ from the interaction of a driven two-level atom (frequency ω_a) with a stationary cavity mode (frequency ω_c) given by

$$\mathbf{H} = \omega_c a^\dagger a + \frac{\omega_a}{2}\sigma_z + F(t)(\sigma_+ \mathrm{e}^{-\mathrm{i}\omega_L t} + \sigma_- \mathrm{e}^{\mathrm{i}\omega_L t}) + G(a\sigma_+ + a^\dagger\sigma_-), \tag{14}$$

where ω_L stands for the frequency of the classical driving field and G denotes the Rabi frequency. The atomic operators are given by $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$ and $\sigma_- = |g\rangle\langle e|$, e and g being the excited and the ground states. We assume the atomic amplification modulated as $F(t) = F_0\cos(\zeta t/2 + \phi)$. In the interaction picture, the transformed Hamiltonian is given by

$$\mathbf{H}_I = G(a\sigma_+ \mathrm{e}^{\mathrm{i}\delta_1 t} + a^\dagger\sigma_- \mathrm{e}^{-\mathrm{i}\delta_1 t}) + F(t)(\sigma_+ \mathrm{e}^{\mathrm{i}\delta_2 t} + \sigma_- \mathrm{e}^{-\mathrm{i}\delta_2 t}), \tag{15}$$

where $\delta_1 = \omega_a - \omega_c$ and $\delta_2 = \omega_a - \omega_L$ are the atom–field and the atom–laser detunings. Next, we define $H_1 = G(a\sigma_+ \mathrm{e}^{\mathrm{i}\delta_1 t} + a^\dagger\sigma_- \mathrm{e}^{-\mathrm{i}\delta_1 t})$ and $H_2 = F(t)(\sigma_+ \mathrm{e}^{\mathrm{i}\delta_2 t} + \sigma_- \mathrm{e}^{-\mathrm{i}\delta_2 t})$, and assume the highly off-resonance laser amplification process, such that $|\delta_2| \gg F_0, \zeta, G, |\delta_1|$ with $G \ll |\delta_1|$. Under this assumption, the strongly oscillating terms of H_2 lead, to a good approximation, to the effective Hamiltonian [27],

$$\mathbf{H}_{\mathrm{eff}} = H_1 - \mathrm{i}H_2(t)\int H_2(\tau)\,\mathrm{d}\tau = \omega_c a^\dagger a + \Omega(t)\sigma_z + g(a\sigma_+ + a^\dagger\sigma_-), \tag{16}$$

where $\Omega(t) = \omega_a/2 + F^2(t)/\delta_1$. The diagonalization of Hamiltonian $\mathbf{H}_{\mathrm{eff}}$ is easily accomplished through the dressed atomic basis $\{|g,n\rangle, |e,n-1\rangle\}$ [28]. Under the usual assumption that $G^2 n \ll \delta_1^2$, we obtain the dispersive atom–field interaction:

$$\mathcal{H} = \omega_c a^\dagger a + \Omega(t)\sigma_z + \Upsilon(t)a^\dagger a\sigma_z, \tag{17}$$

where the adjustment $\phi = \pi/4$ makes $\Upsilon(t) = \Upsilon_1 + \Upsilon_2\sin(\zeta t)$ with $\Upsilon_1 = \left[1 - 3F_0^2/2\delta_1\delta_2\right]G^2/\delta_1$ and $\Upsilon_2 = (G^2/\delta_1)\left(F_0^2/2\delta_1\delta_2\right)$. Evidently, by turning off the laser we obtain the usual Stark shift $\Upsilon a^\dagger a\sigma_z$ with $\Upsilon = G^2/\delta_1$. In a frame rotating with the shifted atomic frequency $\Omega(t)$, obtained through the unitary operator $U = \exp[-\mathrm{i}\widetilde{\Omega}(t)\sigma_z]$ with $\widetilde{\Omega}(t) = \int\Omega(t')\,\mathrm{d}t'$, the state vector associated with the transformed Hamiltonian $\widetilde{\mathcal{H}} = \omega_c a^\dagger a + \Upsilon(t)a^\dagger a\sigma_z$ is given by

$$|\Psi(t)\rangle = \mathrm{e}^{\mathrm{i}\widetilde{\Omega}(t)}|g\rangle|\Phi_g(t)\rangle + \mathrm{e}^{-\mathrm{i}\widetilde{\Omega}(t)}|e\rangle|\Phi_e(t)\rangle, \tag{18}$$

where, in the Fock basis: $|\Phi_\ell(t)\rangle = \sum_n\langle\ell,n|\Psi(t)\rangle|n\rangle$, $\ell = g, e$. Using the orthogonality of the atomic states and equations (17) and (18), we obtain the uncoupled TD Schrödinger equations

