# Networks of dissipative quantum harmonic oscillators: A general treatment

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We present a general treatment of a bosonic dissipative network: a chain of coupled dissipative harmonic oscillators whatever its topology—i.e., whichever the way the oscillators are coupled together, the *strength* of their couplings, and their *natural frequencies*. Starting with a general more realistic scenario where each oscillator is coupled to its own reservoir, we also discuss the case where all the network oscillators are coupled to a common reservoir. We obtain the master equation governing the dynamic of the network states and the associated evolution equation of the Glauber-Sudarshan $P$ function. With these instruments we briefly show how to analyze the decoherence and the evolution of the linear entropy of general states of the network. We also show how to obtain the master equation for the case of distinct reservoirs from that of a common one.

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## I. INTRODUCTION

Over the last few years interest has grown in obtaining a better understanding of the phenomena of coherence and decoherence dynamics in quantum networks, especially in connection to the protocols for quantum-state transference and quantum-state protection for information processing. Beyond the quest for conditions that weaken the system-reservoir coupling [1,2], the search for mechanisms to bypass decoherence started with quantum-error-correction codes [3] and went through the program of engineering reservoirs [4]. Moreover, in a closer contact with quantum networks, the investigation of collective decoherence resulted in what has been called a decoherence-free subspace [5–7]. Interestingly enough, while, in general, the protocols for quantum error-correcting codes presuppose that quantum systems decohere independently, the decoherence-free subspace is generated by distinct quantum systems coupled to a common reservoir.

Regarding state transfer, the controlled coherent transport with splitting of atomic wave packets [8] and the evolution of macroscopically entangled states [9] have been analyzed within the context of optical lattices. The dynamics of Bose-Einstein condensates in a one-dimensional optical lattice is also investigated [10], and a class of spin networks has been proposed for perfect state transfer of any quantum state in a fixed period of time [11]. In the context of systems of coupled harmonic oscillators (HOs), which we focus on in the present work, the dynamics and manipulation of entanglement was analyzed in Ref. [12]. Evidently, the pressure for the implementation of logical operations with an increasingly larger number of quantum systems will decisively drive the quest for controlled coherent transport in quantum networks.

Concerned with a simple network of two coupled resonators, Raimond *et al.* [13] have presented the blueprint of an experiment in which the decoherence of a mesoscopic superposition of radiation states becomes a reversible process. A theoretical model of the proposal in Ref. [13] is given in Ref. [14], where the coupling of the resonators to their environments is taken into account when the reversibility of coherence loss is analyzed. In Ref. [15], the authors assume that only one of the resonators in Refs. [13,14] is interacting with a reservoir to derive a master equation in the case where the resonators are strongly coupled. It is shown that the relaxation term is not simply the standard one, obtained by neglecting the interaction between the cavities; i.e., dissipation is not additive for strongly coupled systems. Finally, in Ref. [16] both resonators are considered to be lossy, as in Refs. [13,14], and the regime of strongly coupled cavities is also analyzed, as done in Ref. [15]. A detailed analysis of the coherence and decoherence dynamics of quantum states is presented in [16], regarding this network of two coupled resonators, including a study of the correlation between the fields in both resonators through the excess entropy. The phenomena of electromagnetically induced transparency and dynamical Stark effect are also analyzed in a network of two coupled dissipative resonators [17].

In the context of complex networks, composed of a large number of subsystems, in Refs. [18,19] the authors present a detailed treatment of the coherence and decoherence dynamics in arrays of coupled dissipative resonators. In Ref. [18] a symmetric network of $N$ interacting resonators is considered, where each oscillator interacts with each other, apart from its own reservoir. A different topology is analyzed in Ref. [19], where a central oscillator is assumed to interact with the remaining $N-1$ peripheral and noninteracting oscillators. In both topologies, the decoherence process is analyzed by focusing on a single resonator which, apart from interacting with its own reservoir, also interacts with the remaining $N-1$ coupled resonators plus their respective reservoirs. Considering all resonators with the same natural frequency $\omega_0$ and all couplings with the same strength $\lambda$, master equations are derived for both weak ($\lambda \ll \omega_0$) and strong ($\lambda \approx \omega_0$) coupling regimes. From such development, a detailed analysis of the emergence of relaxation- and decoherence-free subspaces in networks of weakly and strongly coupled resonators is presented in Ref. [20]. The main result in Ref. [20] is that both subspaces are generated when all the resonators couple with the same group of reservoir modes, thus building up a correlation (among these modes), which has the potential to shield particular network states against relaxation and/or decoherence.

 

It is worth noting that recent results regarding entanglement and the nonclassical effect in collective two-atom systems [21] retain some resemblance with those discussed above for networks of coupled resonators. Beyond the entanglement dynamics, which is a crucial but recurrent ingredient of any network, the collective damping effects coming from two-atom systems [21] can be directly identified with those in a network of dissipative oscillators [16,18–20]. Such collective damping effects are certainly on the basis of the nonadditivity of decoherence rates observed in the network of dissipative oscillators [16,18–20] as well as in superconducting qubits [22].

Since in Refs. [18,19] two different topologies are analyzed independently, for the particular case where all resonators have the same natural frequency $\omega_0$ and all couplings have the same strength $\lambda$, in this work we present a unified approach for treating a bosonic dissipative network. Such an approach holds for whatever the topology of the network—i.e., for whichever (i) the way the resonators are coupled among them, (ii) their coupling strengths, and (iii) natural frequencies.

In Sec. II, towards a derivation of the master equation governing the evolution of the network, we present our model. Considering first a nondissipative network, we show how to derive particular topologies from the general case of a symmetric network where each oscillator interacts with each other. In Sec. III, the evolution equation of the Glauber-Sudarshan $P$ function is obtained as a $c$-number map of the master equation in operator form. Thus, in the context of dissipative networks, we show how to derive particular topologies from a general symmetric dissipative network where each oscillator is coupled to its respective reservoir apart from interacting with each other. Solutions in terms of the Glauber-Sudarshan $P$ function, for general initial states of the network, are given in Sec. IV together with a brief analysis of decoherence and the linear entropy. Finally, the concluding remarks are presented in Sec. VI.

