

Continuous pumping and control of a mesoscopic superposition state in a lossy QED cavity

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Here we consider the continuous pumping of a dissipative QED cavity and derive the time-dependent density operator of a cavity field prepared initially as a superposition of mesoscopic coherent states. The control of the coherence of this superposition is analyzed by considering the injection of a beam of two-level Rydberg atoms through the cavity. Our treatment is compared to other approaches.

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I. INTRODUCTION

In the last two decades a consensus has been established about the importance of the effects of the environment on a macroscopic system in explaining the nonobservation of superposition of quantum states [1–3]. The formal treatment of a nonisolated macroscopic quantum *system of interest* assumes a unitary evolution of the whole system composed of *system of interest + environment + (possibly) measurement apparatus*. In the dynamical process called *decoherence*, the environment drives the macroscopic superposition state into a statistical mixture in a very short time, as compared to the relaxation time [4]. As a matter of fact, even microscopic systems suffer from the effects of the environment since they are not perfectly isolated, but less drastically.

The construction of mesoscopic superposition states of the electromagnetic field (EMF) in a cavity (*cat states*) has attracted attention due to the experimental observation [5,6] of the very short lifetime of the superposition. The most recent proposals for preparing mesoscopic superposition states rely on strategies which aim to keep them in a high degree of purity by precluding noise coming from the reservoir, in order to delay the decoherence process. One proposal for creating and sustaining cat states is based on the confinement of an EMF in a superconducting cavity [7]. In [8–10] the authors proposed and achieved an experiment consisting of the preparation of a cat state (a superposition of coherent states in the microwave region) in a Fabry-Pérot open superconducting cavity of high quality factor then measuring its decoherence time using the interaction with a beam of two-level Rydberg atoms with an electromagnetic field going through the cavity [11]. In such an experiment energy and information loss are unavoidable, not permitting the existence of the cat state for a sufficiently long time, so constituting a drawback for its use for technological purposes; the cavity field ends up in a thermal state [9].

Other proposals for suppressing the action of the environment on the coherence of mesoscopic superposition field states have been presented in recent years. One of them con-

siders a stroboscopic feedback scheme [12–14]: a stream of two-level atoms, all prepared in the same state, cross a cavity where each atom interacts dispersively with the field. If the delay time between sequential atoms is short enough, all atoms will be detected in the same state, e.g., the excited state. This is an indirect measure of the coherence and parity of the initial field superposition state. However, when a cavity photon is lost, the following atom will be detected in the ground state. When this event occurs a subsequent atom prepared in the excited state should be sent to interact in resonance with the field in order to compensate the lost energy and phase, so that hopping restores the original state of the cavity by absorbing a photon from the atom. This procedure needs full control of the atom-field interactions by the experimenter. However, due to the poor efficiency of atomic detectors, the stroboscopic feedback scheme is not fully reliable.

In order to reduce the velocity of the decoherence process of a cat state in a lossy QED cavity (hereafter referred as *C*), in this paper another practical strategy is proposed. It considers the continuous action of a classical pumping field—a single-mode microwave signal—in *C*, during the running time of the experiment. We begin by showing that, under the action of pumping and at temperature $T=0$ K, an arbitrary initial state of the field in *C* goes asymptotically to a coherent state. The pumping action compensates the energy lost to the environment, but not the initial available information about the state (interference of probability amplitudes or coherence) as it is not sensitive to the phase information of the field state. This can only be achieved with the combined action of pumping together with the injection of Rydberg atoms through the cavity, which permits sustaining the energy of the field and reconstructing the initial coherence. This process can also use feedback atoms, as proposed in [12,13], to guarantee full efficiency for maintaining the initial cat state. Another important question raised in the present paper is the following: For an open system constantly fed by an external source how does the decoherence and relaxation process evolve?

This paper is organized as follows. In Sec. II we review the mechanism for generating superposition field states in superconducting cavities. Section III is devoted to obtaining of the Heisenberg equations for the field operators that govern the evolution of the continuously pumped quantum state. In Sec. IV we discuss how a cat state is generated in a cavity

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and how it evolves under the action of the combined pumping field plus environment. Section V is dedicated to the study of the decoherence process of the cavity state. In Sec. VI we propose a strategy that combines the action of atoms and pumping to restore the initial superposition of the field state, and finally in Sec. VII we present a summary of this work.

II. GENERATION OF SCHRÖDINGER-CAT STATES

The experimental apparatus for the generation of field superposition states consists of a beam of Rydberg atoms crossing three cavities, R_1 , R_2 , and C . R_1 and R_2 are low-quality cavities (Ramsey zones), but C is a high- Q superconducting cavity, into which a coherent state was previously injected by a microwave source. The atoms are initially prepared in circular states of principal quantum number of the order of 50, which are well designed for these experiments since their lifetime is over 3×10^{-2} s [9].

The usual method of Ramsey interferometry consists in injecting classical fields into the Ramsey zones R_1 and R_2 during the interaction time with the atoms [7]. The transition between two nearly orthogonal atomic states, $|e\rangle$ (excited) and $|g\rangle$ (ground), is resonant with the R_1 and R_2 fields, and the transition strength is set by selecting the velocity of the atom, which suffers a rotation in the space spanned by state vectors $\{|e\rangle, |g\rangle\}$.

The experiment begins by preparing the Rydberg atom in state $|e\rangle$, which is then rotated in R_1 to the superposition state

$$|\Psi_a\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle). \quad (1)$$

Subsequently the atom interacts with the field in C , whose dynamics is described by the Jaynes-Cummings Hamiltonian

$$H = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\kappa(a\sigma^+ + a^\dagger\sigma^-), \quad (2)$$

where $\sigma_z \equiv |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma^+ \equiv |e\rangle\langle g|$, and $\sigma^- \equiv |g\rangle\langle e|$ are atomic pseudospin operators, a (a^\dagger) is the annihilation (creation) operator for the field mode of frequency ω in C , κ is the atom-field coupling constant, and ω_0 is the atomic transition frequency.

The cavity C is tuned near resonance with the atomic transition frequency ω_i , between states $|e\rangle$ and $|i\rangle$, where $|i\rangle$ is a reference state with energy level above that of $|e\rangle$. The transition frequency ω_i is distinct from any other one involving the state $|g\rangle$. The mode geometry inside the cavity is such that the intensity of the field increases and decreases smoothly along the atomic trajectory inside C . For sufficiently slow atoms and for sufficiently large detuning between ω and ω_i , the atom-field evolution is adiabatic and no photonic absorption or emission occurs [7,11]. However, dispersive effects are very important—the atom crossing C in the state $|e\rangle$ induces a phase shift in the cavity field which can be adjusted by a suitable selection of the atomic velocity (~ 100 m/s). For a phase shift π , a coherent field $|\alpha\rangle$ in C is turned into $|-\alpha\rangle$. On the other hand, an atom in state $|g\rangle$

does not introduce any phase shift on the cavity field. Therefore, an atom in state (1) crossing C will lead the system C + atom into the correlated state

$$\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \otimes |\alpha\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle \otimes |-\alpha\rangle + |g\rangle \otimes |\alpha\rangle). \quad (3)$$

The atom crosses the cavity in a time of the order of 10^{-4} s, which is well below the relaxation time of the field inside C (typically 10^{-3} – 10^{-2} s for niobium superconducting cavities) and below the atomic spontaneous emission time (3×10^{-2} s) [11].

