Phenomenological-operator approach to dissipation in cavity quantum electrodynamics

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We present a phenomenological-operator approach to describe energy dissipation in cavity QED phenomena. This approach, developed for an absolute-zero and a thermal environment, considerably simplifies the introduction of the inevitable errors due to the environmental degrees of freedom when describing processes involving dispersive atom-field interactions. The main result in the present work consists in furnishing a straightforward technique to estimate the fidelity resulting from dispersive atom-field interactions, precluding the necessity of performing the usually extensive *ab initio* calculations. Furthermore, we expect that the present work can help us account for dissipation in resonant atom-field interactions and even help us achieve a general phenomenological approach to estimate the effects of dissipation in whichever system. To illustrate the universal applicability of the present technique, we calculate the fidelity of a mesoscopic quantum superposition state engineered in a lossy cavity, considering also the excited-state spontaneous decay of the required atom. For the case of a stable atomic excited state, the fidelity computed here is in agreement with a recently announced exact calculation.

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I. INTRODUCTION

The physics of open quantum systems has always deserved considerable attention due to the inescapable influence of the environment on microscopic objects [1]. Nowadays, with the advent of interesting new theoretical promises strongly based on quantum coherence and entanglement, the treatment of the inevitable errors introduced by the environment has become increasingly relevant. In fact, the proposals of techniques to overcome these errors, the error correction protocols [2], represent a crucial step for the successful achievement of quantum communication [3], computation [4], and teleportation [5].

The study of dissipative quantum systems is a difficult task due to the infinite number of environmental degrees of freedom. Two appropriate treatments of dissipation in quantum mechanics have been presented by Caldeira and Leggett [6], based on a path-integral formulation, and by Mollow and Glauber [7], using characteristic functions and Glauber's P representation.

Despite the success and accuracy of these two approaches, their application to calculate the fidelity of the above quantum processes turns out to be prohibitive. In fact, the number of quantum systems and steps required by the control-NOT gate, for example, or by the teleportation process of a qubit, makes the exact calculation of the fidelity considerably difficult. With this concern, in this article we present a phenomenological-operator approach which simplifies the accountability of the errors coming from the environment when describing cavity QED phenomena. The essence of this approach is to work in the Schrödinger picture using operators to represent dissipation and decoherence that are assumed rather than derived from first principles. Specifically, we consider the dissipation process of a coherent field in a lossy cavity interacting dispersively with a two-level atom, including the case of an unstable atomic excited state. Such an interaction is the physical process responsible for the preparation of a mesoscopic quantum superposition state (MQSS) trapped in a cavity [8]. This kind of superposition state has recently received particular attention, mainly in the context of estimating the *fidelity* of a prepared or teleported MQSS [9] and of searching for protocols to suppress or, at least, delay its decoherence process [10–12].

Several experiments have been realized in the microwave regime, in which Rydberg atoms are made to interact with a field trapped in a high-Q cavity [8,13]. In the present work we model these realistic experiments by using two-level atoms with an excited state $|e\rangle$ and a ground state $|g\rangle$ which represent circular Rydberg levels with principal quantum numbers 51 and 50, respectively. Using this simplified model, we estimate the fidelity of a MQSS engineered in a high-Q cavity through a particular atomic-state superposition. The experimental engineering setup of Fig. 1 consists of a two-level Rydberg atom A, which crosses a Ramsey-type arrangement, i.e., a high-Q cavity C_1 sandwiched by two Ramsey zones R_1 and R_2. After interacting with the Ramsey-type arrangement, the atom is counted with high efficiency by the ionization detector D.

Concerning the required dispersive atom-field interaction, the Rydberg atoms must be engineered so that the excited state $|e\rangle$ is coupled to a third more excited state $|i\rangle$ with a transition frequency $\omega_0=\omega+\Delta$, ω being the cavity frequency. The $|e\rangle\rightarrow|g\rangle$ transition frequency is far from resonance with the cavity mode. However, the detuning Δ between the $|e\rangle\rightarrow|i\rangle$ transition frequency and the cavity

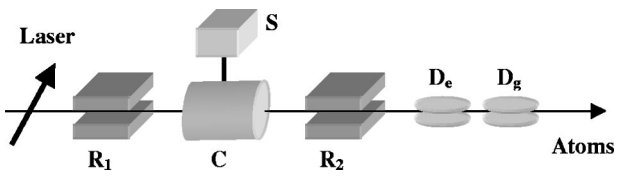

FIG. 1. Sketch of the experimental setup for engineering the MQSS.

frequency is properly adjusted so that only virtual transitions occur between these levels. The effective Hamiltonian describing these dispersive interactions can be written as $H_{\rm eff}=\hbar\chi a^{\dagger}a\sigma_3$, where $a^{\dagger}a$ is the field number operator, $\sigma_3=|i\rangle\langle i|-|e\rangle\langle e|$, and $\chi=\delta^2/2\Delta$, where δ is the atomic dipole moment. This Hamiltonian is derived under the assumption that $\delta^2 n\ll\Delta^2+\kappa^2$, where n is a characteristic photon number and κ is the spontaneous-emission rate [14]. We assume that $\chi=0$ when the atom is outside the cavity and, for simplicity, we suppose that the atom-field coupling is turned on (off) suddenly at the instant the atom enters (leaves) the cavity region.

This paper is organized as follows. In Sec. II we describe the phenomenological-operator approach at absolute-zero temperature. We illustrate this technique for the decay of a free atomic excited state, the decay of a free cavity field, and a coherent field in a lossy cavity interacting dispersively with both, a stable and an unstable, two-level atom. In Sec. III, we generalize the approach of Sec. II to treat the case of a reservoir at finite temperature.

II. ENVIRONMENT AT ABSOLUTE ZERO

We emphasize that, in this section, the treatment of the errors involved in preparing the MQSS will be considered at absolute zero. In fact, this constitutes a good approximation since the required Ramsey-type arrangement R_1-C-R_2 in Fig. 1, where C comprehends a high-Q cavity sandwiched by Ramsey zones R_1 and R_2, is cooled to 0.6 K by a ^{3}He-^{4}He refrigerator to avoid blackbody radiation (0.02 thermal photon on average in C) [13].

A. Decay of the free atomic excited state

Here we observe that the coupling of the atomic states to a surrounding environment $\mathcal{E}$ can be described by the relations

$$|g\rangle|\mathcal{E}\rangle\stackrel{U_t}{\rightarrow}|g\rangle\hat{\mathcal{T}}_0|\mathcal{E}\rangle, \tag{1a}$$

$$|e\rangle|\mathcal{E}\rangle\stackrel{U_t}{\rightarrow}|e\rangle\hat{\mathcal{T}}_e|\mathcal{E}\rangle+|g\rangle\hat{\mathcal{T}}_g|\mathcal{E}\rangle, \tag{1b}$$