## II. GENERAL TREATMENT OF A BOSONIC NETWORK

### A. Model

Supposing from here on that the indices $m$, $m'$, $n$, and $n'$ run from 1 to $N$, we start by considering a general Hamiltonian for a bosonic network, $H=H_S+H_R+H_I$, involving a network of $N$ coupled oscillators,

$$H_S = \hbar\sum_m \omega_m a_m^\dagger a_m + \frac{\hbar}{2}\sum_{m\neq n}\lambda_{mn}(a_m^\dagger a_n + a_m a_n^\dagger), \quad (1)$$

$N$ distinct reservoirs, composed by a set of $k=1,\ldots,\infty$ modes,

$$H_R = \hbar\sum_m\sum_k \omega_{mk} b_{mk}^\dagger b_{mk}, \quad (2)$$

and the coupling between the network oscillators and their respective reservoirs:

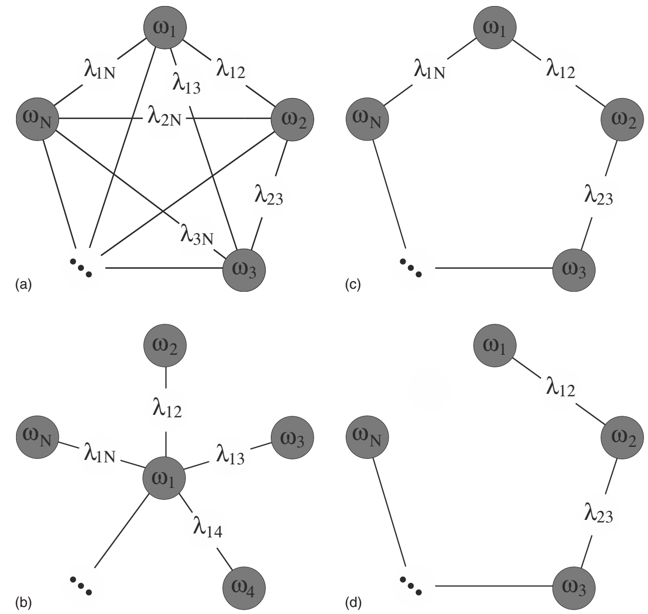


FIG. 1. Sketches of nondissipative symmetric (a), central (b), circular (c), and linear (d) networks.

$$H_I = \hbar\sum_m\sum_k V_{mk}(b_{mk}^\dagger a_m + b_{mk}a_m^\dagger). \quad (3)$$

$b_{mk}^\dagger$ ($b_{mk}$) is the creation (annihilation) operator for the $k$th bath mode $\omega_{mk}$ coupled to the $m$th network oscillator $\omega_m$ whose creation (annihilation) operator reads $a_m^\dagger$ ($a_m$). The coupling strengths between the oscillators are given by the set $\{\lambda_{mn}\}$, while those between the oscillators and their reservoirs by $\{V_{mk}\}$. Before addressing the dissipative process through Hamiltonians (2) and (3), we focus first on Hamiltonian $H_S$ to show how to derive different topologies of a nondissipative network of coupled harmonic oscillators. Rewriting $H_S$ in a matrix form $H_S=\hbar\sum_{m,n}a_m^\dagger\mathcal{H}_{mn}a_n$, its elements are given by

$$\mathcal{H}_{mn} = \begin{cases}\omega_m & \text{if } m=n,\\ \lambda_{mn} & \text{if } m\neq n,\end{cases} \quad (4)$$

whose values characterize whichever the network topology: the way the oscillators are coupled together, the set of coupling strengths $\{\lambda_{mn}\}$, and their natural frequencies $\{\omega_m\}$.

### B. From the general matrix $\mathcal{H}$ to particular nondissipative topologies

To illustrate the procedure to construct particular nondissipative topologies we consider four different cases: the (i) symmetric, (ii) central, (iii) circular, and (iv) linear networks. For the case of a (i) symmetric (*sym*) network, sketched in Fig. 1(a), all the oscillators are coupled together, with all matrix elements of $\mathcal{H}$ being not null:

$$\mathcal{H}_{sym} = \begin{pmatrix} \omega_1 & \lambda_{12} & \lambda_{13} & \cdots & \lambda_{1N} \\ \lambda_{12} & \omega_2 & \lambda_{23} & \cdots & \lambda_{2N} \\ \lambda_{13} & \lambda_{23} & \omega_3 & \cdots & \lambda_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{1N} & \lambda_{2N} & \lambda_{3N} & \cdots & \omega_N \end{pmatrix}. \tag{5}$$

In a (ii) central (*cent*) network, sketched in Fig. 1(b), only one selected oscillator, the central one, is assumed to interact with the remaining $N-1$ noninteracting peripheral oscillators. Labeling the central oscillator by 1, with the peripherals running from 2 to $N$, the matrix $\mathcal{H}$ has the form

$$\mathcal{H}_{cent} = \begin{pmatrix} \omega_1 & \lambda_{12} & \lambda_{13} & \cdots & \lambda_{1N} \\ \lambda_{12} & \omega_2 & 0 & \cdots & 0 \\ \lambda_{13} & 0 & \omega_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{1N} & 0 & 0 & \cdots & \omega_N \end{pmatrix}, \tag{6}$$

where only in the first column and row are all the nondiagonal elements not null. As depicted in Fig. 1(c), in a (iii) circular (*circ*) network the $k$th oscillator is coupled to the $(k\pm 1)$th oscillators, with the additional condition that the $N$th oscillator be coupled to the first one. The matrix $\mathcal{H}$ is given by

$$\mathcal{H}_{circ} = \begin{pmatrix} \omega_1 & \lambda_{12} & 0 & \cdots & \lambda_{1N} \\ \lambda_{12} & \omega_2 & \lambda_{23} & \cdots & 0 \\ 0 & \lambda_{23} & \omega_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{1N} & 0 & 0 & \cdots & \omega_N \end{pmatrix}. \tag{7}$$

Finally, the (iv) linear (*lin*) network follows directly from the circular one by turning off the coupling between the first and $N$th oscillators. The matrix $\mathcal{H}$ obtained for this case has the three-diagonal form

$$\mathcal{H}_{lin} = \begin{pmatrix} \omega_1 & \lambda_{12} & 0 & \cdots & 0 \\ \lambda_{12} & \omega_2 & \lambda_{23} & \cdots & 0 \\ 0 & \lambda_{23} & \omega_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \omega_N \end{pmatrix}. \tag{8}$$

Next, we treat the general situation of the dissipative network, described by the Hamiltonian $H=H_S+H_R+H_I$, through the standard perturbative approach in the system-bath coupling strengths $\{V_{mk}\}$. Since all the particular topologies follow from the case of a symmetric network, choosing appropriately the elements of $\mathcal{H}$, we shall obtain the reduced density operator of the coupled oscillators from this general topology.