When the atom is submitted to a second $\pi/2$ pulse, in R_2 , the total state will be transformed to

$$\begin{aligned} \frac{1}{\sqrt{2}}(|e\rangle \otimes |-\alpha\rangle + |g\rangle \otimes |\alpha\rangle) \rightarrow \frac{1}{\sqrt{2}} \left(|e\rangle \otimes \frac{1}{\sqrt{2}}(|-\alpha\rangle - |\alpha\rangle) \right. \\ \left. + |g\rangle \otimes \frac{1}{\sqrt{2}}(|-\alpha\rangle + |\alpha\rangle) \right). \end{aligned} \quad (4)$$

Therefore, if the atom is detected in the state $|g\rangle$ or $|e\rangle$ the field in C will be projected to the state

$$|\Psi_c\rangle = \frac{1}{N}(|\alpha\rangle + \cos\varphi|-\alpha\rangle), \quad (5)$$

with $\varphi=0$ (π) if the atom is detected in the state $|g\rangle$ ($|e\rangle$). $N = \sqrt{2(1 + (\cos\varphi)e^{-2|\alpha|^2})}$ is the normalization constant and the density operator for the superposition state (5) is given by

$$\begin{aligned} \rho_C &= |\Psi_c\rangle\langle\Psi_c| \\ &= \frac{1}{N^2} [|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha| \\ &\quad + \cos\varphi(|\alpha\rangle\langle-\alpha| + |-\alpha\rangle\langle\alpha|)]. \end{aligned} \quad (6)$$

When such a state is produced inside the cavity, the presence of dissipative effects alters its free evolution, introducing an amplitude damping as well as a coherence loss term. At temperature $T=0$ K the density operator (6) evolves according to

$$\begin{aligned} \rho_C(t) &= \frac{1}{N^2} [|\alpha e^{-\gamma t/2}\rangle\langle\alpha e^{-\gamma t/2}| + |-\alpha e^{-\gamma t/2}\rangle\langle-\alpha e^{-\gamma t/2}| \\ &\quad + \cos\varphi e^{[-2|\alpha|^2(1-e^{-\gamma t})]} (|-\alpha e^{-\gamma t/2}\rangle\langle\alpha e^{-\gamma t/2}| \\ &\quad + |\alpha e^{-\gamma t/2}\rangle\langle-\alpha e^{-\gamma t/2}|)], \end{aligned} \quad (7)$$

where two characteristic times are involved. The first one, the *decoherence time*, is the time in which the pure state Eq. (7) is turned into a statistical mixture,

$$\rho_C(t) \approx \frac{1}{2} [|\alpha e^{-\gamma t/2}\rangle\langle\alpha e^{-\gamma t/2}| + |-\alpha e^{-\gamma t/2}\rangle\langle-\alpha e^{-\gamma t/2}|]; \quad (8)$$

the other one is the *damping* or *relaxation time* of the field, $t_c = \gamma^{-1}$, the characteristic time when the energy dissipation becomes important, driving the field asymptotically to a vacuum state. The decoherence phenomenon is characterized by the factor $\exp[-2|\alpha|^2(1-e^{-\gamma t})]$, and for short times, $\gamma t \ll 1$, it turns to be $\exp(-2|\alpha|\gamma t)$. The decoherence time $t_d = (2\gamma|\alpha|^2)^{-1}$ will be called for future reference, the *free decoherence time*.

III. THEORY OF CLASSICAL PUMPING OF LOSSY CAVITIES

We are going to show how a stationary coherent field state is generated in cavities by the action of continuous pumping and how this can change the decoherence process due to the energy loss. In the experimental apparatus discussed in the last section the pumping consists in maintaining the microwave radiation in C during the experimental running time.

A single EM mode in C interacts with the reservoir modes, represented by a vast number of harmonic oscillators [15,16], accounting for the energy dissipation of the field in C . In the rotating wave approximation the total Hamiltonian is

$$H = \hbar\omega_0 a^\dagger a + \sum_k \hbar\omega_k b_k^\dagger b_k + \hbar \sum_k (\lambda_k a^\dagger b_k + \lambda_k^* a b_k^\dagger) + \hbar(F e^{-i\omega t} a^\dagger + F^* e^{i\omega t} a), \quad (9)$$

where ω_0 is the mode frequency of the cavity, ω_k is the frequency of the k th mode of the reservoir, λ_k is the field-reservoir coupling constant, and F is the coupling constant between the cavity and pumping fields, proportional to the pumping field amplitude. Operators a^\dagger (a) and b_k^\dagger (b_k) are the bosonic creation (annihilation) operators of the field mode and the reservoir, respectively. Let us suppose that initially the quantum field and reservoir are uncoupled,

$$|\Psi_T; t=0\rangle \equiv |\psi_F\rangle \otimes |\phi_R\rangle, \quad (10)$$

where $|\psi_F\rangle$ is the state of the field and $|\phi_R\rangle$ is the state of the reservoir.

The Heisenberg equations for a and b_k are given by

$$\dot{a} = \frac{1}{i\hbar} [a, H] = -i\omega_0 a - i \sum_k \lambda_k b_k - i F e^{-i\omega t}, \quad (11)$$

$$\dot{b}_k = \frac{1}{i\hbar} [b_k, H] = -i\omega_k b_k - i\lambda_k^* a, \quad (12)$$

and the formal solution to Eq. (12) is

$$b_k(t) = e^{-i\omega_k t} b_k(0) - i\lambda_k^* \int_0^t a(t') e^{i\omega_k(t'-t)} dt'. \quad (13)$$

The rapid oscillation of the free field evolution can be eliminated by introducing in Eq. (11) the operator of slow variation in time $A \equiv e^{-i\omega_0 t} a$, whose equation of motion is

$$\dot{A} = -i \sum_k \lambda_k b_k e^{i\omega_0 t} - i F e^{i\omega_0 t} e^{-i\omega t}. \quad (14)$$

Substituting Eq. (13) into Eq. (14) we get an equation for A only,

$$\begin{aligned} \dot{A} = & -i \sum_k \lambda_k b_k(0) e^{-i(\omega_k - \omega_0)t} \\ & - \sum_k |\lambda_k|^2 \int_0^t A(t') e^{-i(\omega_k - \omega_0)(t'-t)} dt' - i F e^{i\omega_0 t} e^{-i\omega t}. \end{aligned} \quad (15)$$

Using the Wigner-Weisskopf approximation [15,16] in the above equation (see details of calculations in the Appendix) and after some algebraic manipulation, the solution of the Heisenberg equation for the operator a can be written as

$$a(t) = u(t) a(0) + \sum_k v_k(t) b_k(0) + w(t), \quad (16)$$

where

$$u(t) = e^{-\gamma t/2} e^{-i\omega_0 t}, \quad (17)$$

$$v_k(t) = -\lambda_k e^{-i\omega_k t} \frac{(1 - e^{-\gamma t/2} e^{i(\omega_k - \omega_0)t})}{\omega_0 - \omega_k - i\gamma/2}, \quad (18)$$

and

$$w(t) = F e^{-i\omega t} \frac{(1 - e^{-\gamma t/2} e^{i(\omega - \omega_0)t})}{\omega - \omega_0 + i\gamma/2}. \quad (19)$$

γ (defined in the Appendix) is the damping constant.

A. Characteristic function and field state representation

Any density operator can be spanned by the overcomplete basis of coherent states having the associated Glauber-Sudarshan P distribution,

$$\rho(t) = \int d^2\gamma P(\gamma; t) |\gamma\rangle \langle \gamma|. \quad (20)$$

The normal ordered characteristic function (CF) associated with $\rho(t)$ is given by

$$\chi_N(\eta, t) = \text{Tr}[\rho(t) e^{\eta a^\dagger} e^{-\eta^* a}] = \text{Tr}[\rho(0) e^{\eta a^\dagger(t)} e^{-\eta^* a(t)}], \quad (21)$$

where the term in the middle is written in the Schrödinger picture and the last one is in the Heisenberg picture. The P Glauber-Sudarshan distribution [18,19] is related to the normal ordered CF by a double Fourier transform (FT),

$$P(\gamma; t) = \frac{1}{\pi^2} \int d^2\eta e^{\gamma\eta^* - \gamma^*\eta} \chi_N(\eta, t), \quad (22)$$

whereas the Wigner function [20] is defined as a double Fourier transform of the symmetric ordered CF by

$$W(\zeta; t) = \frac{1}{\pi^2} \int d^2 \eta e^{\zeta \eta^* - \zeta^* \eta} \chi_S(\eta, t). \quad (23)$$

The two CF's are related through

$$\chi_S(\eta, t) \equiv \text{Tr}[\rho(t) e^{\eta a^\dagger - \eta^* a}] = e^{|\eta|^2/2} \chi_N(\eta, t). \quad (24)$$