where $|\mathcal{E}\rangle$ denotes the initial state of the environment and the operators $\hat{\mathcal{T}}$, acting on this state, account for the atom-environment coupling. In the microwave regime required by the cavity-QED setup, the transition frequency between the atomic Rydberg levels being $\nu=51.099$ GHz, it is reasonable to approximate $|\mathcal{E}\rangle$ by the vacuum state. Moreover, as mentioned above, the mean number of thermal photons in the cavity which the atom crosses is exceedingly small. Accordingly, we assume that $\hat{\mathcal{T}}_0=\mathbf{1}$, $\hat{\mathcal{T}}_e=\mathbf{f}(t)=e^{-\kappa t}\mathbf{1}$, $\hat{\mathcal{T}}_g=\Sigma_j\mathbf{g}_j(t)\hat{b}_j^{\dagger}$, with $\Sigma_j|\mathbf{g}_j(t)|^2=1-e^{-2\kappa t}$, κ denoting the spontaneous decay rate of the atomic excited state, $b_j^{\dagger}$ and b_j are, respectively, the creation and annihilation operators of the jth oscillator mode of the environment, and t is the required time for the accomplishment of a given process [15]. It can be readily seen that the superposition $(|g\rangle+|e\rangle)/\sqrt{2}$ leads to the reduced density operator:

$$\rho=\frac{1}{2}\{\exp(-2\kappa t)|e\rangle\langle e|+[2-\exp(-2\kappa t)]|g\rangle\langle g| +\exp(-\kappa t)(|e\rangle\langle g|+|g\rangle\langle e|)\}, \tag{2}$$

the relaxation and decoherence times being exactly the same.

Let us note that the evolution (1) is consistent with the well-known result, following from the Weisskopf-Wigner approximation, that an unstable atomic state decays exponentially. In this case, the phenomenological-operator evolution leads to the same atomic density operator as the one we obtain using an *ab initio* master equation approach.

B. Decay of the free field

We can also triumph over brute force when treating the decay of the free field described by the Hamiltonian

$$H_{f,\mathcal{E}}=\hbar\omega a^{\dagger}a+\sum_k\hbar\omega_k b_k^{\dagger}b_k+\sum_k\hbar(\lambda_k a^{\dagger}b_k+\lambda_k^{*}ab_k^{\dagger}), \tag{3}$$

where the subscripts $f,\mathcal{E}$ refer to the field-environment coupling. The operators $a^{\dagger}$ and a are, respectively, the creation and annihilation operators for the cavity mode of frequency ω, whereas $b_k^{\dagger}$ and b_k are the analogous operators for the kth bath oscillator mode, whose corresponding frequency and coupling constant write ω_k and λ_k, respectively.

We first invoke the following remarkable result by Mollow and Glauber [7]: (i) that an initial coherent state remains coherent despite losing excitation when coupled to an environment at absolute zero; its field amplitude at time t is given by $\alpha_t=\alpha_0 e^{-\Gamma t/2}$, where Γ is the cavity-field decay rate and α_0 is the initial field amplitude. Accordingly, we assume that, under the time evolution $U_t=e^{-iH_{f,\mathcal{E}}t/\hbar}$ dictated by Hamiltonian (3), $|\pm\alpha_0\rangle|\mathcal{E}\rangle\stackrel{U_t}{\rightarrow}|\pm\alpha_t\rangle\hat{\mathcal{L}}_{\pm}|\mathcal{E}\rangle$, where the operators $\hat{\mathcal{L}}_{\pm}$ act on the initial environmental state $|\mathcal{E}\rangle$. These operators are assumed to account for the effects of the field-environment coupling, while $|\mathcal{E}\rangle$ represents the coherent state of the set of N initially decoupled harmonic oscillators $|\beta_1,\beta_2,\ldots,\beta_N\rangle=|\{\beta_k\}\rangle$, simulating the cavity damping mechanism. We next observe that: (ii) a superposition field state always gets entangled to its environment, and it follows that $\hat{\mathcal{L}}_{\pm}|\mathcal{E}\rangle=|\mathcal{E}_{\pm,t}\rangle$, with $\mathcal{E}_{\pm,t}=|\{\pm\beta_{k,t}\}\rangle=|\pm\beta_{1,t},\pm\beta_{2,t},\ldots,\pm\beta_{N,t}\rangle$. We also observe, a result that can be formally proved [16,17], that (iii) at absolute zero the environment coherently receives the excitation lost by the field, i.e., $b|\beta_{k,t}\rangle=\beta_{k,t}|\beta_{k,t}\rangle$. We thus proceed to describe the field dissipation through the evolution

$$|\pm\alpha\rangle|\mathcal{E}\rangle\stackrel{U_t}{\rightarrow}|\pm\alpha_t\rangle|\{\pm\beta_{k,t}\}\rangle. \tag{4}$$

Finally, it can be noted that (iv) the excitation operator $a^{\dagger}a+\Sigma_k b_k^{\dagger}b_k$ is a conserved quantity, implying that $\Sigma_k|\beta_{k,t}|^2=|\alpha_0|^2-|\alpha_t|^2=|\alpha_0|^2(1-e^{-\Gamma t})$.

It is straightforward to conclude that, under the evolution (4), an initially prepared MQSS $\mathcal{N}(|\alpha\rangle \pm e^{i\varphi}|-\alpha\rangle)$ implies a corresponding density operator which is exact, that is, the one obtained from *ab initio* techniques [7]

$$\rho(\tau)=\frac{1}{2}[|\alpha_t\rangle\langle\alpha_t|+|-\alpha_t\rangle\langle-\alpha_t|\pm e^{-2|\beta_t|^2} \times(e^{-i\varphi}|\alpha_t\rangle\langle-\alpha_t|+e^{i\varphi}|-\alpha_t\rangle\langle\alpha_t|)], \quad (5)$$

the decoherence time being $\tau_D \cong 1/(2|\alpha|^2\Gamma)$.

C. Coherent field in a lossy cavity interacting dispersively with a two-level atom

1. The case of a stable atomic excited state

The Hamiltonian including the dispersive interaction between a lossy cavity field and a two-level atom with a stable excited state is described, through Glauber's assumption of a linear field-environment coupling, as

$$H_{a,f,\mathcal{E}}=\frac{\hbar\omega_0}{2}\sigma_z+H_{f,\mathcal{E}}+H_{\rm eff}, \quad (6)$$

where the subscripts $a,f,\mathcal{E}$ refer to the atom-field-environment entanglement generated by the field-environment and atom-field couplings. The Hamiltonian $H_{f,\mathcal{E}}$ is defined in Eq. (3), while the effective dispersive interaction between the atom and the field $H_{\rm eff}$ is defined in the Introduction. As also mentioned in the Introduction, ω_0 is the atomic $|e\rangle \rightarrow |g\rangle$ transition frequency, which is far from resonance with the cavity mode ω. Finally, $\sigma_z=|e\rangle\langle e|-|g\rangle\langle g|$. The Schrödinger state vector associated with the Hamiltonian above can be written as

$$|\Psi(t)\rangle=e^{i\omega_0 t/2}|g\rangle|\Phi_g(t)\rangle+e^{-i\omega_0 t/2}|e\rangle|\Phi_e(t)\rangle, \quad (7)$$

with

$$|\Phi_\ell(t)\rangle=\int\frac{d^2\alpha}{\pi}\int\left\{\frac{d^2\beta_k}{\pi}\right\}\mathcal{A}_l(\alpha,\{\beta_k\},t)|\alpha,\{\beta_k\}\rangle, \quad (8)$$