### C. The master equation of a bosonic dissipative network: Direct and indirect dissipative channels

To obtain the master equation of the network we first diagonalize the Hamiltonian $\mathcal{H}$ through a canonical transformation

$$A_m = \sum_n C_{mn} a_n, \tag{9}$$

where the coefficients of the $m$th line of matrix $\mathbf{C}$ define the eigenvectors associated with the eigenvalues $\varpi_m$ of matrix (5). With $\mathbf{C}$ being an orthogonal matrix, in that $\mathbf{C}^T=\mathbf{C}^{-1}$, the commutation relations $[A_m,A_n^\dagger]=\delta_{mn}$ and $[A_m,A_n]=0$ follow, enabling the Hamiltonian $H$ to be rewritten as $\widetilde{H}=H_0+V$, with $a_m=\Sigma_n A_n C_{nm}$ and

$$H_0 = \hbar\sum_m \varpi_m A_m^\dagger A_m + \hbar\sum_m\sum_k \omega_{mk} b_{mk}^\dagger b_{mk}, \tag{10a}$$

$$V = \hbar\sum_{m,n}\sum_k C_{nm}V_{mk}(b_{mk}^\dagger A_n + b_{mk}A_n^\dagger). \tag{10b}$$

With the diagonalized Hamiltonian $H_0$ we are ready to introduce the interaction picture, defined by the transformation $U(t)=\exp(-iH_0t/\hbar)$, in which

$$V(t) = \hbar\sum_{m,n}[\mathcal{O}_{mn}(t)A_n^\dagger + \mathcal{O}_{mn}^\dagger(t)A_n], \tag{11}$$

where $\mathcal{O}_{mn}(t)=C_{nm}\Sigma_k V_{mk}\exp[-i(\omega_{mk}-\varpi_n)t]b_{mk}$. Next, we assume the interactions between the resonators and the reservoirs to be weak enough in order to perform a second-order perturbation approximation followed by tracing out the reservoir degrees of freedom. We also assume a Markovian reservoir such that the density operator of the global system can be factorized as $\rho_{1,\ldots,N}(t)\otimes\rho_R(0)$. Under these assumptions we obtain the reduced density operator of the network of $N$ dissipative coupled resonators given by

$$\frac{d\rho_{1,\ldots,N}(t)}{dt} = -\frac{1}{\hbar^2}\int_0^t dt'\,\mathrm{Tr}_R[V(t),[V(t'),\rho_R(0)\otimes\rho_{1,\ldots,N}(t)]]. \tag{12}$$

Since for a thermal reservoir $\langle b_{mk}b_{nk'}\rangle=\langle b_{mk}^\dagger b_{nk'}^\dagger\rangle=0$, we have to solve the integrals appearing in Eq. (12), related to correlation functions of the form

$$\int_0^t dt'\langle\mathcal{O}_{mn}^\dagger(t)\mathcal{O}_{m'n'}(t')\rangle = C_{nm}C_{n'm'}\int_0^t dt'\sum_{k,k'}V_{mk}V_{m'k'}\langle b_{mk}^\dagger b_{m'k'}\rangle \times\exp\{i[(\omega_{mk}-\varpi_n)t-(\omega_{m'k'}-\varpi_{n'})t']\}. \tag{13}$$

Considering that the reservoir frequencies are very closely spaced to allow a continuum summation and defining the average excitation of the $m$th mode associated to the $k$th reservoir $\mathbf{N}_m(\nu)$ as $\langle b_{mk}^\dagger(\nu)b_{m'k'}(\nu')\rangle=2\pi\delta_{mm'}\mathbf{N}_m(\nu)\delta(\nu-\nu')$, we obtain

$$\int_0^t dt' \langle \mathcal{O}_{mn}^{\dagger}(t)\mathcal{O}_{m'n'}(t')\rangle = \delta_{mm'} C_{nm} C_{n'm} e^{i(\varpi_{n'}-\varpi_n)t} \times \int_0^t dt' \int_0^{\infty} \frac{d\nu}{2\pi}[V_m(\nu)\sigma_m(\nu)]^2 \mathbf{N}_m(\nu) e^{-i(\nu-\varpi_{n'})(t'-t)}, \tag{14}$$

with $\sigma_m(\omega_{mk})$ being the density of states of the $m$th reservoir. Assuming, as usual, that $V_m(\varpi_n)$, $\sigma_m(\varpi_n)$, and $\mathbf{N}_m(\varpi_n)$ are slowly varying functions, we obtain after the variable transformations $\varepsilon=\nu-\varpi_{n'}$ and $\tau=t-t'$ the simplified form

$$\int_0^t dt' \langle \mathcal{O}_{mn}^{\dagger}(t)\mathcal{O}_{m'n'}(t')\rangle = \frac{N}{2}\delta_{mm'} C_{nm} C_{n'm} \gamma_m(\varpi_{n'}) \mathbf{N}_m(\varpi_{n'}) \exp[i(\varpi_{n'}-\varpi_n)t], \tag{15}$$

where we have defined the damping rates as

$$\gamma_m(\varpi_n) = \frac{1}{N}[V_m(\varpi_n)\sigma_m(\varpi_n)]^2 \int_{-R_n}^{\infty} \delta(\varepsilon) d\varepsilon. \tag{16}$$

Back to the Schrödinger picture and to the original field operators $a_m$, we finally obtain from the steps outlined above the master equation

$$\frac{d\rho_{1,\ldots,N}(t)}{dt} = \frac{i}{\hbar}[\rho_{1,\ldots,N}(t),H_0] + \frac{N}{2}\sum_{m,n,n'} C_{n'm}C_{nn'}\gamma_m(\varpi_{n'}) \times (\mathbf{N}_m(\varpi_{n'})\{[a_n^{\dagger}\rho_{1,\ldots,N}(t),a_m] + [a_m^{\dagger},\rho_{1,\ldots,N}(t)a_n]\} + [\mathbf{N}_m(\varpi_{n'})+1] \times \{[a_n\rho_{1,\ldots,N}(t),a_m^{\dagger}] + [a_m,\rho_{1,\ldots,N}(t)a_n^{\dagger}]\}). \tag{17}$$

From here on we shall focus on the case of reservoirs at 0 K, leaving for the last but one section a brief analysis of the effect of finite temperatures. Defining the effective damping matrix whose elements are

$$\Gamma_{mn} = N\sum_{n'} C_{n'm}\gamma_m(\varpi_{n'})C_{n'n}, \tag{18}$$

the master equation for the reservoirs at 0 K simplifies to the generalized Lindblad form

$$\frac{d\rho_{1,\ldots,N}(t)}{dt} = \frac{i}{\hbar}[\rho_{1,\ldots,N}(t),H_0] + \sum_{m,n}\frac{\Gamma_{mn}}{2}\{[a_n\rho_{1,\ldots,N}(t),a_m^{\dagger}] + [a_m,\rho_{1,\ldots,N}(t)a_n^{\dagger}]\} \equiv \frac{i}{\hbar}[\rho_{1,\ldots,N}(t),H_0] + \sum_{m,n}\mathcal{L}_{mn}\rho_{1,\ldots,N}(t), \tag{19}$$

where $\mathcal{L}_{mn}\rho_{1,\ldots,N}(t)$ are the Liouville operators accounting for the direct ($m=n$) and indirect ($m\neq n$) dissipative channels, respectively. Through the direct dissipative channels the oscillators lose excitation to their own reservoirs, whereas through the indirect channels they lose excitation to all the other reservoirs but not to their own. We observe that for Markovian white noise reservoirs, where the spectral densities of the reservoirs are invariant over translation in frequency space, such that $\gamma_m(\varpi_{n'})=\gamma_m$, expression (18) reduces to $\Gamma_{mn}=N\gamma_m\delta_{mn}$. For these particular reservoirs the indirect channels disappear. We also observe that, in the weak coupling regime where $N\{\lambda_{mn}\}\ll\{\omega_{m'}\}$ [18,19] and consequently $\gamma_m(\varpi_{n'})\approx\gamma_m(\omega_m)$, we obtain $\Gamma_{mn}=N\gamma_m(\omega_m)\delta_{mn}$, such that the indirect channels again disappear. Therefore, it is worth nothing that the indirect channels play a significant role only in the strong coupling regime where $N\{\lambda_{mn}\}\approx\{\omega_{m'}\}$.