Substituting Eq. (24) into Eq. (23) and using the inverse FT of Eq. (22), we relate the Wigner function to the P distribution by

$$W(\zeta; t) = \frac{2}{\pi} \int d^2 \gamma e^{-2|\zeta - \gamma|^2} P(\gamma, t). \quad (25)$$

The symmetric ordered CF associated with the ω_0 mode in cavity C is given, in the Heisenberg picture, by

$$\chi_S^F(\eta, t) = \text{Tr}_{F+R}[\rho_{F+R}(0) e^{\eta a^\dagger(t) - \eta^* a(t)}], \quad (26)$$

where the trace operation runs over the field and reservoir coordinates and the subsystems are assumed initially uncorrelated,

$$\rho_{F+R}(0) = \rho_F(0) \otimes \rho_R(0). \quad (27)$$

Inserting operator (16) and its Hermitian conjugate into Eq. (26), the CF for the field can be written

$$\begin{aligned} \chi_S^F(\eta, t) &= \text{Tr}_{F+R} \left\{ \rho_{F+R}(0) \exp \left[\eta \left(u^*(t) a^\dagger + \sum_k v_k^*(t) b_k^\dagger + w^*(t) \right) - \text{H.c.} \right] \right\} \\ &= \text{Tr}_{F+R} \left[\rho_{F+R}(0) e^{\eta w^*(t) - \eta^* w(t)} \exp[\eta u^*(t) a^\dagger - \eta^* u(t) a] \exp \left(\sum_k [\eta v_k^*(t) b_k^\dagger - \eta^* v_k(t) b_k] \right) \right] \\ &= e^{\eta w^*(t) - \eta^* w(t)} \text{Tr}_F \{ \rho_F(0) \exp[\eta u^*(t) a^\dagger - \eta^* u(t) a] \} \times \text{Tr}_R \left[\rho_R(0) \exp \left(\sum_k [\eta v_k^*(t) b_k^\dagger - \eta^* v_k(t) b_k] \right) \right] \\ &= e^{\eta w^*(t) - \eta^* w(t)} \chi_S^F(\eta u^*(t), 0) \text{Tr}_R \left[\rho_R(0) \exp \left(\sum_k [\eta v_k^*(t) b_k^\dagger - \eta^* v_k(t) b_k] \right) \right], \end{aligned} \quad (28)$$

with

$$\chi_S^F(\eta u^*(t), 0) \equiv \text{Tr}_F[\rho_F(0) e^{\eta u^*(t) a^\dagger - \eta^* u(t) a}]. \quad (29)$$

For a thermalized reservoir the state is given by

$$\rho_R(0) = \int \prod_k d^2 \beta_k \frac{1}{\pi \langle n_k \rangle} e^{-|\beta_k|^2 / \langle n_k \rangle} |\beta_k\rangle \langle \beta_k|, \quad (30)$$

where $\langle n_k \rangle$ is the mean occupation number of the k th oscillator mode. So Eq. (29) can be written as

$$\begin{aligned} \chi_S^F(\eta, t) &= e^{\eta w^*(t) - \eta^* w(t)} \chi_S^F(\eta u^*(t), 0) \prod_k e^{-|\eta|^2 |v_k(t)|^2 / 2} \\ &\quad \times \int d^2 \beta_k \frac{1}{\pi \langle n_k \rangle} e^{-|\beta_k|^2 / \langle n_k \rangle} \exp[\eta v_k^*(t) \beta_k^\dagger \\ &\quad - \eta^* v_k(t) \beta_k]. \end{aligned} \quad (31)$$

The integral in Eq. (31) is easily solved with the help of the identity

$$\frac{1}{\pi} \int d^2 \eta e^{-\mu |\eta|^2 + \lambda \eta + \nu \eta^*} = \frac{1}{\mu} e^{\lambda \nu / \mu} [\text{Re}(\mu) > 0] \quad (32)$$

and the CF is written

$$\begin{aligned} \chi_S^F(\eta, t) &= e^{\eta w^*(t) - \eta^* w(t)} \chi_S^F(\eta u^*(t), 0) \prod_k \\ &\quad \times e^{-|\eta|^2 |v_k(t)|^2 [(1/2) + \langle n_k \rangle]} \\ &= e^{\eta w^*(t) - \eta^* w(t)} \chi_S^F(\eta u^*(t), 0) \\ &\quad \times e^{-|\eta|^2 \sum_k |v_k(t)|^2 [(1/2) + \langle n_k \rangle]}. \end{aligned} \quad (33)$$

Substituting the discrete sum by an integration we obtain after minor algebra [16]

$$\sum_k |v_k(t)|^2 \left(\frac{1}{2} + \langle n_k \rangle \right) = (1 - e^{-\gamma t}) \left(\frac{1}{2} + \bar{n} \right), \quad (34)$$

with $\bar{n} \equiv (e^{\beta \omega_0} - 1)^{-1}$ and $\beta = (k_B T)^{-1}$, where k_B is the usual Boltzmann constant and T is the reservoir temperature. Substituting Eq. (34) into Eq. (33) we obtain

$$\chi_S^F(\eta, t) = \chi_S^F(\eta u^*(t), 0) e^{\eta w^*(t) - \eta^* w(t)} \exp \left[-|\eta|^2 (1 - e^{-\gamma t}) \times \left(\frac{1}{2} + \bar{n} \right) \right]. \quad (35)$$

For a reservoir at $T=0$ K, $\bar{n}=0$, the symmetrically ordered CF becomes

$$\begin{aligned} \chi_S^F(\eta, t) &= \text{Tr}_F[\rho_F(0) e^{\eta u^*(t) a^\dagger - \eta^* u(t) a}] e^{\eta w^*(t) - \eta^* w(t)} \\ &\quad \times \exp \left[-\frac{1}{2} |\eta|^2 (1 - e^{-\gamma t}) \right] \\ &= \text{Tr}_F[\rho_F(0) e^{\eta u^*(t) a^\dagger} e^{-\eta^* u(t) a}] e^{\eta w^*(t) - \eta^* w(t)} \\ &\quad \times e^{-|\eta|^2/2}. \end{aligned} \quad (36)$$

Comparing the right-hand side (RHS) of the second equality with the normal ordered CF, Eq. (24), we identify the following relation:

$$\chi_N^F(\eta, t) = \chi_S^F(\eta u^*(t), 0) e^{\eta w^*(t) - \eta^* w(t)}. \quad (37)$$

At this point it is important to emphasize that we have not yet mentioned the initial state of the field inside the cavity. Equation (37) allows one to obtain the density operator evolved for an arbitrary initial state. The dynamics of the system cavity field + reservoir correlates the initial states of the subsystems, entailing energy dissipation and loss of coherence during evolution. In the next section we show that when the reservoir is at $T=0$ K, both the cavity field and the reservoir states remain uncorrelated in the course of the evolution. In the absence of pumping, the field state is called a dissipative coherent state.

IV. GENERATION OF STATES IN THE DISSIPATIVE CAVITY

A. Coherent states

Let us first consider the situation when the initial state of the field in the cavity C is

$$\rho_C(0) = |\alpha\rangle\langle\alpha|, \quad (38)$$

introducing this into the normal ordered CF Eq. (21) we have

$$\begin{aligned} \chi_N^F(\eta u^*(t), 0) &= \text{Tr}_F[|\alpha\rangle\langle\alpha| e^{\eta u^*(t) a^\dagger} e^{-\eta^* u(t) a}] \\ &= \langle\alpha| e^{\eta u^*(t) a^\dagger} e^{-\eta^* u(t) a} |\alpha\rangle \\ &= e^{\eta u^*(t) \alpha^* - \eta^* u(t) \alpha}, \end{aligned} \quad (39)$$

and substituting into Eq. (37) one gets

$$\chi_N^F(\eta, t) = e^{\eta[u^*(t)\alpha^* + w^*(t)] - \eta^*[u(t)\alpha + w(t)]}. \quad (40)$$