where $\ell=g,e$, the complex quantities α and β_k stand for the eigenvalues of a and b_k, respectively, and $\mathcal{A}_\ell(\alpha,\{\beta_k\},t)$ are the expansion coefficients for $|\Phi_\ell(t)\rangle$ in the basis of coherent-state products $\{|\alpha,\{\beta_k\}\rangle\}$. Using the orthogonality of the atomic states and Eqs. (6) and (7) we obtain the uncoupled time-dependent Schrödinger equations

$$i\hbar\frac{d}{dt}|\Phi_\ell(t)\rangle=\mathcal{H}_\ell|\Phi_\ell(t)\rangle, \quad (9)$$

with

$$\mathcal{H}_\ell=\hbar\omega_\ell a^\dagger a+\sum_k\hbar\omega_k b_k^\dagger b_k+\sum_k\hbar(\lambda_k a^\dagger b_k+\lambda_k^* a b_k^\dagger), \quad (10)$$

where $\omega_g=\omega$ and $\omega_e=(\omega-\chi)$. Note that the problem has been reduced to that of the free dissipation of a cavity field whose frequency is now ω_ℓ. As a consequence, the dissipating field evolves according to Eq. (4). However, in this case the field amplitude $\alpha_{\ell,t}$ is given by [7]

$$\alpha_{\ell,t}=\alpha_0 e^{-\Gamma t/2}e^{-i\omega_l t}. \quad (11)$$

Defining $\tilde{\alpha}_t=\alpha_0 e^{-\Gamma t/2}e^{-i\omega t}$, we get $\alpha_{g,t}=\tilde{\alpha}_t$ and $\alpha_{e,t}=\tilde{\alpha}_t e^{i\chi t}$. Observation (i) seems to be less natural when including the factor $ei\xi t$, which can only be derived from the exact evolution of the state of the system under Hamiltonian (6). However, as explained above, our aim is to provide a secure and simple approach to account for the errors introduced by the environment when computing the fidelity of complex processes for which *ab initio* calculations turn out to be prohibitive. For dispersive atom-field interactions the factor $e^{-i\omega t}$ plays a crucial role in the evolution of the complete-system state

$$|\ell\rangle|\alpha\rangle|\mathcal{E}\rangle\overset{U_t}{\rightarrow}|\ell\rangle|\alpha_{\ell,t}\rangle|\beta_{\ell,t}\rangle, \quad (12)$$

where $\beta_{g,t}=\beta_t$ and $\beta_{e,t}=e^{i\chi t}\beta_t$ [β_t being the same as that in Eq. (4)], follows immediately from observations (ii) and (iii). By considering the evolution (12) it is straightforward to account for the preparation of the MQSS through the experiment sketched in Fig. 1. Let us assume that at instant t_0 the atom is prepared, through Ramsey zone R_1, in an arbitrary superposition $c_g|g\rangle+c_e|e\rangle$ simultaneously to the injection of the coherent field α in cavity C. Until t_1, when the atom enters the cavity, the initial state of the whole system evolves to $c_g|g\rangle|\tilde{\alpha}_t\rangle|\beta_t\rangle+c_e|e\rangle|\tilde{\alpha}_t\rangle|\beta_t\rangle$. During the time interval the atom spends inside the cavity, from instant $t_1>0$ to t_2, when χ is not zero, we get $c_g|g\rangle|\tilde{\alpha}_{g,t}\rangle|\beta_{g,t}\rangle+c_e|e\rangle|\tilde{\alpha}_{e,t}\rangle|\beta_{e,t}\rangle$. As a final step, from instant t_2 to t_3, when the atom is supposed to reach the detection chamber D, we have, for the atom-field-environment entanglement $c_g|g\rangle|\tilde{\alpha}_t\rangle|\beta_t\rangle+c_e|e\rangle|e^{-i\chi\tau}\tilde{\alpha}_t\rangle|e^{-i\chi\tau}\beta_t\rangle$, where $\tau=t_2-t_1$. Once the atom has been detected in D, after undergoing a $\pi/2$ rotation in R_2, the prepared MQSS is written as

$$|\psi_p\rangle=\mathcal{N}_p[c_g|\tilde{\alpha}_t\rangle|\beta_t\rangle\pm c_e|e^{-i\chi\tau}\tilde{\alpha}_t\rangle|e^{-i\chi\tau}\beta_t\rangle], \quad (13)$$

where $\mathcal{N}_p$ is a normalization constant and the signal $+$ $(-)$ referring to the detection of the g (e) state. Fixing $\chi\tau=\pi$, the density operator for the MQSS entangled to its environment reads

$$\rho=|\mathcal{N}_p|^2[|c_g|^2|\tilde{\alpha}_t\rangle\langle\tilde{\alpha}_t|+|c_e|^2|-\tilde{\alpha}_t\rangle\langle-\tilde{\alpha}_t| \pm e^{-2|\beta_t|^2}(c_g c_e^*|\tilde{\alpha}_t\rangle\langle-\tilde{\alpha}_t|+c_g^* c_e|-\tilde{\alpha}_t\rangle\langle\tilde{\alpha}_t|)]. \quad (14)$$

The fidelity of the engineering process, the overlap between the desired state vector $|\psi_d\rangle=\mathcal{N}_d[c_g|\tilde{\alpha}_t^{\Gamma=0}\rangle\pm c_e|-\tilde{\alpha}_t^{\Gamma=0}\rangle]$, where $\mathcal{N}_d$ is a normalization constant, and the above-described prepared state, leads, for $\Gamma t\ll 1$, to the expression

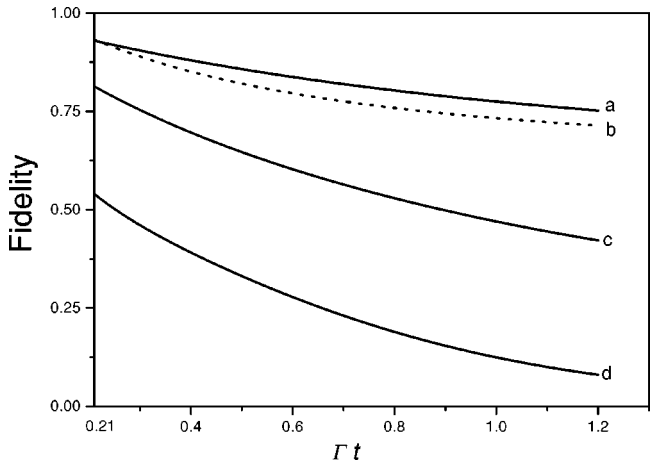

FIG. 2. Fidelity for the prepared MQSS for the case of a stable atom. Curves *a* and *b* display the results obtained, for the field amplitude $\alpha=1$, through the present approach and through a recently exact calculation (Ref. [9]), respectively. In curves *c* and *d* we find the behavior of the fidelity, due to the present approach, for $\alpha=1.5$ and 3.0, respectively. We have employed the realistic parameter $\kappa=10^2\ \mathrm{s}^{-1}$.

$$\mathcal{F}=\|\langle\psi_p|\psi_d\rangle\|^2=|\mathcal{N}_p\mathcal{N}_d|^2\{1-2(1+e^{-4|\alpha|^2})|c_g|^2|c_e|^2 \pm e^{-2|\alpha|^2}(c_g c_e^*+c_g^* c_e)+e^{-2|\alpha|^2\Gamma t}[2|c_g|^2|c_e|^2 + e^{-2|\alpha|^2}[(c_g^* c_e+e^{-2|\alpha|^2}(c_g^* c_e)^2]+\text{c.c.})]\}. \quad (15)$$