## III. GLAUBER-SUDARSHAN $P$ FUNCTION

The evolution equation for the Glauber-Sudarshan $P$ function, derived from the master equation (19), is given by

$$\frac{dP_{1,\ldots,N}(\{\eta_{m'}\},t)}{dt} = \sum_m\left(\frac{\Gamma_{mm}}{2} + \sum_n \mathcal{H}_{mn}^D\eta_n\frac{\partial}{\partial\eta_m} + \text{c.c.}\right) \times P_{1,\ldots,N}(\{\eta_{m'}\},t), \tag{20}$$

where we have defined the matrix $\mathcal{H}^D$, with the elements

$$\mathcal{H}_{mn}^D = \Gamma_{mn}/2 + i\mathcal{H}_{mn}, \tag{21}$$

thus generalizing the former matrix $\mathcal{H}$, Eq. (5), to account for the dissipative ($D$) process. With the transformation $P_{1,\ldots,N}(\{\eta_{m'}\},t)=\tilde{P}_{1,\ldots,N}(\{\eta_{m'}\},t)\exp(\Sigma_m\Gamma_{mm}t)$, and assuming a solution of Eq. (20) of the form $\tilde{P}(\{\eta_n\},t)=\tilde{P}(\{\eta_n(t)\})$, we obtain the differential equation

$$\frac{d\tilde{P}_{1,\ldots,N}(\{\eta_{m'}(t)\})}{dt} = \sum_m\left(\frac{\partial\eta_m(t)}{\partial t}\frac{\partial}{\partial\eta_m} + \text{c.c.}\right)\tilde{P}_{1,\ldots,N}(\{\eta_{m'}\},t) = \sum_m\left(\sum_n\mathcal{H}_{mn}^D\eta_n\frac{\partial}{\partial\eta_m} + \text{c.c.}\right) \times \tilde{P}_{1,\ldots,N}(\{\eta_{m'}\},t), \tag{22}$$

which makes it possible to calculate the time evolution of the parameters $\eta_m(t)$ through the physical quantities of the system appearing on the elements $\mathcal{H}_{mn}^D$, as

$$\frac{\partial\eta_m(t)}{\partial t} = \sum_n\mathcal{H}_{mn}^D\eta_n. \tag{23}$$

Through the transformation $\tilde{\eta}_m(t)=\Sigma_n D_{mn}^{-1}\eta_m(t)$, we diagonalize the matrix $\mathcal{H}^D$, thus reducing Eq. (23) to the diagonal form $\partial\tilde{\eta}_m(t)/\partial t=\Omega_m\tilde{\eta}_m$, whose solution is $\tilde{\eta}_m(t)=\mathcal{A}_m\exp(\Omega_m t)$. Therefore, back to the parameters $\eta_m(t)$ we obtain

$$\eta_m(t) = \sum_n D_{mn} \tilde{\eta}_n(t) = \sum_n D_{mn} \exp(\Omega_n t) \mathcal{A}_n, \quad (24)$$

where the elements of the $m$th column of matrix $\mathbf{D}$ define the $m$th eigenvector associated with the eigenvalue $\Omega_m$ of matrix $\mathcal{H}^D$, and by setting the initial condition $\eta_m(t=0) \equiv \eta_m^0$, we verify from Eq. (24) that $\Sigma_n D_{mn} \mathcal{A}_n = \eta_m^0$. Therefore, $\mathcal{A}_n = \Sigma_m D_{nm}^{-1} \eta_m^0$ and, consequently,

$$\eta_m(t) = \sum_{m',n} D_{mn} \exp(\Omega_n t) D_{nm'}^{-1} \eta_{m'}^0, \quad (25)$$

leading to the solution for the Glauber-Sudarshan $P$ function:

$$P_{1,\ldots,N}(\{\eta_n\},t) = \exp\left(\sum_m \Gamma_{mm} t\right) P_{1,\ldots,N}(\{\eta_n\},t=0)\big|_{\{\eta_n\}\to\{\eta_n(t)\}}. \quad (26)$$

Therefore, having the $P$ function at time $t=0$, we immediately obtain it at any other time by substituting the set $\{\eta_n\}$ by $\{\eta_n(t)\}$.

### From the general matrix $\mathcal{H}^D$ to particular dissipative topologies

In Sec. II B we illustrate how to construct particular nondissipative topologies from a general symmetric network described by matrix (5). Now, after introducing the generalized matrix $\mathcal{H}^D$ we are in the position to enlarge the focus by constructing networks entirely composed of dissipative oscillators or, in a more general fashion, composed of mixed nondissipative and dissipative oscillators. Back to the symmetric, central, circular, and linear networks, when considering that they are all composed of dissipative oscillators, each one coupled to its respective reservoir, we obtain for the matrices $\mathcal{H}^D$ exactly the same structure as those in Sec. II B. However, from Eq. (21) we verify that the matrix elements $\mathcal{H}_{mn}^D$ follow from those of $\mathcal{H}_{mn}$ multiplied by the imaginary $i$ apart from the correction $\Gamma_{mn}/2$ coming from the dissipative process. As an example, for the symmetric network composed entirely of dissipative oscillators, as sketched in Fig. 2(a), the Hamiltonian $\mathcal{H}^D$ assumes the form

$$\mathcal{H}_{sym}^D = i\mathcal{H}_{sym} + \frac{1}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \cdots & \Gamma_{1N} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \cdots & \Gamma_{2N} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \cdots & \Gamma_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Gamma_{N1} & \Gamma_{N2} & \Gamma_{N3} & \cdots & \Gamma_{NN} \end{pmatrix}. \quad (27)$$

Let us consider, instead, the case of a mixed symmetric (*mix-sym*) network, composed of an even total number of oscillators, $N$, where those designated by odd (even) numbers are nondissipative (dissipative), as sketched in Fig. 2(b). In this case, the Hamiltonian $\mathcal{H}^D$ is given by