However, since this result must be identical to the normal ordered CF obtained in the Schrödinger picture, it follows that

$$\begin{aligned} \chi_N^F(\eta, t) &= \text{Tr}_F[\rho_F(t) e^{\eta a^\dagger} e^{-\eta^* a}] \\ &= \langle\psi_F(t)| e^{\eta a^\dagger} e^{-\eta^* a} |\psi_F(t)\rangle, \end{aligned} \quad (41)$$

with $\rho_F(t) = |\psi_F(t)\rangle\langle\psi_F(t)|$. Then, if we compare the term on the left-hand side of the second equality of Eq. (41) to Eq. (40), it can be directly verified that one gets

$$|\psi_F(t)\rangle = |u(t)\alpha + w(t)\rangle, \quad (42)$$

as a consequence of the disentanglement between the field and the reservoir states only at $T=0$ K. Thus the density operator for the continuously pumped field is given by

$$\begin{aligned} \rho_F(t) &= |u(t)\alpha + w(t)\rangle\langle u(t)\alpha + w(t)| \\ &= |e^{-\gamma t/2} e^{-i\omega_0 t} \alpha + w(t)\rangle\langle e^{-\gamma t/2} e^{-i\omega_0 t} \alpha + w(t)|, \end{aligned} \quad (43)$$

where

$$w(t) = F e^{-i\omega t} \frac{[1 - e^{-\gamma t/2} e^{i(\omega - \omega_0)t}]}{\omega - \omega_0 + i\frac{\gamma}{2}}. \quad (44)$$

By adjusting the pumping field in resonance with the cavity field ($\omega = \omega_0$), we have

$$w(t) = -i \frac{2F}{\gamma} e^{-i\omega_0 t} (1 - e^{-\gamma t/2}), \quad (45)$$

and the density operator becomes

$$\begin{aligned} \rho_F(t) &= \left| e^{-i\omega_0 t} \left[e^{-\gamma t/2} \alpha - i \frac{2F}{\gamma} (1 - e^{-\gamma t/2}) \right] \right\rangle \\ &\quad \times \left\langle e^{-i\omega_0 t} \left[e^{-\gamma t/2} \alpha - i \frac{2F}{\gamma} (1 - e^{-\gamma t/2}) \right] \right|. \end{aligned} \quad (46)$$

Setting the relation between the system parameters, $F = i\alpha\gamma/2$, all the terms multiplying $\exp(-\gamma t/2)$ cancel and the field in the cavity remains coherent, oscillating at frequency ω_0 ,

$$\rho_C(t) = |e^{-i\omega_0 t} \alpha\rangle\langle e^{-i\omega_0 t} \alpha|. \quad (47)$$

In this way, despite the dissipative effect, the pumping action compensates the lost energy, establishing a stationary coherent field state in the cavity. This result is independent of the cavity quality factor $Q \equiv \omega_0/\gamma$, showing that coherent fields are quite stable.

For the generation of another coherent state it is sufficient to adjust the pumping field amplitude. Asymptotically the field state is stationary,

$$\lim_{t \rightarrow \infty} \rho_F(t) \approx \left| e^{-i(\omega_0 t + \pi/2)} \frac{2F}{\gamma} \right\rangle \left\langle e^{-i(\omega_0 t + \pi/2)} \frac{2F}{\gamma} \right|, \quad (48)$$

even if the field in the cavity is initially in the vacuum state $\alpha=0$. This result shows how a lossy cavity fills up coherently when pumped by a classical source of EM radiation [17].

B. Superposition state

Let us consider now that the state (47) is sustained in the cavity and that the experiment described in Sec. II is going on. With the pumping field acting continuously we consider the adiabatic passage of a Rydberg atom across the cavity C , i.e., the time of flight of the atom is very small compared to the relaxation time of the field. It is worth noting that, if the detuning between the atomic transition frequency ω_i and the cavity field is sufficiently large, the atomic presence inside the cavity does not much change its frequency mode distribution. For an atom prepared initially in the state $|e\rangle$ the density operator of the system atom+field is written as

$$\rho_{F+A} = \rho_A \otimes \rho_F = |e\rangle\langle e| \otimes \rho_F. \quad (49)$$

The resonant interaction of the atom with the field in R_1 rotates the atomic state by $\pi/2$,

$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle), \quad (50)$$

and the joint density operator is written as

$$\rho_{F+A} = \frac{1}{2}(|e\rangle + |g\rangle)(\langle e| + \langle g|) \otimes \rho_F. \quad (51)$$

Due to the dispersive interaction of the atom with the field in C the joint state is given by [10]

$$\begin{aligned} \rho_{F+A} = & \frac{1}{2}(|e\rangle\langle e| e^{-i\pi a^\dagger a} \rho_F e^{i\pi a^\dagger a} + |g\rangle\langle g| \rho_F + |e\rangle \\ & \times \langle g| e^{-i\pi a^\dagger a} \rho_F + |g\rangle\langle e| \rho_F e^{i\pi a^\dagger a}), \end{aligned} \quad (52)$$

since the state $|e\rangle$ is always associated with the phase shift operator $\exp(-i\pi a^\dagger a)$ in this experiment [9,10]. Then the outgoing atom passing through R_2 is submitted to a new $\pi/2$ rotation and the joint state becomes

$$\begin{aligned} \rho_{F+A} = & \frac{1}{4}(|e\rangle + |g\rangle)(\langle e| + \langle g|) e^{-i\pi a^\dagger a} \rho_F e^{i\pi a^\dagger a} \\ & + (-|e\rangle + |g\rangle)(-\langle e| + \langle g|) \rho_F + (|e\rangle + |g\rangle) \\ & \times (-\langle e| + \langle g|) e^{-i\pi a^\dagger a} \rho_F + (-|e\rangle + |g\rangle) \\ & \times (\langle e| + \langle g|) \rho_F e^{i\pi a^\dagger a}. \end{aligned} \quad (53)$$

If the atom is detected in the state $|g\rangle$ or $|e\rangle$, the field state collapses instantaneously to

$$\begin{aligned} \rho_F^{(g)} = & \frac{1}{4}[e^{-i\pi a^\dagger a} \rho_F e^{i\pi a^\dagger a} + \rho_F + \cos \varphi \\ & \times (e^{-i\pi a^\dagger a} \rho_F + \rho_F e^{i\pi a^\dagger a})], \end{aligned} \quad (54)$$

where

$$\rho_F^g = \langle g| \rho_{F+A} |g\rangle, \quad \rho_F^e = \langle e| \rho_{F+A} |e\rangle, \quad (55)$$

and $\varphi=0$ or π depending on whether the atom is detected in state $|g\rangle$ or $|e\rangle$, respectively. The final state can be obtained from Eq. (54) when the initial state is known; for example, if $\rho_F = |\alpha\rangle\langle\alpha|$ is the initial state of the field in C , we have from Eq. (47) (α containing the time-dependent phase $e^{-i\omega_0 t}$)

$$\begin{aligned} \rho_F^{(g)} = & \frac{1}{N^2}[|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha| \\ & + \cos \varphi(|-\alpha\rangle\langle\alpha| + |\alpha\rangle\langle-\alpha|)], \end{aligned} \quad (56)$$

with $N = \sqrt{2(1 + \cos \varphi e^{-2|\alpha|^2})}$. Thus, immediately after atomic detection, the collapsed state of the field decoheres due to the effects of pumping and energy dissipation. Its explicit time dependence is obtained by first constructing the CF by substituting Eq. (56) into Eq. (37),

$$\begin{aligned} \chi_N(\eta, t) = & \frac{1}{N^2}[e^{\eta[u^*(t)\alpha^* + w^*(t)] - \eta^*[u(t)\alpha + w(t)]} \\ & + e^{-\eta[u^*(t)\alpha^* - w^*(t)] + \eta^*[u(t)\alpha - w(t)]} \\ & \times (\cos \varphi) e^{-2|\alpha|^2} (e^{\eta[u^*(t)\alpha^* + w^*(t)] + \eta^*[u(t)\alpha - w(t)]} \\ & + e^{-\eta[u^*(t)\alpha^* - w^*(t)] - \eta^*[u(t)\alpha + w(t)]}], \end{aligned} \quad (57)$$