It can be verified that, for $t=0$, one has $\mathcal{F}=1$, while the fidelity decreases exponentially as a function of time. In Fig. 2 we display the fidelity decay for the prepared MQSS considering the case of a stable atom according to Eq. (15). The result obtained through the present phenomenological-operator approach, shown by curve *a*, is compared with a recent exact calculation [9], shown by curve *b* (dotted line), computed through characteristic functions. We observe that curve *a* displays a slower decay as compared to the exact calculation leading to curve *b*, developed in Ref. [9]. This effect is due to the fact that the present approach does not take into account the cavity-field damping mechanism during the atom-field interaction time. In fact, it is a usual procedure in the literature to disregard the cavity-field losses during the considerably short time during which the atom crosses the cavity, about 10^{-4} s. Moreover, as mentioned above, the time the atom spends inside the cavity is about 1/20 the time it spends outside. On the other hand, the exact calculation of the prepared-MQSS fidelity in Ref. [9] takes into account the cavity damping mechanism even during the time the atom spends within the cavity. Quantitatively, the argument of the damping factor multiplying the off-diagonal terms in Eq. (14), i.e., $-2|\beta_t|^2$, is corrected, by the exact calculation of Ref. [9], by

$$-|\beta_t|^2-\int_0^t dt' e^{-i\Delta t'}\frac{d}{dt'}[\alpha_{e,t'}\alpha_{g,t'}^* e^{i\Delta t'}], \quad (16)$$

where $\Delta=\omega_e-\omega_g=-\chi$, and $\alpha_{\not\in,t}$ satisfies Eq. (11). By solving Eq. (16) step by step from $t=0$ to t_3, when the atom is supposed to reach the detection chamber *D*, recalling that $\chi\neq 0$ during the interaction time $\tau=t_2-t_1$, we obtain

$$-|\alpha_0|^2(1-e^{-\Gamma t})-|\alpha_0|^2\left\{1-e^{-\Gamma t_1}\left[\frac{i\Delta}{\Gamma+i\Delta} + e^{-(\Gamma+i\Delta)\tau}\left(\frac{2\Gamma+i\Delta}{\Gamma+i\Delta}-e^{-\Gamma(t-t_2)}\right)\right]\right\}. \quad (17)$$

When disregarding the cavity-field dissipation during the time the atom crosses the cavity, which, in a rough way, can be done by taking the exponential function out of the derivative in Eq. (16), we recover exactly the decoherence time deduced from the present approach: $-2|\beta_t|^2=-2|\alpha_0|^2(1-e^{-\Gamma t})$. As we see from Fig. 2, the error in this approach, compared to the exact calculation, increases with time and is about 4% when $\Gamma t=1.2$.

Finally, curves *c* and *d* display the fidelity decay for the case of a stable atom when considering the field amplitudes $\alpha=1.5$ and 3.0, respectively. As expected, as the field excitation increases, the fidelity decays faster. This figure employs the realistic atomic damping rate $\kappa=10^2\ \mathrm{s}^{-1}$.

2. The case of an unstable atomic excited state

Finally, we now account for the process in which both atom and field are unstable. It is reasonable to suppose that the atom and the field are coupled to different environments: the errors introduced in the field are mainly due to the lossy cavity, while the atom will be mainly subjected to the external surroundings of the cavity. It is also clear that the dispersive atom-field interaction will couple together both environments, leading to complicated dynamics of the states

$$|\tilde{\Phi}_{\not\in}(t)\rangle=\int\frac{d^2\alpha}{\pi}\int\left\{\frac{d^2\beta_k}{\pi}\right\}\int\left\{\frac{d^2\gamma_k}{\pi}\right\} \times\mathcal{A}_{\not\in}(\alpha,\{\beta_k\},t)|\alpha,\{\beta_k\},\{\gamma_k\}\rangle, \quad (18)$$

where $\{\gamma_k\}$ is the set of the coherent-state amplitudes of the oscillators constituting the environment of the atom. Following the steps outlined in Eqs. (7)–(10), we obtain coupled differential equations which mix together the states $|\tilde{\Phi}_g(t)\rangle$ and $|\tilde{\Phi}_e(t)\rangle$. In fact, when the atomic decay is considered, the rate of occupation of the ground state $|g\rangle$ turns out to be also dependent on $|e\rangle$. Thus, to simplify the calculations we must assume that the atomic decay does not occur inside the cavity, i.e., during the time interval $\tau=t_2-t_1$ we assume that the atom is stable. This is a good approximation, since two out of three atoms, at least, are able to travel the distance of the whole setup without decaying. In fact, since a Rydberg-atom excited state has a lifetime on the order of 10^{-2} s, the probability of staying in this state is about 2/3 for an experiment duration of about 2×10^{-3} s. Such an argument helps sustain the preceding approximation of taking into account only the errors introduced by the cavity dissipation mechanism, assuming the atom as a stable system. Moreover, in the

present case, the probability for the spontaneous emission to occur exactly inside the cavity is about 5%, since the time the atom spends inside the cavity is about 1/20 the time it spends outside, thus justifying the approximation that the atom does not decay inside the cavity.

Under the above approximation, by considering separately the evolutions in Eqs. (1) and (12) one can account for the preparation of the MQSS when both the atom and the field are subject to errors introduced by the environment. From the time the system is prepared in the state $(c_g|g\rangle + c_e|e\rangle)|\alpha\rangle$, until t_1, when the atom enters the cavity, the state of the whole system evolves to $(c_g|g\rangle\hat{\mathcal{T}}_0 + c_e|e\rangle\hat{\mathcal{T}}_e + c_e|g\rangle\hat{\mathcal{T}}_g)|\mathcal{E}\rangle|\tilde{\alpha}_t\rangle|\beta_t\rangle$. Dispersively interacting inside the cavity, from instant $t_1>0$ to t_2, when χ is not zero, the system becomes entangled as $(c_g\hat{\mathcal{T}}_0 + c_e\hat{\mathcal{T}}_g)|\mathcal{E}\rangle|g\rangle|\tilde{\alpha}_t\rangle|\beta_t\rangle + c_e|e\rangle\hat{\mathcal{T}}_e|\mathcal{E}\rangle|\tilde{\alpha}_{e,t}\rangle|\beta_{e,t}\rangle$. Here we consider that the environmental operators evolve continuously inside the cavity, despite the atomic excited state being assumed stable during such a short time interval τ. From t_2 to t_3, when the atom reaches the detection chamber D, we have, for the atom-field-environment entanglement, $(c_g\hat{\mathcal{T}}_0 + c_e\hat{\mathcal{T}}_g)|\mathcal{E}\rangle|g\rangle|\tilde{\alpha}_t\rangle|\beta_t\rangle + c_e|e\rangle\hat{\mathcal{T}}_e|\mathcal{E}\rangle|e^{-i\chi\tau}\tilde{\alpha}_t\rangle|e^{-i\chi\tau}\beta_t\rangle$. Once the atom has been detected in D, after undergoing a $\pi/2$ rotation in R_2, the prepared MQSS reads