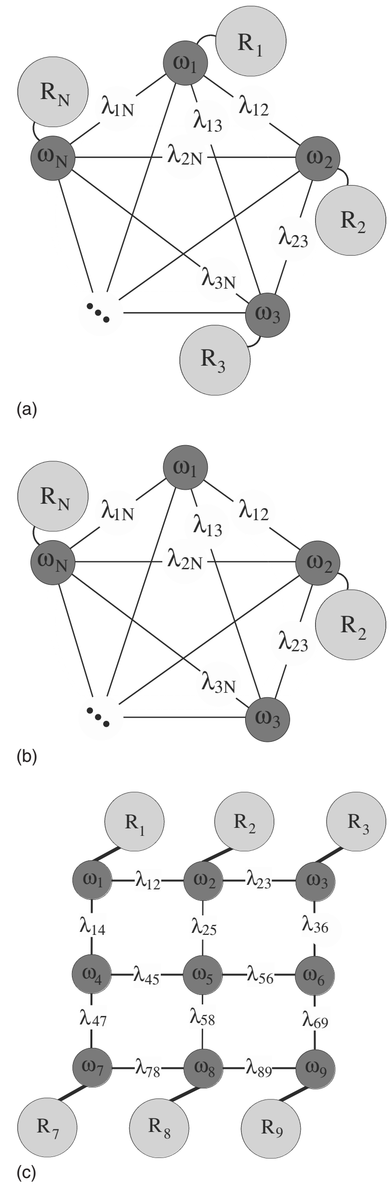


FIG. 2. Sketches of a dissipative symmetric network (a), a mixed-symmetric network (b) composed of dissipative and nondissipative oscillators, and a mixed network (c) composed of dissipative and nondissipative chains of oscillators.

$$\mathcal{H}_{mix\text{-}sym}^D = i\mathcal{H}_{sym} + \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \cdots & \Gamma_{2N} \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Gamma_{N1} & \Gamma_{N2} & \Gamma_{N3} & \cdots & \Gamma_{NN} \end{pmatrix}. \quad (28)$$

As a last example of a mixed network we consider the topology sketched in Fig. 2(c), where three linear chains of coupled oscillators are connected together, with the central (lateral) chain(s) being composed by nondissipative (dissipative) oscillators. In this case, labeling the oscillators as in Fig. 2(c), we obtain

$$\mathcal{H}_{mix}^{D} = i\begin{pmatrix} \omega_1 & \lambda_{12} & 0 & \lambda_{14} & & & & & \\ \lambda_{12} & \omega_2 & \lambda_{23} & 0 & \lambda_{25} & & & 0 & \\ 0 & \lambda_{23} & \omega_3 & 0 & 0 & \lambda_{36} & & & \\ \lambda_{14} & 0 & 0 & \omega_4 & \lambda_{45} & 0 & \lambda_{47} & & \\ & \lambda_{25} & 0 & \lambda_{45} & \omega_5 & \lambda_{56} & 0 & \lambda_{58} & \\ & & \lambda_{36} & 0 & \lambda_{56} & \omega_6 & 0 & 0 & \lambda_{69} \\ & & & \lambda_{47} & 0 & 0 & \omega_7 & \lambda_{78} & 0 \\ & 0 & & & \lambda_{58} & 0 & \lambda_{78} & \omega_8 & \lambda_{89} \\ & & & & & \lambda_{69} & 0 & \lambda_{89} & \omega_9 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \cdots & \Gamma_{19} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \cdots & \Gamma_{29} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \cdots & \Gamma_{39} \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \Gamma_{71} & \Gamma_{72} & \Gamma_{73} & \cdots & \Gamma_{79} \\ \Gamma_{81} & \Gamma_{82} & \Gamma_{83} & \cdots & \Gamma_{89} \\ \Gamma_{91} & \Gamma_{92} & \Gamma_{93} & \cdots & \Gamma_{99} \end{pmatrix}. \tag{29}$$

Therefore, for a given topology, the diagonalization of the matrix $\mathcal{H}^D$, leading to the eigenvalues $\Omega_m$ and eigenvectors composing the matrix $\mathbf{D}$, enable us to obtain the evolved Glauber-Sudarshan $P$ function and, consequently, the reduced density operator of the network, given by

$$\rho_{1,\ldots,N}(t) = \left[\bigotimes_m \int d^2\eta_m^0 |\eta_m^0\rangle\langle\eta_m^0|\right] P_{1,\ldots,N}(\{\eta_n\},t). \tag{30}$$

## IV. INITIAL STATES, DECOHERENCE, AND LINEAR ENTROPY

Starting from two different initial states of the network, given by general superpositions of coherent and Fock states, in this section we obtain the evolved reduced density operator (30) for both cases.

### A. General superposition of coherent states

Let us assume that the initial pure state of the network is given by

$$\rho(0) = \mathcal{N}^2 \sum_{r,s=1}^{Q} \Lambda_r \Lambda_s^* |\{\beta_m^r\}\rangle\langle\{\beta_m^s\}|, \tag{31}$$

where $\mathcal{N}$ is the normalization factor, $\Lambda_r$ is the probability amplitudes of the product state $|\{\beta_m^r\}\rangle = \otimes_{m=1}^N |\beta_m^r\rangle$, and the labels $r$ and $s$ run from 1 to the integer $Q$. The superscript $r$ stands for the $r$th state of the superposition while the subscript $m$ stands for the coherent state of $m$th oscillator. We stress that the discrete sum of product states in Eq. (31) can be substituted by the continuum sum $|\psi(0)\rangle = \mathcal{N}\int d\theta \Lambda(\theta)|\{\beta_m(\theta)\}\rangle$ with no further complication. After some algebra, we verify that the state (31) evolves to the $N$-oscillator density operator

$$\rho_{1,\ldots,N}(t) = \mathcal{N}^2 \sum_{r,s} \Lambda_r \Lambda_s^* \frac{\langle\{\beta_m^s\}|\{\beta_m^r\}\rangle}{\langle\{\zeta_m^s(t)\}|\{\zeta_m^r(t)\}\rangle} |\{\zeta_m^r(t)\}\rangle\langle\{\zeta_m^s(t)\}|, \tag{32}$$

where the excitation of the $m$th oscillator is given by

$$\zeta_m^r(t) = \sum_n \Theta_{mn}(t)\beta_n^r, \tag{33}$$

with the time-dependent matrix elements

$$\Theta_{mn}(t) = \sum_{m'} D_{mm'} \exp(-\Omega_{m'}t) D_{m'n}^{-1}. \tag{34}$$

For the reduced density operator of the $m$th oscillator we obtain

$$\rho_m(t) = \mathcal{N}^2 \sum_{r,s} \Lambda_r \Lambda_s^* \frac{\langle\{\beta_n^s\}|\{\beta_n^r\}\rangle}{\langle\zeta_m^s(t)|\zeta_m^r(t)\rangle} |\zeta_m^r(t)\rangle\langle\zeta_m^s(t)|, \tag{35}$$

where the influence of all the other oscillators of the network is present explicitly in the product $\langle\{\beta_n^s\}|\{\beta_n^r\}\rangle$ and implicitly in the states $|\zeta_m^r(t)\rangle$.