Then, by comparing again the expressions in both the Schrödinger and Heisenberg pictures, we obtain the density operator for the field state,

$$\begin{aligned} \rho_F(t) = & \frac{1}{N^2}[|e^{-\gamma t/2} \alpha + w(t)\rangle\langle e^{-\gamma t/2} \alpha + w(t)| \\ & + |-e^{-\gamma t/2} \alpha - w(t)\rangle\langle -e^{-\gamma t/2} \alpha - w(t)|] \\ & + (\cos \varphi) e^{-2|\alpha|^2(1 - e^{-\gamma t})} (\exp\{-e^{-\gamma t}[\alpha^* w(t) \\ & - \alpha w^*(t)]\} |-e^{-\gamma t/2} \alpha - w(t)\rangle\langle e^{-\gamma t/2} \alpha + w(t)| \\ & + \exp\{e^{-\gamma t}[\alpha^* w(t) - \alpha w^*(t)]\} |e^{-\gamma t/2} \alpha + w(t)\rangle \\ & \times \langle -e^{-\gamma t/2} \alpha - w(t)|]). \end{aligned} \quad (58)$$

When the amplitude of the field is adjusted to $F = i\alpha\gamma/2$ we have

$$w(t) = \alpha(1 - e^{-\gamma t/2}), \quad (59)$$

and

$$\begin{aligned} \rho_F(t) = & \frac{1}{N^2}\{|\alpha\rangle\langle\alpha| + |\alpha(1 - 2e^{-\gamma t/2})\rangle\langle\alpha(1 - 2e^{-\gamma t/2})| \\ & + (\cos \varphi) \exp[-2|\alpha|^2(1 - e^{-\gamma t})][|\alpha(1 - 2e^{-\gamma t/2})\rangle \\ & \times \langle\alpha| + |\alpha\rangle\langle\alpha(1 - 2e^{-\gamma t/2})|]\}, \end{aligned} \quad (60)$$

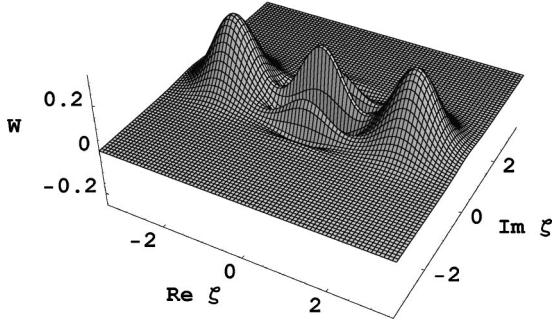


FIG. 1. Wigner distribution function for the superposition state $(|\alpha\rangle + |-\alpha\rangle)/\sqrt{2}$ with $|\alpha|^2=5$. The central structure represents the coherence of the quantum state.

which shows the time evolution of the quantum state. Asymptotically this state goes to the coherent state $\rho_F(t) = |\alpha\rangle\langle\alpha|$, which acts as an attractor for other initial quantum states. Independently of the initial amplitude α , the pumping field supplies energy continuously to the cavity, sustaining the field in a pure coherent state, Eq. (48).

V. DECOHERENCE OF A CONTINUOUSLY PUMPED FIELD

Now we analyze the evolution of the density operator at times far from the asymptotic regime, in which case we can observe the effects of the classical pumping on the evolution of the state. From Eq. (58) we observe that the coherence terms (nondiagonal) are modified, in comparison to the pumping-free decoherence, by a factor $\exp\{\pm e^{-\gamma t}[\alpha^* w(t) - \alpha w^*(t)]\}$. However, due to the oscillatory character of these factors, the nondiagonal terms do not sustain the coherence of the field, which is continuously attenuated by the damping factor $\exp[-2|\alpha|^2(1-e^{-\gamma})]$, as shown in Sec. II.

The quantum characteristic of a field state can be visualized when it is represented by a Wigner function [20], obtained from the Fourier transform of the symmetrically ordered CF Eq. (23). The state given by Eq. (58) has as Wigner function

$$\begin{aligned}
 W(\zeta, t) = & \frac{2}{N^2 \pi} [\exp[-2|\zeta - e^{-\gamma t/2} \alpha - w(t)|^2] \\
 & + \exp[-2|\zeta + e^{-\gamma t/2} \alpha - w(t)|^2] 2(\cos \varphi) \\
 & \times e^{-2|\zeta - w(t)|^2} \exp[-2|\alpha|^2(1-e^{-\gamma})] \cos \\
 & \times (4e^{-\gamma t/2} \text{Im}\{\zeta - w(t)\} \alpha^*)], \quad (61)
 \end{aligned}$$

where the first two exponential functions are Gaussians centered at $[e^{-\gamma t/2} \alpha + w(t)]$ and $-[e^{-\gamma t/2} \alpha - w(t)]$, respectively, representing the two distinct states $|e^{-\gamma t/2} \alpha + w(t)\rangle$ and $|e^{-\gamma t/2} \alpha - w(t)\rangle$. The (third) coherence term is composed of three factors, a Gaussian centered at $w(t)$, a sinusoidal modulation $\cos(4e^{-\gamma t/2} \text{Im}\{\zeta - w(t)\} \alpha^*)$ and the factor responsible for the decoherence, $\exp[-2|\alpha|^2(1-e^{-\gamma})]$. The

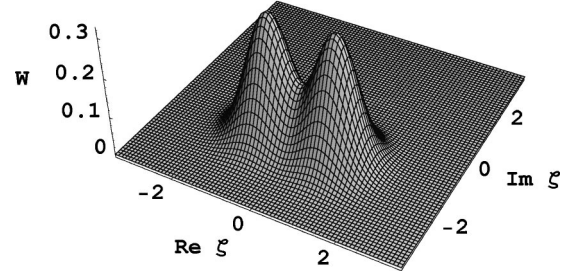


FIG. 2. Wigner distribution function for the state of Fig. 1 evolved to $\gamma t=1$. The original coherence was suppressed by the environment action and the state suffers a continuous displacement due to the pumping field.

modulation and the time of decoherence given by the last factor depend on the intensity of the state. The larger $|\alpha|^2$, the faster is the decoherence.

In Figs. 1–3 three configurations of the Wigner function (61) for $|\alpha|^2=5$ are shown at three distinct times, $t=0$, $t=\gamma^{-1}$, and $t\rightarrow\infty$. For $F=1$ we observe the progressive evolution of the superposition state, driven continuously to a stationary coherent state, Fig. 3, representing

$$\lim_{t\rightarrow\infty} W(\zeta, t) = \frac{2}{\pi} \exp\left(-2\left|\zeta - e^{-i(\omega_0 t + \pi/2)} \frac{2F}{\gamma}\right|^2\right). \quad (62)$$

The coherence term is suppressed in a time shorter than the time of relaxation of the state, still given by the free decoherence time $t_d = (2|\alpha|^2 \gamma)^{-1}$, the effect of the pumping on the coherence terms being null.

The evolution of the superposition state shows the decoherence and relaxation processes (loss of purity) as analyzed through the linear entropy,

$$\begin{aligned}
 S = \text{Tr}_F[\rho_F(t) - \rho_F^2(t)] = & 1 - \frac{2}{N^4} (1 + 4e^{-2|\alpha|^2} \\
 & + \exp(-4|\alpha|^2 e^{-\gamma t}) + \exp[-4|\alpha|^2(1-e^{-\gamma})] \\
 & + e^{-4|\alpha|^2} \exp(-2|\alpha|^2 e^{-\gamma t} \cos\{2e^{-\gamma t/2} \text{Im}\{w(t)\} \alpha^*\})). \quad (63)
 \end{aligned}$$

In Fig. 4 we have plotted S against γt for $|\alpha|^2=5$, where the state is initially pure. As the decoherence goes on the state

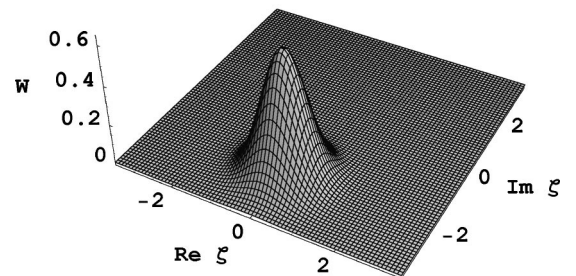


FIG. 3. Asymptotic Wigner distribution function for the state of Fig. 1. The original superposition state evolved asymptotically to a coherent state due to the classical pumping.