$$|\psi_p\rangle = \mathcal{N}_p[(c_g\hat{\mathcal{T}}_0 + c_e\hat{\mathcal{T}}_g)|\mathcal{E}\rangle|\tilde{\alpha}_t\rangle|\beta_t\rangle \pm c_e\hat{\mathcal{T}}_e|\mathcal{E}\rangle|e^{-i\chi\tau}\tilde{\alpha}_t\rangle|e^{-i\chi\tau}\beta_t\rangle], \tag{19}$$

where $\mathcal{N}_p$ is a normalization constant and the signal $+$ $(-)$ refers to the detection of the state g (e). Fixing $\chi\tau = \pi$, the density operator for the MQSS entangled to its environment is derived from Eq. (19):

$$\rho = |\mathcal{N}_p|^2[(1-e^{-2\kappa t}|c_e|^2)|\tilde{\alpha}_t\rangle\langle\tilde{\alpha}_t| + e^{-2\kappa t}|c_e|^2|-\tilde{\alpha}_t\rangle\langle-\tilde{\alpha}_t| \pm e^{-2|\beta_t|^2}e^{-2\kappa t}(c_g c_e^*|\tilde{\alpha}_t\rangle\langle-\tilde{\alpha}_t| + c_g^* c_e|-\tilde{\alpha}_t\rangle\langle\tilde{\alpha}_t|)]. \tag{20}$$

We note that if $\kappa=0$, the result above recovers Eq. (14), as expected. The fidelity of the engineering process, the overlap between the desired state vector $|\psi_d\rangle$ and the above-described prepared state leads, for $\Gamma t \ll 1$, to the expression

$$\mathcal{F} = |\mathcal{N}_p\mathcal{N}_d|^2|c_e|^4 e^{-2\kappa t}\{1-(1-|c_e|^{-2}e^{2\kappa t})[\mathcal{R}+e^{-4|\alpha|^2} + 2\sqrt{\mathcal{R}}e^{-2|\alpha|^2}\cos\phi] + e^{-2|\alpha|^2}(\mathcal{R}e^{-2|\alpha|^2} + 2\sqrt{\mathcal{R}}\cos\phi) + 2\mathcal{R}e^{-2\Gamma t}[1+\cos(2\phi)+|c_e|^{-1}e^{-2|\alpha|^2}\cos\phi]\}, \tag{21}$$

where $\mathcal{R}=|c_g|^2/|c_e|^2$ and ϕ indicates the phase difference between the complex coefficients c_e and c_g. As expected, for $t=0$ one has $\mathcal{F}=1$. Moreover, it can be easily verified that, as the ratio $\mathcal{R}$ increases, resulting in a smaller probability to find the atomic ground state and thus, a superposition state, the fidelity decay rate decreases.

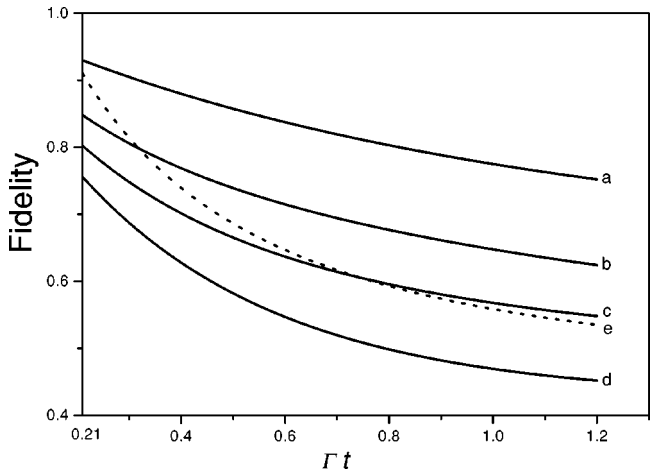

FIG. 3. Fidelity of the prepared MQSS for the cases of a stable (curves *a* and *e*) and an unstable atom (curves *b*, *c*, and *d*). Curve *a* is exactly that shown in Fig. 2 and it is repeated here to compare the behavior displayed by both a stable and an unstable atom. Curves *b*, *c*, and *d* display the fidelity decay when $=2.0$, 1.0, and 0.1, respectively. Curve *e* shows the fidelity decay for a MQSS preprared in a thermal environment, when considering the photon average number $\langle n_{\omega\ell}\rangle \simeq 0.02$. All curves consider $\alpha=1$ and $\kappa = 10^2\ \mathrm{s}^{-1}$.

In Fig. 3, obtained for $\alpha_0=1$ and employing the realistic atomic damping rate $\kappa=10^2\ \mathrm{s}^{-1}$, we display the fidelity of the prepared MQSS considering both cases of a stable (curves *a* and *e*) and an unstable atomic system (curves *b*, *c*, and *d*). In Fig. 3, curve *a*, the same shown in Fig. 2, is repeated to compare the behavior displayed by both cases of a stable and an unstable excited atomic state. Curve *e* (dotted line), also related to a stable atomic state, but considering an environment at nonzero temperature, will be discussed later. The fidelity for the prepared MQSS, considering an unstable excited atomic state, is displayed through curves *b*, *c*, and *d*, for three values of the rate $\mathcal{R}$: 2.0, 1.0, and 0.1, respectively. The behavior of curve *c*, for $\mathcal{R}=1.0$, is to be compared with that of a stable atom (curve *a*), also obtained for $\mathcal{R}=1.0$. As expected, curve *c* exhibits a faster decay than *a*, since it also includes the atomic decay. When $\mathcal{R}=0.1$ (curve *d*), the fidelity decay is even faster since the smaller the ratio $\mathcal{R}$, the higher the probability to find an atomic photon and, thus, the probability of atomic decay. However, the larger the ratio $\mathcal{R}$, making the probability to find an atomic photon negligible, the slower the fidelity decay, as we can conclude from curve *b*, obtained for $\mathcal{R}=2.0$.

III. RESERVOIR AT A FINITE TEMPERATURE

The recently reported experiments [8,13] can be grouped in a category well described through the zero-temperature approximation of Sec. II. It is possible, however, to develop a phenomenological-operator treatment in the case of dissipation at finite temperature. The main idea, distinguished from the program of Sec. II, is to introduce generalized phe-

nomenological operators which account for the thermal mixing of the states describing the system of interest $\mathcal{A}$ together with its environment $\mathcal{E}$. Let us suppose that before time zero $\mathcal{A}$ is isolated from $\mathcal{E}$, that is, they are prevented from interacting. (For example, in cavity QED, $\mathcal{A}$ is taken to be the cavity field and $\mathcal{E}$, the cavity and its surroundings; time zero is defined as the instant at which the field is created inside the cavity.) Since $\mathcal{A}$ and $\mathcal{E}$ are initially not interacting, each is in a pure state at time zero. Afterwards, when interaction is on, the compound system $\mathcal{A}+\mathcal{E}$ evolves to an entangled state.

To describe the evolution of the system of interest together with its environment in a phenomenological-operator approach, we consider an operator U_t that acts on the tensor-product Hilbert space of the compound system $\mathcal{A}+\mathcal{E}$. We define this phenomenological operator by requiring that a pure state of the product Hilbert space $|\Phi_{\mathcal{A}}\rangle|\Psi_{\mathcal{E}}\rangle$ be transformed by U_t into the entangled state representing $\mathcal{A}+\mathcal{E}$ at time t.