### B. General superposition of Fock states

In the Fock basis we consider the state

$$|\varphi(0)\rangle = \sum_{n_1,\ldots,n_N} C_{n_1,\ldots,n_N} |n_1,\ldots,n_N\rangle, \tag{36}$$

where $n_m$ stands for the photon number of $m$th oscillator and $C_{n_1,\ldots,n_N}$ is the probability amplitude of each state in the superposition. After a lengthy calculation we verify that the $N$-oscillator density operator is given by

$$\begin{aligned}\rho_{1,\ldots,N}(t) = &\sum_{n_1,\ldots,n_N}\sum_{m_1,\ldots,m_N} C_{m_1,\ldots,m_N}^* C_{n_1,\ldots,n_N} \\ &\times\left(\prod_m \sum_{q_m=0}^{n_m}\sum_{k_m=0}^{\infty} \frac{(-1)^{k_m}\sqrt{m_m! n_m!}}{(n_m-q_m)! k_m!}\right) \\ &\times |\mathcal{F}(\{q_\ell\},\{k_\ell\},t)\rangle\langle\mathcal{F}(\{m_\ell-n_\ell+q_\ell\},\{k_\ell\},t)|,\end{aligned} \tag{37}$$

where

$$|\mathcal{F}(\{q_\ell\},\{k_\ell\},t)\rangle = \otimes_m \sum_{j_m=0}^{\infty} \frac{(j_m+k_m)!}{\sqrt{j_m!}} \sum_{\mu_{m,1}=0}^{j_m+k_m} \frac{[\Theta_{m,1}(t)]^{j_m+k_m-\mu_{m,1}}[\Theta_{m,N}(t)]^{\mu_{m,N-1}}}{(j_m+k_m-\mu_{m,1})!\,\mu_{m,N-1}!}\left(\prod_{i=2}^{N-1}\sum_{\mu_{m,i}=0}^{\mu_{m,i-1}} \frac{[\Theta_{m,i}(t)]^{\mu_{m,i-1}-\mu_{m,i}}}{(\mu_{m,i-1}-\mu_{m,i})!}\right) \times \delta\left\{\sum_n [j_n+k_n-\mu_{n,m}(1-\delta_{m,N})] - \sum_{r=1}^{m} q_r\right\}|j_m\rangle, \tag{38}$$

where $\delta(x)$ equals unity for $x=0$, being null otherwise.

### C. State transfer and recurrence dynamics

From the reduced density operators $\rho_m(t)$ following from Eqs. (32) and (37) it is a direct matter to verify the transfer of an initial state prepared in the $m$th oscillator to the remaining one of the network, followed by the recurrence of this state back to the $m$th oscillator. The probability of recurrence of an initial state $\rho_m(0)$ prepared in the $m$th oscillator is given by the expression

$$\mathcal{P}_R(t) \equiv \mathrm{Tr}[\rho_m(t)\rho_m(0)], \tag{39}$$

which is also a measurement of the fidelity of the initial state $\rho_m(0)$, expected to decrease due to the dissipative process. For the probability of transfer of the initial state $\rho_m(0)$ to a particular $n$th oscillator picked up from the remaining $N-1$ of the network, we get

$$\mathcal{P}_T(t) \equiv \mathrm{Tr}[\rho_n(t)\rho_m(0)]. \tag{40}$$

From Eqs. (39) and (40) it can be verified—as analyzed in detail in Refs. [18,19] for the particular symmetric and central topologies, respectively—that an initial superposition prepared in the $m$th oscillator bounces between its original oscillators and all the remaining oscillators of the network. Evidently, the dynamics of a given prepared state through the network can be manipulated through the choice of the topology.

### D. Decoherence

From a given initial superposition of coherent states (31) and the density operator of the network (32), we can estimate the decoherence time of an arbitrary chosen off-diagonal element of the density operator relatively to the relaxation time of the diagonal elements. In fact, the literature concerned with the decoherence of $N$-dimensional superpositions deals only with relative time-decay measurements by which the larger the distance from the main diagonal of the matrix elements, the smaller are their decay time [24]. From Eq. (32) we verify that such relative time-decay measurements follow from the real part of the coefficients $\langle\{\beta_m^r\}|\{\beta_m^s\}\rangle/\langle\{\zeta_m^r(t)\}|\{\zeta_m^s(t)\}\rangle$ which is directly computed from the initial state of the network together with Eq. (33).

However, additional ingredients concerning the decoherence dynamics arise when considering a network of dissipative quantum systems. In Ref. [16], where a minimal network of two dissipative oscillators is analyzed, it is demonstrated that the decoherence time of a "Schrödinger-cat"-like state prepared in one of the oscillators can be doubled compared to that when the same state is prepared in an isolated dissipative oscillator. This result follows when the decay rate of the oscillator, where the state is prepared, is significantly larger than the other one composing the network. A generalized analysis of decoherence for the case of a symmetric network of dissipative oscillators is presented in Ref. [20], where the physical ingredients that enable the emergence of relaxation-free and decoherence-free subspaces are exposed. In this regard, a detailed study of the optimum topologies leading to maximum decoherence times of superposition states prepared in particular oscillators of dissipative networks will be presented elsewhere [23]. The memory devices presented in Ref. [23], which follow from the general formalism presented here, combine both ingredients: (i) the large decay rate of the storage oscillators of the network—those except the one where the state to be protected is prepared—and (ii) specific dynamics of this state through the network, achieved by engineering particular topologies.

### E. Linear entropy

From the density operator in Eq. (32), we are able to calculate the linear entropies for the mixed states of the whole network, $\mathcal{S}_{1,\ldots,N}(t)$, of oscillator 1 (or any other particular oscillator), $\mathcal{S}_1(t)$, and of all the remaining $N-1$ oscillators, $\mathcal{S}_{2,\ldots,N}(t)$, which are given by

$$\begin{aligned}\mathcal{S}_{1,\ldots,N}(t) &= 1-\mathrm{Tr}_{1,\ldots,N}[\rho_{1,\ldots,N}(t)]^2\\ &= 1-\mathcal{N}^4\sum_{r,s,p,q}\langle\{\beta_m^r\}|\{\beta_m^s\}\rangle\langle\{\beta_m^p\}|\{\beta_m^q\}\rangle\\ &\quad\times\exp\left[-\sum_n(\beta_n^s-\beta_n^q)(\beta_n^r-\beta_n^p)^*\sum_m|\Theta_{mn}(t)|^2\right],\end{aligned} \tag{41a}$$

$$\begin{aligned}\mathcal{S}_1(t) &= 1-\mathrm{Tr}_1[\rho_1(t)]^2\\ &= 1-\mathcal{N}^4\sum_{r,s,p,q}\langle\{\beta_m^r\}|\{\beta_m^s\}\rangle\langle\{\beta_m^p\}|\{\beta_m^q\}\rangle\\ &\quad\times\exp\left[-\sum_n(\beta_n^s-\beta_n^q)(\beta_n^r-\beta_n^p)^*|\Theta_{1n}(t)|^2\right],\end{aligned} \tag{41b}$$

$$\mathcal{S}_{2,\ldots,N}(t) = 1 - \mathrm{Tr}_{2,\ldots,N}[\rho_{2,\ldots,N}(t)]^2 = 1 - \mathcal{N}^4 \sum_{r,s,p,q} \langle\{\beta_m^r\}|\{\beta_m^s\}\rangle\langle\{\beta_m^p\}|\{\beta_m^q\}\rangle \times\exp\left[-\sum_n (\beta_n^s - \beta_n^q)(\beta_n^r - \beta_n^p)^* \sum_{\ell=2}^N |\Theta_{\ell n}(t)|^2\right], \tag{41c}$$

where the second equality follows from the initial state (31) and, like $r$ and $s$, the labels $p$ and $q$ run from 1 to the integer $Q$. With these expressions we can analyze, as discussed in Refs. [18–20], the evolution of the correlation between the reduced state of oscillator 1 and that of all the remaining $N-1$ oscillators, through the excess entropy, defined as

$$\mathcal{E}(t) \equiv \mathcal{S}_1(t) + \mathcal{S}_{2,\ldots,N}(t) - \mathcal{S}_{1,\ldots,N}(t). \tag{42}$$