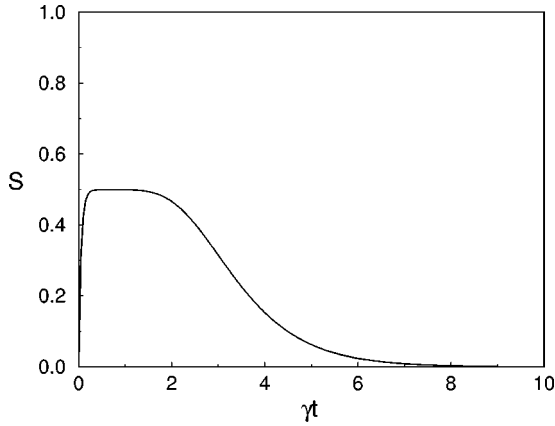


FIG. 4. The evolution in time (in units of γ^{-1}) of the linear entropy for the continuously pumped initial superposition state. The pumping does not affect the coherence terms; the state evolves from a pure state to a mixture and then to a pure state again, as in the absence of the pumping, but the final state is a coherent state instead of a vacuum state.

evolves into a mixture, $\text{Tr} \rho^2 < 1$, and the entropy increases, meaning that there is a flux of information to the reservoir. Although the pumping field is able to restore the energy lost by the cavity field, it is not able to reestablish the *information* of the original superposition state encoded by the coherence terms. Certainly here the process of decoherence is tied to the loss of energy of the field to the reservoir; however, there are situations where the information transfer does not occur necessarily together with an energy transfer [9,21]. The information transfer strongly depends on the phase relation of the superposition of quantum states [21]. Although reversible subsystems can exhibit decoherence and recoherence at constant mean energy [21], this characteristic ceases to be true for irreversible subsystems. Decoherence still occurs in a characteristic time, which is dependent on the field relaxation time and the field energy, as shown in Sec. II. An open question remains: Is the information flow (decoherence) always accompanied by an energy flow, or is this valid only for open irreversible systems? We emphasize that the time-irreversible character of these models of reservoirs follows from the introduction of approximations such as the Wigner-Weisskopf and Markov approximations.

VI. ATOMS AND PUMPING

The attempt to sustain the field, against decoherence, in a superposition of coherent states by using a classical pumping field is not effective because the insertion of photons to compensate for those lost to the reservoir is not phase sensitive. The pumping is sufficient to reestablish only the energy lost to the reservoir and not the original superposition state. Asymptotically only a stationary coherent state is established in the cavity. However, maintenance of the superposition state could be possible if an additional process accounting for reestablishing the original coherence were considered. In the experiment proposed in [8,9], once the superposition is created in C , the field interacts with atoms sent sequentially through C . The authors argue that this procedure refreshes

the initial coherence. Here we analyze the same process of sending atoms through the cavity, but with the pumping field included.

At time T , after the detection of the first atom, the state of the field in C is given by $\rho_F(T)$, Eq. (58); then a second atom is released, going through the same interaction process as the former. After crossing R_1 , the joint state of second atom + C field is given by

$$\rho_{F+A_2}(T) = \frac{1}{2}(|e\rangle + |g\rangle)_2 (\langle e| + \langle g|)_2 \otimes \rho_F(T), \quad (64)$$

and the dispersive interaction in the C field produces the entangled joint state

$$\begin{aligned} \rho_{F+A_2}(T) = & \frac{1}{2} [|e\rangle \langle e|_2 e^{-i\pi a^\dagger a} \rho_F(T) e^{i\pi a^\dagger a} + |g\rangle \langle g|_2 \rho_F(T) \\ & + |e\rangle \langle g|_2 e^{-i\pi a^\dagger a} \rho_F(T) + |g\rangle \langle e|_2 \rho_F(T) e^{i\pi a^\dagger a}]. \end{aligned} \quad (65)$$

After crossing the cavity R_2 , the joint state suffers a new transformation, becoming

$$\begin{aligned} \rho_{F+A_2}(T) = & \frac{1}{4} [(|e\rangle + |g\rangle)_2 (\langle e| + \langle g|)_2 e^{-i\pi a^\dagger a} \rho_F(T) e^{i\pi a^\dagger a} \\ & + (-|e\rangle + |g\rangle)_2 (-\langle e| + \langle g|)_2 \rho_F(T) \\ & + (|e\rangle + |g\rangle)_2 (-\langle e| + \langle g|)_2 e^{-i\pi a^\dagger a} \rho_F(T) \\ & + (-|e\rangle + |g\rangle)_2 (\langle e| + \langle g|)_2 \rho_F(T) e^{i\pi a^\dagger a}]. \end{aligned} \quad (66)$$

If the atom is detected in the $|g\rangle$ or $|e\rangle$ state the field will collapse instantaneously to

$$\begin{aligned} \rho_F^{(g)}(T) = & \frac{1}{4} [e^{-i\pi a^\dagger a} \rho_F(T) e^{i\pi a^\dagger a} + \rho_F(T) \pm e^{-i\pi a^\dagger a} \rho_F(T) \\ & \pm \rho_F(T) e^{i\pi a^\dagger a}], \end{aligned} \quad (67)$$

with the signal $+$ ($-$) standing for $|g\rangle$ ($|e\rangle$).

Substituting Eq. (58), for $\rho_F(T)$ in Eq. (67) we obtain the conditional expression for $\rho_F^{(g)}(T)$. In short, the probability for the second atom be detected in either state $|g\rangle$ or $|e\rangle$ is given by

$$\begin{aligned} P_{(g)}(T) = & \text{Tr}_F [\rho_F^{(g)}(T)] = \frac{1}{2} (1 \pm \text{Re}\{\text{Tr}[e^{-i\pi a^\dagger a} \rho_C(T)]\}) \\ = & \frac{1}{2} \left(1 \pm \frac{e^{-2|w(T)|^2}}{1 + (\cos \varphi) e^{-2|\alpha|^2}} \right. \\ & \times (\exp(-2|\alpha|^2 e^{-\gamma}) \cosh\{4e^{-\gamma T/2} \text{Re}[\alpha w^*(T)]\} \\ & + (\cos \varphi) \exp[-2|\alpha|^2(1 - e^{-\gamma T})] \\ & \left. \times \cos\{4e^{-\gamma T/2} \text{Im}[\alpha w^*(T)]\} \right), \end{aligned} \quad (68)$$

where $\varphi=0$ (π) for the first atom detected in the state $|g\rangle_1$ ($|e\rangle_1$) and the signal + (−) is for the second atom detected in the state $|g\rangle_2$ ($|e\rangle_2$). Analyzing Eq. (68), one verifies that if the second atom is detected instantaneously after the first one, $\gamma T \ll 1$, one gets

$$P_{(e)}^g = \frac{1}{2} \left(1 \pm \frac{e^{-2|\alpha|^2} + \cos \varphi}{1 + (\cos \varphi) e^{-2|\alpha|^2}} \right), \quad (69)$$

and, for $|\alpha| \gg 1$,

$$P_{(e)}^g = \frac{1}{2} (1 \pm \cos \varphi), \quad (70)$$

which is the result obtained in [10] without pumping: If the first atom is detected in $|g\rangle$ or $|e\rangle$, the field in C collapses to an even or odd cat field state, $|\Psi_C\rangle = (1/N)(|\alpha\rangle + \cos \varphi |-\alpha\rangle)$, $\varphi=0$ or π , respectively.

Now let us suppose that the first atom is detected in the state $|e\rangle$, and thus an odd cat state is generated in C . For $T \ll t_d$ (the time interval between sequentially emitted atoms being quite small) the second atom can be detected either in the state $|g\rangle$, with conditional probability $P(e,g,T) \geq 0$, or in the state $|e\rangle$, with conditional probability $P(e,e,T) \leq 1$, and so on for the subsequent atoms crossing the apparatus. In this manner the atoms crossing the apparatus sustain (approximately) the superposition state. The measurement of the field state in C by the atoms refreshes its superposition character, thus rendering the environment-induced decoherence almost ineffective. Thus, if an experiment can be done where $T \ll t_d$, a kind of Zeno effect will take place in a continuous measurement process.