Underlying the scheme of the previous paragraph is the assumption that the totality of the degrees of freedom of the environment $\mathcal{E}$ is the union of two subsets: a reservoir $\mathcal{R}_{\mathcal{E}}$, which interacts with $\mathcal{A}$, and the surroundings of this reservoir, $\mathcal{S}_{\mathcal{E}}$, which does not interact with $\mathcal{A}$, but only with $\mathcal{R}_{\mathcal{E}}$. Moreover, we suppose that $\mathcal{R}_{\mathcal{E}}$ and $\mathcal{S}_{\mathcal{E}}$ are in thermodynamical equilibrium, that is, considering the density operator of $\mathcal{E}$, if we trace out the degrees of freedom of one of the subsystems ($\mathcal{S}_{\mathcal{E}}$ or $\mathcal{R}_{\mathcal{E}}$), we obtain the reduced density operator of the other, representing a mixed state at temperature T. We further assume that the number of degrees of freedom of $\mathcal{A}$ is so small as compared with that of $\mathcal{R}_{\mathcal{E}}$ that this reservoir always remains at temperature T, regardless of its interaction with $\mathcal{A}$.

A. Decay of the free atomic excited state

To consider the effects of atomic spontaneous emission at a finite temperature, it is necessary to account for the possibility of excitation of an atom initially in its ground state, i.e.,

$$|g\rangle|\mathcal{E}\rangle \overset{U_t}{\rightarrow} |g\rangle\hat{\mathfrak{T}}_0|\mathcal{E}\rangle + |e\rangle\hat{\mathfrak{T}}_1|\mathcal{E}\rangle, \tag{22a}$$

$$|e\rangle|\mathcal{E}\rangle \overset{U_t}{\rightarrow} |e\rangle\hat{\mathfrak{T}}_e|\mathcal{E}\rangle + |g\rangle\hat{\mathfrak{T}}_g|\mathcal{E}\rangle, \tag{22b}$$

where both $\hat{\mathfrak{T}}_0$ and $\hat{\mathfrak{T}}_e$ are proportional to the unity operator **1**, whereas $\hat{\mathfrak{T}}_1=\Sigma_j \mathfrak{f}_j(\tau)\hat{b}_j$, and $\hat{\mathfrak{T}}_e=\Sigma_j \mathfrak{g}_j(\tau)\hat{b}_j^\dagger$, with $b_j^\dagger$ and b_j denoting, respectively, the creation and annihilation operators of the jth oscillator mode of the environment. We can find the relevant properties of the functions $\mathfrak{f}_j(\tau)$ and $\mathfrak{g}_j(\tau)$ by imposing the relations

$$\langle\mathcal{E}|\hat{\mathfrak{T}}_0^\dagger\hat{\mathfrak{T}}_1|\mathcal{E}\rangle=\langle\mathcal{E}|\hat{\mathfrak{T}}_0^\dagger\hat{\mathfrak{T}}_g|\mathcal{E}\rangle=0, \quad \langle\mathcal{E}|\hat{\mathfrak{T}}_0^\dagger\hat{\mathfrak{T}}_e|\mathcal{E}\rangle=e^{-(\langle n\rangle+1/2)\kappa t},$$

$$\langle\mathcal{E}|\hat{\mathfrak{T}}_1^\dagger\hat{\mathfrak{T}}_g|\mathcal{E}\rangle=\langle\mathcal{E}|\hat{\mathfrak{T}}_1^\dagger\hat{\mathfrak{T}}_e|\mathcal{E}\rangle=\langle\mathcal{E}|\hat{\mathfrak{T}}_e^\dagger\hat{\mathfrak{T}}_g|\mathcal{E}\rangle=0,$$

$$\langle\mathcal{E}|\hat{\mathfrak{T}}_g^\dagger\hat{\mathfrak{T}}_g|\mathcal{E}\rangle=(\langle n\rangle+1)(\kappa/\xi)\sinh(\xi t)e^{-(\langle n\rangle+1/2)\kappa t},$$

$$\langle\mathcal{E}|\hat{\mathfrak{T}}_e^\dagger\hat{\mathfrak{T}}_e|\mathcal{E}\rangle=[\cosh(\xi t)-(\kappa/2\xi)\sinh(\xi t)]e^{-(\langle n\rangle+1/2)\kappa t}, \tag{23}$$

$$\langle\mathcal{E}|\hat{\mathfrak{T}}_0^\dagger\hat{\mathfrak{T}}_0|\mathcal{E}\rangle=[\cosh(\xi t)+(\kappa/2\xi)\sinh(\xi t)]e^{-(\langle n\rangle+1/2)\kappa t},$$

$$\langle\mathcal{E}|\hat{\mathfrak{T}}_1^\dagger\hat{\mathfrak{T}}_1|\mathcal{E}\rangle=\frac{\xi[1+(\kappa/2\xi)^2]}{(\langle n\rangle+1)\kappa}\sinh(\xi t)e^{-(\langle n\rangle+1/2)\kappa t},$$

where $\xi=\kappa[1/4+\langle n\rangle(\langle n\rangle+1)]^{1/2}$ and the thermal average of the boson number associated with the atomic frequency is $\langle n\rangle=[\exp(\hbar\omega_0/k_BT)-1]^{-1}$, where k_B is the Boltzmann constant and T is the reservoir temperature. Through Eqs. (22a), (22b), and (23), it can be readily seen that, for an atom initially in the superposition state $(|g\rangle+|e\rangle)/\sqrt{2}$, the atomic density-matrix elements satisfy the expected equations [17]

$$\dot{\rho}_{ee}=\langle e|\dot{\rho}_{\text{atom}}|e\rangle=-(\langle n\rangle+1)\kappa\rho_{ee}+\langle n\rangle\kappa\rho_{gg},$$

$$\dot{\rho}_{eg}=\dot{\rho}_{ge}^*=-(\langle n\rangle+1/2)\kappa\rho_{eg}, \tag{24}$$

$$\dot{\rho}_{gg}=-\langle n\rangle\kappa\rho_{gg}+(\langle n\rangle+1)\kappa\rho_{ee}.$$

It is evident that for a reservoir at absolute zero, $\langle n\rangle=0$, the result in Eq. (2) is recovered.

B. Decay of the free field

To model the effects of the environmental temperature, observation (i) must be modified: the description of the evolution of the field state must now account for the spreading of the initial coherent state. According to the discussion presented in the beginning of this Sec. III, we consider the whole system as consisting of our cavity field, system $\mathcal{A}$, and whatever remains, which we call the environment. An initial coherent superposition field state $\Sigma_i c_i|\alpha_i\rangle$ thus evolves to a mixed state according to

$$\sum_i c_i|\alpha_i\rangle|\mathcal{E}\rangle \overset{U_t}{\rightarrow} \sum_i c_i\hat{\mathfrak{L}}_t|\alpha_i\rangle|\mathcal{E}\rangle, \tag{25}$$

where the action of the operator $\hat{\mathfrak{L}}_t$, to be defined below, mixes the field states with those of the environment. From Eq. (25) the density operator of the whole system is

$$\rho=\sum_{ij} c_ic_j^*\hat{\mathfrak{L}}_t|\alpha_i\rangle|\mathcal{E}\rangle\langle\mathcal{E}|\langle\alpha_j|\hat{\mathfrak{L}}_t^\dagger. \tag{26}$$