The excess entropy can also reveals [through a residual value of $E(t)$] the development of an inevitable background correlation between all the network oscillators which thus become permanently entangled [18–20]. This background correlation arises from two different mechanisms: First, the action of the indirect channels $[\mathcal{L}_{mn}\rho_{1,\ldots,N}(t)]$ and, second, the action of the direct channels $[\mathcal{L}_{mm}\rho_{1,\ldots,N}(t)]$, when the decay rates $\Gamma_{mn}$ are different from each other. For equal decay rates, the direct decay channels do not contribute to the development of the background correlation. The indirect channels thus play an important role in the entanglement process in dissipative networks.

## V. CASE OF A COMMON RESERVOIR FOR THE WHOLE NETWORK

In this section we extend our analysis to contemplate the case where all the oscillators of the network are coupled to one common reservoir (at 0 K). For a particular symmetric network, the different results following from both cases of distinct or a common reservoir have been discussed in Ref. [20] in connection with the emergence of relaxation-free and/or decoherence-free subspaces. In Refs. [16,18–20] a brief discussion is also provided of the rather unusual scenario of a common reservoir.

The Hamiltonian for the case of a common reservoir is given by

$$H = \hbar\sum_m \omega_m a_m^\dagger a_m + \frac{\hbar}{2}\sum_{m\neq n} \lambda_{mn}(a_m^\dagger a_n + a_m a_n^\dagger) + \sum_k \omega_k b_k^\dagger b_k + \hbar\sum_m\sum_k V_{mk}(b_k^\dagger a_m + b_k a_m^\dagger). \tag{43}$$

Following the same steps as in Sec. II, we obtain from Hamiltonian (43) the same master equation (19) derived previously for the case of distinct reservoirs, but with the effective damping matrix (18) replaced by

$$\Gamma_{mn} = N \sum_{m',n'} \xi_{mm'}(\varpi_{n'}) C_{n'm'} C_{n'n}, \tag{44}$$

where

$$\xi_{mn}(\varpi_{m'}) = \int_0^t d\tau \int_0^\infty \frac{d\nu}{\pi}\sigma^2(\nu) V_m(\nu) V_n(\nu) e^{-i(\nu-\varpi_{m'})(t-\tau)}. \tag{45}$$

The correlation factor $\xi_{mn}$ arises from the fact that both network oscillators $m$ and $n$ may interact indirectly through their common reservoir. To analyze more closely such a correlation, we assume (as usual for the case of weak coupling between the system and the reservoir) that the network oscillators only interact with the reservoir modes in the neighborhood of their normal modes. Under this assumption the maximum correlation takes place when both oscillators $m$ and $n$ are identically coupled with the same group of reservoir modes—i.e., when $V_m(\nu)=V_n(\nu)$. Otherwise, a partial correlation arises when the coupling between the oscillators with the reservoir modes turns out not to be identical—i.e., $\{V_m(\nu)\}\cap\{V_n(\nu)\}\neq\emptyset$. In this case, the oscillators may still be coupled with the same group of reservoir modes, but with different strengths, or be coupled with different groups of reservoir modes apart from a common intersection of them. The correlation between the oscillators disappears only when $\{V_m(\nu)\}\cap\{V_n(\nu)\}=\emptyset$—i.e., when there is practically no intersection of common reservoir modes coupled to both oscillators.

It is particularly interesting to note that in the case where $\{V_m(\nu)\}\cap\{V_n(\nu)\}=\emptyset$ and consequently $\xi_{mn}=0$, only the self-correlation $\xi_{mm}$ survives, which reduces to the damping factor $\delta_{mn}\gamma_m$ in Eq. (16), apart from the unique frequency distribution $\sigma(\nu)$ of the common reservoir. In its turn, when assuming the coupling strengths $V_m$ between the oscillators and the common reservoir to be all different to compensate the unique $\sigma(\nu)$, the effective damping factor (44) arising from the self-correlations reduces to that of the case of distinct reservoirs in Eq. (18). Therefore, it is possible to derive the master equation for the case of distinct reservoir starting from that of a common one, under the condition that no correlation between two oscillators be induced by their common reservoir. Conversely, it is also possible to shift from the case of distinct reservoir to that of a common one assuming that all the reservoirs present the same frequency distribution $\sigma(\nu)$ and the limit of strong interactions between the network oscillators. In this limit, as discussed in Refs. [16,18–20], the condition $N\lambda_{mn}\gtrsim\omega_{m'}$ must be satisfied. The interesting aspect of such a condition is that it can be fulfilled for coupling strengths $\lambda_{mn}\ll\omega_{m'}$, as long as a sufficiently large network is provided $(N\gtrsim\omega_{m'}/\lambda_{mn})$.

## VI. CASE OF RESERVOIRS AT FINITE TEMPERATURES

Since a formal approach for the case where the reservoirs are at finite temperatures is somewhat demanding, here we shall present a brief qualitative analysis of this case. Our analysis focus on the normal-mode oscillators $\varpi_m$ (represented by the operators $A_m$ and $A_m^\dagger$) under the assumption that all the coupling strengths between the original oscillators $\omega_m$ (represented by $a_m$ and $a_m^\dagger$) and their respective reservoirs are around the same. We first note that the $N$ normal-mode oscillators are decoupled from each other, whereas each one interacts with all the $N$ reservoirs as described by Hamiltonian in Eq. (10). Evidently, when the couplings $\lambda_{mn}$ between the original oscillators are all turned off, the normal-mode oscillators degenerate into the original one. Moreover, when the coupling strengths are significantly smaller than the natural frequencies of the original oscillators—i.e., $N\{\lambda_{mn}\} \ll \{\omega_{m'}\}$—the magnitude of the interaction between the $m$th normal-mode oscillator with the $m$th reservoir is significantly larger than those with the remaining $N-1$ reservoirs. In fact, when $N\{\lambda_{mn}\} \ll \{\omega_{m'}\}$, the indirect channels are not quite effective as pointed out above. Otherwise, when $N\{\lambda_{mn}\} \approx \{\omega_{m'}\}$, the indirect channels become as effective as the direct one and the magnitude of the interactions between the $m$th normal-mode oscillator with all the reservoirs becomes quite the same. From the above qualitative observations we next discuss the effects of temperature in both cases of distinct reservoirs and a common reservoir.