When the second atom is detected in a different state from the first, the original cat state changes its parity. If one wishes to maintain the parity of the original cat state a resonant interaction can be used to restore the state of the field to its initial state. Such a process could be outlined as the feedback process reported in [12], once the resonant interaction time can be controlled to produce a single-photon exchange between the atom and field. When the cavity field loses a photon the state of the field flips from odd to even cat state and vice versa. As the initial field state (prepared by the first atom) is an odd cat state and the second atom is detected at $|g\rangle$, then a conditional feedback process must be activated and the field flips to the even cat state.

The case $T \geq t_d$ is better understood by observing the behavior of the conditional probabilities $P(g,e,T)$ and $P(e,e,T)$ in Figs. 5 and 6, where these quantities are plotted as functions of γT for several values of pumping field intensity $|F|^2$ and for $|\alpha|^2=5$. In the absence of pumping both conditional probabilities go to zero asymptotically because the field in C ends in a vacuum state. However, pumping modifies this trend: the higher the pumping field intensity the faster $P(g,e,T)$ and $P(e,e,T)$ will attain 1/2, an upper limit that does not depend on the intensity of the pumping. This limit means that the pumping action drives the cavity field to a coherent state and with the next atomic interaction another superposition state is generated. It has 50% probability to be an even or odd cat state depending on the state in which the second atom is measured. This process that guarantees a 50%

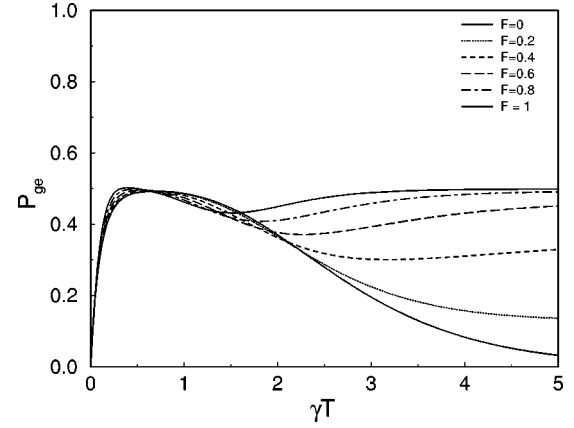


FIG. 5. The conditional probability $P(g,e,T)$ (first atom in $|e\rangle$ and second in $|g\rangle$) increases with the interaction time T (in units of γ^{-1}) as the pumping field intensity increases, saturating at 0.5.

efficiency for the detection of the same initial superposition is not very useful if we do not introduce a supplementary process. The efficiency can be increased if a conditional measurement is used for assuring that for each “wrong” result (the atom not being detected in the required state), a resonant feedback atom is sent through the cavity to flip the parity of the field state. It is worth mentioning that this process, which guarantees an efficiency for generation of the same superposition state up to 93%, was proposed in [22] for controlling the parity of a field cat state in a quantum logic gate encoding.

It is important to note that classical pumping acts on the cavity-field relaxation time. The stronger the pumping intensity $|F|^2$, the faster will be the relaxation of any initial state to a coherent state. For $|F|^2=1$, the time delay between sequentially emitted atoms should be about $\gamma T \geq 3$, defining a minimum time interval for state reconstruction. While the feedback process [12,22] is fully dependent on the atomic detector efficiency, the proposed process for delaying the cavity-field decoherence is not. Thus, this process is feasible

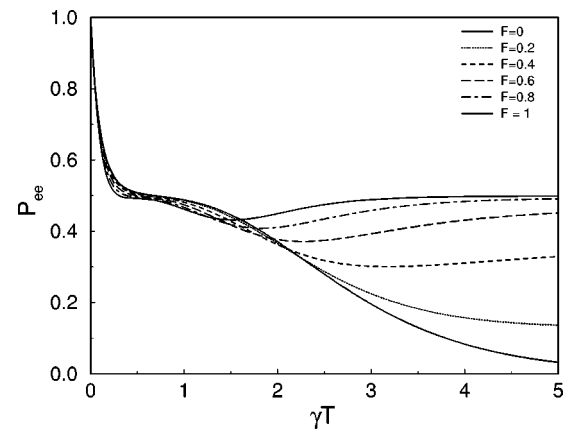


FIG. 6. As like as $P(g,e,T)$ in Fig. 5, the conditional probability $P(e,e,T)$ (first and second atoms in $|e\rangle$) increases with the interaction time T as the pumping field intensity increases, saturating at 0.5. This means that the cavity field state has a 50% chance to be left in an even or odd cat state.

as soon as each atom of the sequence is prepared in the required state and time, as discussed above. Actually, nowadays it is not an easy task to achieve efficient control of atomic injection for sending exactly one atom at a time into the cavity [13]. For instance, sending a single atom into a cavity means sending an atomic pulse with an average number of 0.2 atoms, making negligible the chance of finding two atoms in the cavity simultaneously [23]. However, the required technology for energy supply—feeding the cavity continuously with a classical source—is already available, since it is employed in current experiments [8,9].

VII. SUMMARY AND DISCUSSION

The proposed scheme of the paper shows how a classical pumping field drives any initial state prepared in a lossy cavity into a stationary coherent state. The pumping compensates the lost energy due to the cavity damping mechanism; however, due to the phase insensitivity, this energy feeding does not reestablish the initial superposition of two coherent states destroyed during the decoherence process. The pumping does not change the time of decoherence of an initial cat state, which remains the same as in the free decoherence case, showing that the information flows from the cavity field to the environment at the same rate independently of the amount of supplied energy. However, the combined action of pumping together with a sequential injection of atoms interacting dispersively with the cavity field (atomic quantum nondemolition measurement) can be used for partially conserving an initial cat state in the cavity. This state can be partially conserved by an atom “measuring” the cavity field state and thus partially reestablishing its original coherence. This result is to be compared with that in [10], where the mechanism of atomic quantum nondemolition measurement is used without pumping the cavity. In Figs. 5 and 6 we show that for large enough delay times between sequentially injected atoms the action of pumping ($F \neq 0$) contributes to resetting the initial cat state. This may be important in a practical implementation of quantum processors.

The importance of seeking a process that may sustain the coherence of a superposition state is based on the possibility of encoding information in the field state. We expect that even and odd cat states could be used for this purpose because they constitute an orthogonal basis, which should be a sufficient condition to encode qubits. As reported in [24], we can consider the even cat state as being the 0 qubit and the odd cat state as the 1 qubit,

$$|0\rangle_L \equiv \frac{1}{N_+}(|\alpha\rangle + |-\alpha\rangle) \quad \text{and} \quad |1\rangle_L \equiv \frac{1}{N_-}(|\alpha\rangle - |-\alpha\rangle).$$

These states can be used to encode qubits only while they are pure states; however, dissipation precludes their existence as such. In conclusion, finding strategies to suppress or at least to delay the decoherence is therefore extremely important for technological purposes and worth pursuing.