Tracing out the environmental degrees of freedom gives the cavity-field reduced density operator, which, in the diagonal Glauber's representation, is given by

$$\rho=\sum_{ij} c_i c_j^* \left(\prod_{k'} \int \frac{d^2\beta_{k'}}{\pi}\right) \hat{\mathfrak{L}}_t(\{\beta_k\},\mathcal{E})|\alpha_i\rangle\langle\alpha_j|\hat{\mathfrak{L}}_t^\dagger(\{\beta_k\},\mathcal{E})$$

$$\equiv\sum_{ij} c_i c_j^* \int \frac{d^2\chi}{\pi} P(\chi,\chi^*,\alpha_i,\alpha_j^*;t)|\chi\rangle\langle\chi|, \tag{27}$$

where $\hat{\mathfrak{L}}_t(\{\beta_k\},\mathcal{E})\equiv\langle\{\beta_k\}|\hat{\mathfrak{L}}_t|\mathcal{E}\rangle$ and $P(\chi,\chi^*,\alpha_i,\alpha_j^*;t)$ can be thought as a conditional quasi-probability density for the occurrence of a coherent state of amplitude χ, at time t, given the amplitude α_i, at time zero. Heuristically, the function P, leading to the spread of the initial coherent state α_i, must be, for the diagonal elements $i=j$ in Eq. (27), of the form $\exp(-|\chi-\alpha_i f(t)|^2/D(t))/D(t)$, where $f(t)$ accounts for a steadily decreasing mean value of the amplitude $\alpha_i(t)$, while its dispersion coefficient $D(t)$ increases. This observation together with Eq. (27) leads us to infer the result, which can also be formally deduced:

$$\left(\prod_{k'} \int \frac{d^2\beta_{k'}}{\pi}\right) \hat{\mathfrak{L}}_t(\{\beta_k\},\mathcal{E})|\alpha_i\rangle\langle\alpha_j|\hat{\mathfrak{L}}_t^\dagger(\{\beta_k\},\mathcal{E})$$

$$=\langle\alpha_j|\alpha_i\rangle \int \frac{d^2\chi}{\pi}\frac{1}{D(t)} \times e^{-[\chi-u(t)\alpha_i](\chi^*-u^*(t)\alpha_j^*)/D(t)}|\chi\rangle\langle\chi|, \tag{28}$$

where the factor $\langle\alpha_j|\alpha_i\rangle$ could be heuristically inferred from the exponential decay, considering an environment at absolute zero, of the off-diagonal terms in Eq. (14). For the diagonal elements it follows, from the weight function $P(\chi,\chi^*,\alpha_i,\alpha_i^*;t)$, the Glauber's Gaussian form for the conditional quasi-probability density

$$P(\chi,t|\alpha_i,0)=\frac{1}{D(t)}\exp\left(-\frac{|\chi-u(t)\alpha_i|^2}{D(t)}\right). \tag{29}$$

The dispersion of this Gaussian function is given by [7]

$$D(t)=\sum_k \langle n_k\rangle|v_k(t)|^2,$$

while its mean value is $\alpha_i u(t)$. The function $u(t)$, to be taken as $f(t)$ in the abovementioned expression for the spreading of the coherent field, together with $v_k(t)$, describes the time evolution of the amplitude $\alpha_i(t)=u(t)\alpha_i+\Sigma_k v_k(t)\beta_k$ [7]. The quantity $u(t)$ gives exactly the decay of the cavity field amplitude α_i for an environment at absolute zero [as indicated in Eq. (11)], since in this case the conditional quasi-probability density reduces to $P(\chi,t|\alpha_i,0)=\delta^{(2)}(\chi-u(t)\alpha_i)$. As regards the off-diagonal elements, at absolute zero Eq. (28) reduces exactly to the expected result, to be compared with the off-diagonal elements in expression (14):

$$\langle\alpha_j|\alpha_i\rangle\int\frac{d^2\chi}{\pi}[e^{-u(t)\alpha_i(\partial/\partial\chi)}e^{-u^*(t)\alpha_j^*(\partial/\partial\chi^*)}\delta^{(2)}(\chi)]|\chi\rangle\langle\chi|$$

$$=\frac{\langle\alpha_j|\alpha_i\rangle}{\langle u(t)\alpha_j|u(t)\alpha_i\rangle}|u(t)\alpha_i\rangle\langle u(t)\alpha_j|. \tag{30}$$

Hence, the expression in Eq. (25) together with expression (28), which defines the action of the operator $\hat{\mathfrak{L}}$ on the states of both the field and the environment, give exactly the evolution of the complete state of the system. When considering the MQSS $\mathcal{N}(|\alpha\rangle\pm e^{i\varphi}|-\alpha\rangle)$ coupled to an environment at nonzero temperature, the mixed state of the system turns out to be

$$\rho=\sum_{i=1}^{2}\int\frac{d^2\chi}{\pi}P(\chi,t|\alpha_i,0)|\chi\rangle\langle\chi| + \sum_{i\neq j=1}^{2}\langle\alpha_j|\alpha_i\rangle\int\frac{d^2\chi}{\pi}P(\chi,\chi^*,\alpha_i,\alpha_j^*;t)|\chi\rangle\langle\chi|. \tag{31}$$

Figures 4(a)–4(c) display the evolution of the conditional quasiprobability density for the amplitude of the cavity-field, the sum of the four weight functions P in Eq. (31), considering $\alpha=1$, and the realistic parameter $\Gamma=10^2\ \mathrm{s}^{-1}$. The axes q and p represent the real and imaginary parts of χ, respectively. We have also considered a constant mean occupation number $\langle n_k\rangle=\langle n_{\omega_\mathscr{A}}\rangle$, calculated at the cavity-field frequency, which for the abovementioned realistic temperature of 0.6 K turns out to be $\langle n_{\omega_\mathscr{A}}\rangle\simeq 0.02$. Figure 4(a) shows the MQSS immediately after its preparation, i.e., $\Gamma t=0.21$, which indeed, to the best of our knowledge, is represented by a statistical mixture due to the effects introduced by the environment during the engineering time. This fact is evident from Fig. 4(a), since the typical positive and negative ripples characterizing coherence already begin to mix together the peaks associated to both coherent states in the ''superposition.'' In Fig. 4(b) we find the MQSS statistical mixture when $\Gamma t=0.32$, whose rotation in phase space comes from the factor $\exp(-i\omega_\mathscr{A} t)$. In this figure we observe both the effects coming from the environment: the diffusion, due to nonzero temperature, of the initial ''coherent states'' which characterize the ''superposition'' and the diffusion of the positive and negative ripples of the interference terms associated with the original prepared mixture on its way to decoherence. In Fig. 4(c) we finally observe, for $\Gamma t=0.1$, the original ''coherent states'' falling onto each other in a peak whose small amplitude has decreased steadily due to the diffusion process.