### A. Distinct reservoirs

For the case of $N$ distinct reservoirs at finite temperatures $T_m$, in the regime where $N\{\lambda_{mn}\} \ll \{\omega_{m'}\}$, we thus conclude that the (non-null) mean energy $\langle E_m \rangle$ of the $m$th normal-mode oscillator will practically be defined by its associated $m$th reservoir. Consequently, in the steady-state configuration, each of the normal-mode oscillators presents a different mean energy which is defined by the equilibrium reached with all the reservoirs, but mostly with its associated reservoir. As expected, in the regime where $N\{\lambda_{mn}\} \approx \{\omega_{m'}\}$, all the normal-mode oscillators present approximately the same mean energy, since the magnitudes of their couplings with the different reservoirs are approximately the same. Summarizing, in spite of the different temperatures of the reservoirs, in the steady-state configuration of the regime where $N\{\lambda_{mn}\} \approx \{\omega_{m'}\}$, all the normal-mode oscillators vibrate with approximately the same mean energy, whereas in the regime $N\{\lambda_{mn}\} \ll \{\omega_{m'}\}$, each normal-mode oscillator exhibits a different mean energy.

### B. Common reservoir

When a common reservoir is considered, the scenario is much like that arising in the case of distinct reservoirs in the regime $N\{\lambda_{mn}\} \approx \{\omega_{m'}\}$. In fact, as the coupling strengths between the normal-mode oscillators and their common reservoir are around the same, in the steady-state configuration all the normal-mode oscillators exhibit around the same mean energy.

## VII. CONCLUDING REMARKS

Motivated by the necessity to better understand the coherence and decoherence dynamics of quantum states in networks composed of a large number of dissipative quantum systems, we have studied chains of dissipative harmonic oscillators. We presented previous results related to different topologies by starting with the simplest case of only two coupled oscillators [16] and generalizing the analysis to the case of $N$ coupled oscillators in a symmetric [18] and a central-oscillator network [19]. The developments in Refs. [18,19] were extended to the analysis of the physical ingredients responsible for the emergence of decoherence-free subspace [20].

Since in Refs. [18,19] we have treated two particular topologies independently, in the present contribution we have presented a general formalism to treat whatever the topology of a chain of dissipative harmonic oscillators. Starting from a symmetric network, were all the oscillators are coupled together, apart from being coupled to their respective reservoirs (or to a common one), we have derived the master equation and the associated evolution equation of the Glauber-Sudarshan $P$ function. We thus have showed how to particularize such results for whichever the specific network. We also shown how to obtain the master equation for the case where each oscillator is coupled to its respective reservoir starting from that where all the oscillators are coupled to a common reservoir.

The presented formalism is quite general and can be used to compute the decoherence time of pure or mixed states prepared in a particular oscillator of the network or even in a cluster of oscillators of the network. The correlation between the states of the network can also be computed through the excess entropy defined for a bipartite system [16,18,19].

It is worth stressing that an exact model to treat a dissipative quantum oscillator can be extracted from the present treatment. To this end, we have to pick up a single oscillator of the network—our system of interest—and couple it to all the other oscillators which play the role of the reservoir. Thus, discounting the reservoirs which we have treated here through the standard perturbative approach, we end up with an exact formalism to account for the dissipative effects over a harmonic oscillator. Evidently, such an exact formalism can also be pursued for dissipative systems other than a harmonic oscillator. An extensive analysis of the effects coming from the exact treatment of dissipation emerging from this work will be presented elsewhere.

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[1] R. Landauer, Proc. R. Soc. London, Ser. A **353**, 367 (1995).
[2] W. G. Unruh, Phys. Rev. A **51**, 992 (1995).
[3] P. W. Shor, Phys. Rev. A **52**, R2493 (1995); A. M. Steane, Phys. Rev. Lett. **77**, 793 (1996). For a review see, e.g., A. M. Steane, in *Introduction to Quantum Computation and Information*, edited by H. K. Lo, S. Popescu, and T. P. Spiller (Word Scientific, Singapore, 1999), pp. 184–212.
[4] J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **77**,

4728 (1996).
[5] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997).
[6] D. A. Lidar, I. L. Chuang, and K. B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998); E. Knill, R. Laflamme, and L. Viola, *ibid.* **84**, 2525 (2000); A. R. Bosco de Magalhães and M. C. Nemes, Phys. Rev. A **70**, 053825 (2004).
[7] D. A. Lidar and K. B. Whaley, e-print arXiv:quant-ph/0301032.
[8] O. Mandel, M. Greiner, A. Widera, T. Rom, T. W. Hänsch, and I. Bloch, Phys. Rev. Lett. **91**, 010407 (2003).
[9] A. Polkovnikov, Phys. Rev. A **68**, 033609 (2003).
[10] R. G. Scott, A. M. Martin, S. Bujkiewicz, T. M. Fromhold, N. Malossi, O. Morsch, M. Cristiani, and E. Arimondo, Phys. Rev. A **69**, 033605 (2004).
[11] M. Christandl, N. Datta, A. Ekert, and A. J. Landahl, Phys. Rev. Lett. **92**, 187902 (2004).
[12] M. B. Plenio, J. Hartley, and J. Eisert, New J. Phys. **6**, 36 (2004).
[13] J. M. Raimond, M. Brune, and S. Haroche, Phys. Rev. Lett. **79**, 1964 (1997).
[14] S. G. Mokarzel, A. N. Salgueiro, and M. C. Nemes, Phys. Rev. A **65**, 044101 (2002).
[15] H. Zoubi, M. Orenstien, and A. Ron, Phys. Rev. A **62**, 033801 (2000).
[16] M. A. de Ponte, M. C. de Oliveira, and M. H. Y. Moussa, Ann. Phys. (N.Y.) **317**, 72 (2004).
[17] M. A. de Ponte, C. J. Villas-Bôas, R. M. Serra, and M. H. Y. Moussa, Europhys. Lett. **72**, 383 (2005).
[18] M. A. de Ponte, M. C. de Oliveira, and M. H. Y. Moussa, Phys. Rev. A **70**, 022324 (2004).
[19] M. A. de Ponte, M. C. de Oliveira, and M. H. Y. Moussa, Phys. Rev. A **70**, 022325 (2004).
[20] M. A. de Ponte, S. S. Mizrahi, and M. H. Y. Moussa, Ann. Phys. (N.Y.) (to be published).
[21] Z. Ficek and R. Tanas, Phys. Rep. **372**, 369 (2002).
[22] G. Burkard and F. Brito, Phys. Rev. B **72**, 054528 (2005).
[23] M. A. de Ponte, S. S. Mizrahi, and M. H. Y. Moussa (unpublished).
[24] M. H. Y. Moussa, S. S. Mizrahi, and A. O. Caldeira, Phys. Lett. A **221**, 145 (1996).