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APPENDIX: SOLUTION OF THE HEISENBERG EQUATION

The solution to Eq. (15) follows closely along the lines of Louisell [15]. Its Laplace transform is

$$\begin{aligned} \mathcal{L}(\dot{A}) \equiv \int_0^\infty e^{-st} \dot{A} dt &= -i \sum_k \lambda_k b_k(0) \int_0^\infty e^{-st} e^{-i(\omega_k - \omega_0)t} dt \\ &\quad - \sum_k |\lambda_k|^2 \int_0^\infty e^{-st} dt \int_0^t A(t') e^{i(\omega_k - \omega_0)(t' - t)} dt' \\ &\quad - iF \int_0^\infty e^{-st} e^{-i(\omega - \omega_0)t} dt. \end{aligned} \quad (\text{A1})$$

The integrals give

$$\int_0^\infty e^{-st} dt \int_0^t A(t') e^{i(\omega_k - \omega_0)(t' - t)} dt' = \frac{\tilde{A}(s)}{s + i(\omega_k - \omega_0)}, \quad (\text{A2})$$

$$\int_0^\infty e^{-st} e^{-i(\omega_k - \omega_0)t} dt = \frac{1}{s + i(\omega_k - \omega_0)}, \quad (\text{A3})$$

$$\int_0^\infty e^{-st} e^{-i(\omega - \omega_0)t} dt = \frac{1}{s + i(\omega - \omega_0)}, \quad (\text{A4})$$

and

$$\int_0^\infty e^{-st} \frac{d}{dt} [A(t)] dt = s\tilde{A}(s) - A(0), \quad (\text{A5})$$

with $\tilde{A}(s) \equiv \mathcal{L}(A(t))$. Substituting these in Eq. (A1), after a little algebra one gets

$$\tilde{A}(s) = \frac{A(0) - iF/[s + i(\omega - \omega_0)] - i \sum_k \lambda_k b_k(0)/[s + i(\omega_k - \omega_0)]}{\left[s + \sum_k |\lambda_k|^2/[s + i(\omega_k - \omega_0)] \right]}. \quad (\text{A6})$$

The Wigner-Weisskopf approximation [15] assumes that in the denominator of the left-hand side in the above equation the frequency spectrum of the reservoir is densely distributed around the cavity characteristic frequency ω_0 , such that one can replace the discrete sum by an integral over the reservoir frequencies having a distribution $g(\omega)$, and do the so-called pole approximation,

$$\begin{aligned} \sum_k \frac{|\lambda_k|^2}{s+i(\omega_k-\omega_0)} &= -i \sum_k \frac{|\lambda_k|^2}{(\omega_k-\omega_0)-is} \\ &= \lim_{s \rightarrow 0} \left(-i \int_0^\infty d\omega' \frac{g(\omega') |\lambda(\omega')|^2}{(\omega'-\omega_0)-is} \right). \end{aligned} \quad (\text{A7})$$

Considering only the first-order shift in the simple pole in ω_0 in the above integral, we have the Wigner-Weisskopf approximation for $s \rightarrow 0$,

$$\begin{aligned} \sum_k \frac{|\lambda_k|^2}{s+i(\omega_k-\omega_0)} &= -i \int d\omega' g(\omega') |\lambda(\omega')|^2 \\ &\quad \times \left(\frac{1}{(\omega'-\omega_0)} + i\pi \delta(\omega'-\omega_0) \right) \\ &= \frac{\gamma}{2} + i\Delta\omega, \end{aligned} \quad (\text{A8})$$

where

$$\gamma \equiv 2\pi g(\omega_0) |\lambda(\omega_0)|^2 \quad (\text{A9})$$

is the damping constant and

$$\Delta\omega \equiv - \int d\omega' \frac{g(\omega') |\lambda(\omega')|^2}{\omega'-\omega_0} \quad (\text{A10})$$

is the frequency shift. So Eq. (A6) can be written as

$$\begin{aligned} \tilde{A}(s) &= \frac{1}{s+\gamma/2+i\Delta\omega} A(0) \\ &\quad - i \sum_k \frac{\lambda_k}{[s+i(\omega_k-\omega_0)](s+\gamma/2+i\Delta\omega)} b_k(0) \\ &\quad - i \frac{F}{[s+i(\omega-\omega_0)](s+\gamma/2+i\Delta\omega)}. \end{aligned} \quad (\text{A11})$$

Now the calculation of the inverse Laplace transform

$$\begin{aligned} A(t) &= \frac{A(0)}{2\pi i} \oint e^{st} \frac{1}{s+\gamma/2+i\Delta\omega} ds \\ &\quad - \frac{1}{2\pi} \sum_k \lambda_k b_k(0) \oint e^{st} \\ &\quad \times \frac{1}{[s+i(\omega_k-\omega_0)]} \frac{1}{(s+\gamma/2+i\Delta\omega)} ds \\ &\quad - \frac{1}{2\pi} F \oint e^{st} \frac{1}{[s+i(\omega-\omega_0)]} \frac{1}{(s+\gamma/2+i\Delta\omega)} ds, \end{aligned} \quad (\text{A12})$$

where $A(t) = e^{-i\omega_0 t} a(t)$ and disregarding the small frequency shift $\Delta\omega$, gives after a little algebra the solution to the Heisenberg equation (11),

$$a(t) = u(t)a(0) + \sum_k v_k(t)b_k(0) + w(t), \quad (\text{A13})$$

where

$$u(t) = e - \gamma t/2 e^{-i\omega_0 t}, \quad (\text{A14})$$

$$v_k(t) = -\lambda_k e^{-i\omega_k t} \frac{[1 - e - \gamma t/2 e^{i(\omega_k - \omega_0)t}]}{\omega_0 - \omega_k - i\gamma/2}, \quad (\text{A15})$$

and

$$w(t) = F e^{-i\omega t} \frac{[1 - e^{-\gamma t/2} e^{i(\omega - \omega_0)t}]}{\omega - \omega_0 + i\gamma/2}. \quad (\text{A16})$$

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- [1] E. Schrödinger, *Naturwissenschaften* **23**, 807 (1935); **23**, 823 (1935); **23**, 844 (1935) [English translation by J. D. Trimmer, *Bull. Am. Phys. Soc.* **124**, 3235 (1980)].
- [2] *Mathematische Grundlagen der Quantenmechanik*, edited by J. von Neumann (Springer-Verlag, Berlin, 1932).
- [3] E. P. Wigner, *Am. J. Phys.* **31**, 6 (1963).
- [4] W. H. Zurek, *Phys. Today* **44** (10), 36 (1991); *Phys. Rev. D* **24**, 1516 (1981); **26**, 1862 (1982).
- [5] S. Haroche, *Phys. Today* **51** (7), 36 (1998).
- [6] D. J. Wineland, C. Monroe, W. M. Itano, D. Liebfried, B. E. King, and D. M. Meekhof, *J. Res. Natl. Inst. Stand. Technol.* **103**, 259 (1998).
- [7] S. Haroche and J. M. Raimond, in *Cavity Quantum Electrodynamics*, edited by P. Berman (Academic Press, New York, 1994).
- [8] J. M. Raimond, M. Brune and S. Haroche, *Phys. Rev. Lett.* **79**, 1964 (1997).
- [9] M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, *Phys. Rev. A* **45**, 5193 (1992).
- [10] L. Davidovich, M. Brune, J. M. Raimond, and S. Haroche, *Phys. Rev. A* **53**, 1295 (1996).
- [11] P. Nussenzveig, Ph.D. thesis, École Normale Supérieure, Paris, 1994 (unpublished).
- [12] D. Vitali, P. Tombesi, and G. J. Milburn, *Phys. Rev. Lett.* **79**, 2442 (1997).
- [13] D. Vitali, P. Tombesi, and G. J. Milburn, *Phys. Rev. A* **57**, 4930 (1998).
- [14] M. Fortunato, J. M. Raimond, P. Tombesi, and D. Vitali, *Phys. Rev. A* **60**, 1687 (1999).
- [15] W. H. Louisell, *Quantum Statistical Properties of Radiation*

- (John Wiley & Sons, New York, 1990).
- [16] M. H. Moussa, S. S. Mizrahi, and A. O. Caldeira, *Phys. Lett. A* **221**, 145 (1996).
- [17] H. J. Carmichael, R. J. Brecha, M. G. Raizen, H. J. Kimble, and P. R. Rice, *Phys. Rev. A* **40**, 5516 (1989).
- [18] B. R. Mollow and R. J. Glauber, *Phys. Rev.* **160**, 1076 (1967).
- [19] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1995).
- [20] E. Wigner, *Phys. Rev.* **40**, 749 (1932).
- [21] M. C. de Oliveira, S. S. Mizrahi, and V. V. Dodonov, *J. Opt. B* **1**, 610 (1999).
- [22] M. C. de Oliveira and W. J. Munro, *Phys. Rev. A* (to be published).
- [23] M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J. M. Raymond, and S. Haroche, *Phys. Rev. Lett.* **77**, 4887 (1996).
- [24] P. T. Cochrane, G. J. Milburn, and W. J. Munro, *Phys. Rev. A* **59**, 2631 (1999).