C. Initially coherent field in a lossy cavity interacting dispersively with a two-level atom

1. Case of a stable atomic excited state

Here, similarly to the case of an environment at absolute zero, it follows from Eqs. (7)–(10) (with the expansion coefficients $\mathcal{A}_\mathscr{A}$ now depending on the temperature) that the

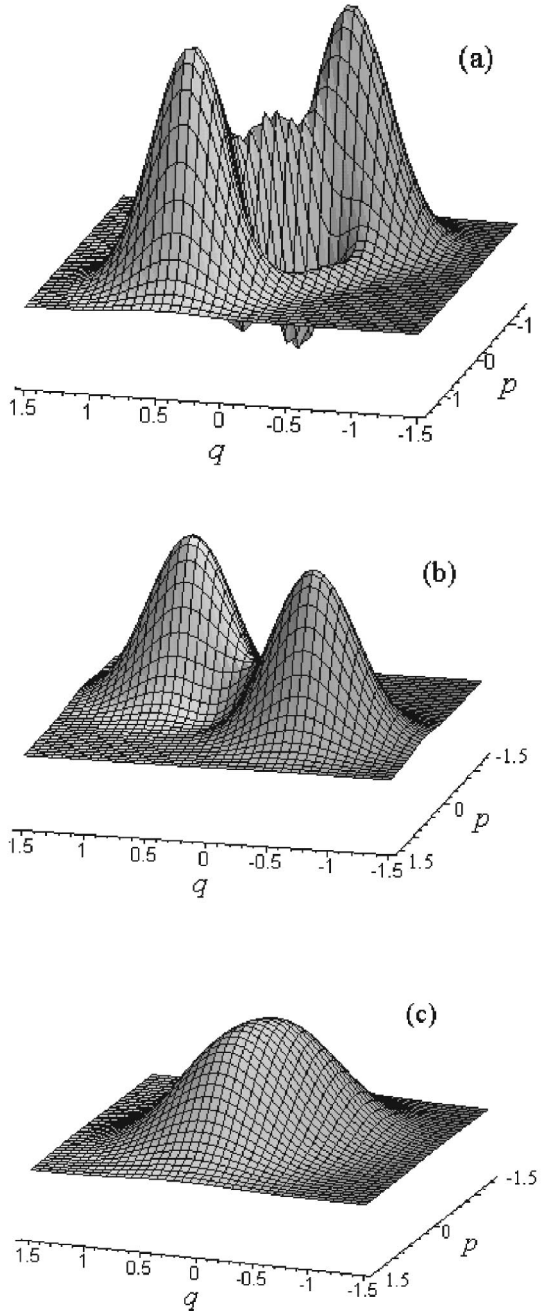

FIG. 4. Evolution of the conditioned quasi-probability density for the amplitude of the cavity field, the sum of the four weight functions P of Eq. (31). We have considered $\alpha=1$, $\kappa=10^2\ \mathrm{s}^{-1}$ and a photon average number $\langle n_{\omega_\mathscr{C}}\rangle\simeq 0.02$. (a)–(c) display the MQSS statistical mixture when $\Gamma t=0.1$, 0.21, and 0.32 s, respectively.

problem is reduced to that of the free dissipation of a cavity field, whose frequency ω has been shifted by $-\chi$ when interacting with the atomic excited state $|e\rangle$. In this way, from Eq. (25) we obtain

$$|\mathscr{C}\rangle\sum_i c_i|\alpha_i\rangle|\mathcal{E}\rangle\overset{U_T}{\rightarrow}\sum_i c_i\hat{\mathfrak{L}}_t|\mathscr{C}\rangle|\alpha_i\rangle|\mathcal{E}\rangle. \tag{32}$$

The reduced cavity-field density operator, in the diagonal Glauber's representation, results in

$$\rho=\sum_{ij} c_i c_j^*\left(\prod_{k'}\int\frac{d^2\beta_{k'}}{\pi}\right)\hat{\mathfrak{L}}_t(\{\beta_k\},\mathcal{E})|\alpha_{i,\mathscr{C}}\rangle \times\langle\alpha_{j,\mathscr{C}}|\hat{\mathfrak{L}}_t^\dagger(\{\beta_k\},\mathcal{E}) \tag{33}$$

$$\equiv\sum_{ij} c_i c_j^*\int\frac{d^2\chi}{\pi}P(\chi,\chi^*,\alpha_{i,\mathscr{C}},\alpha_{j,\mathscr{C}}^*;t)|\chi\rangle\langle\chi|, \tag{34}$$

with

$$P(\chi,\chi^*,\alpha_{i,\mathscr{C}},\alpha_{j,\mathscr{C}}^*;t)=\frac{1}{D(t)}e^{-[\chi-u_\mathscr{C}(t)\alpha_i][\chi^*-u_\mathscr{C}^*(t)\alpha_j^*]/D(t)}. \tag{35}$$

Now, the function $u_\mathscr{C}(t)$, which, together with $v_{\mathscr{C},k}(t)$ describes the time evolution of the amplitude $\alpha_{i,\mathscr{C}}(t)$, reads $u_\mathscr{C}(t)=e^{-\Gamma t/2}e^{-i\omega_\mathscr{C} t}$. Thus, the whole procedure adopted previously remains the same for considering a dissipative initially coherent field interacting dispersively with a stable two-level atom, with the exception that the evolution of the amplitude $\alpha_{i,\mathscr{C}}(t)$ must be appropriately corrected. In Fig. 2, the fidelity decay for the prepared MQSS in a thermal environment follows curve *e*, showing how the temperature effects on phase coherence can be drastic. We have considered the photon average number $\langle n_{\omega_\mathscr{C}}\rangle\simeq 0.02$. In fact, this curve, obtained for $\mathcal{R}=1.0$, crosses the curve accounting for an unstable atom with the same value of the ratio $\mathcal{R}$.

2. Case of an unstable atomic excited state

For a dissipative initially coherent field interacting dispersively with an unstable two-level atom, we must proceed according to the same assumptions of Sec. II. In short, we assume that the atomic decay does not take place inside the cavity. Under this approximation, both relations (25) and (32) can be considered independently when accounting for the errors introduced in a given cavity QED process.

IV. CONCLUSION

We have presented in this work a phenomenological-operator approach to account for dissipation in cavity QED. The development of this approach is possible due to the previous formal work in quantum dissipation, from which we have recovered the main results concerning energy loss in both systems, a coherent field trapped in a cavity and a two-level atom. Thus, we have adjusted these results to account for the noise effects in a dispersive atom-field interaction, the cavity QED process responsible for the preparation of a MQSS. The present phenomenological-operator approach, developed either for an absolute zero or a thermal environment, considerably simplifies the introduction of the noise effects when describing processes resulting from dispersive atom-field interaction. The main error source in these processes concerns the cavity dissipative mechanism and the atom spontaneous decay. Here we have treated, to illustrate the universal applicability of our technique, the generation of a cavity-field MQSS. Since the errors due to the coupling of

the quantum systems to the environment will always be present during the engineering process of the MQSS, the prepared state will necessarily present a non-unit fidelity. The results we have obtained, a good estimate compared to the exact calculation, as shown in Fig. 1, agree quite well with our expectation. So, as pointed out in the abstract, the main result in the present work consists exactly in furnishing a straightforward technique to estimate the fidelity resulting from dispersive atom-field interaction, precluding the necessity of performing the typically extensive *ab initio* calculations. Furthermore, we expect that the present work can help us develop a technique to account for dissipation in resonant atom-field interactions and even achieve a general estimate of the effects of dissipation in whichever system. Of course, we do not expect from this approach an accurate description of the effects coming from dissipation. We just envisage a technique to estimate the fidelity and the decoherence time of a process. After all, as mentioned above, the recent interesting theoretical promises strongly based on the quantum coherence and entanglement properties, which can open the way for a new technology, depend crucially on estimating fidelities and decoherence times.

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