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\Phi_\ell(t)\rangle = \widetilde{\mathcal{H}}_\ell|\Phi_\ell(t)\rangle, \tag{19}$$

$$\widetilde{\mathcal{H}}_\ell = \omega_\ell(t)a^\dagger a, \tag{20}$$

where $\omega_g = \omega_c - \Upsilon(t)$ and $\omega_e = \omega_c + \Upsilon(t)$. Therefore, when preparing the atom in the fundamental state, we obtain the TD frequency $\omega(t) = \omega_0 - \chi\sin(\zeta t)$ where, from interaction (17), $\omega_0 = \omega_c + \Upsilon_1$ and $\chi = \Upsilon_2$. Note that the atom crosses the cavity remaining in its ground state (due to its off-resonance interactions with both the cavity mode and the classical field), so that there is no injection of noise coming from the atomic decay to the engineered nonstationary cavity mode. Assuming typical values for the parameters involved in cavity QED experiments [29, 30]: $G \sim 3 \times 10^5\ \mathrm{s}^{-1}$, $|\delta_1| \sim 10^6\ \mathrm{s}^{-1}$, $|\delta_2| \sim 10^7\ \mathrm{s}^{-1}$ and $\Gamma_0 \sim 10^3\ \mathrm{s}^{-1}$, it follows, with the intensity $F_0 \sim 10 \times G$, that $\chi \sim 4 \times 10^4\ \mathrm{s}^{-1}$ with $\zeta \lesssim 10^6\ \mathrm{s}^{-1}$. Since it is reasonable to assume $\xi \sim \Gamma_0$, the value 1/10 for the parameters κ and $\varkappa$ employed to obtain the dotted line of figure 2, is easily accomplished.

Evidently, to circumvent the difficulties introduced by the small time interval of atom–field interaction, it would be interesting to engineer the nonstationary mode through a sequential interaction of atoms, one by one, with the cavity mode. The trapping of an atom inside the cavity, along the lines suggested in [31], is also a possibility to be analysed. Otherwise, nonstationary modes can also be achieved by other schemes as the mechanical motion of the cavity walls [32], suitable for our purpose since the frequency attainable is in the gigahertz range, or even more sophisticated schemes where the effective motion of the walls is generated by the excitation of a plasma in a semiconductor [33].

4. Concluding remarks

We have thus presented in this paper a scheme which practically switches off the reservoir of a cavity field by engineering a suitable nonstationary mode $\omega(t)$. As pointed out, instead of interfering in the system in timescales less than the bath correlation time, as in the dynamical decoupling schemes presented previously, in our technique we exploit the preexisting natural frequency of the system ω_0, adding to it a small amplitude χ (apart from the rate of variation ζ) to meet the timescales of the bath correlation time, around $\omega_0 + \chi$.

We reach the same goal of the dynamic decoupling schemes without directly interfering in the system within such a small timescale. Instead, our procedure relies on the manipulation of frequencies (χ and ζ) whose timescales are considerable larger than the bath correlation time. Besides analysing the physical parameters required to accomplish this process, we also demonstrated how to engineer such a nonstationary mode through its dispersive interaction with a driven atomic system. Evidently, the scheme presented here for a time-dependent cavity mode applies directly to any oscillatory system such as trapped ions, nanomechanical oscillators, and superconducting transmission lines; it can also be extended to any nonstationary quantum system. We believe that both techniques presented here, to protect quantum states through nonstationary quantum systems and to engineer such systems, can play an essential role in quantum information theory.

Beyond the information theory, we believe that the present work can directly contribute to the field of cavity quantum electrodynamics, specifically in the less-explored topic of the interaction of a two-level atom with a nonstationary mode in the adiabatic regime. In fact, as the engineering of a nonstationary mode is relatively easy to accomplish in the adiabatic regime—through the mechanical motion of the cavity walls [32] or atom–field interaction, as demonstrated in this work—typical quantum-optical phenomena may be investigated in this particular context.

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