

Generation of decoherence-free displaced squeezed states of radiation fields and a squeezed reservoir for atoms in cavity QED

T. Werlang,¹ R. Guzmán,² F. O. Prado,¹ and C. J. Villas-Bôas¹

¹*Departamento de Física, Universidade Federal de São Carlos, 13565-905, São Carlos, Brazil*

²*Departamento de Ciencias Físicas, Universidad de La Frontera, Casilla Postal 54-D, Temuco, Chile*

(Received 24 April 2008; published 15 September 2008)

We present a way to engineer an effective anti-Jaynes-Cumming and a Jaynes-Cumming interaction between an atomic system and a single cavity mode and show how to employ it in reservoir engineering processes. To construct the effective Hamiltonian, we analyze the interaction of an atomic system in a Λ configuration, driven by classical fields, with a single cavity mode. With this interaction, we first show how to generate a decoherence-free displaced squeezed state for the cavity field. In our scheme, an atomic beam works as a reservoir for the radiation field trapped inside the cavity, as employed recently by S. Pielawa *et al.* [Phys. Rev. Lett. **98**, 240401 (2007)] to generate an Einstein-Podolsky-Rosen entangled radiation state in high- Q resonators. In our scheme, all the atoms have to be prepared in the ground state and, as in the cited article, neither atomic detection nor precise interaction times between the atoms and the cavity mode are required. From this same interaction, we can also generate an ideal squeezed reservoir for atomic systems. For this purpose we have to assume, in addition to the engineered atom-field interaction, a strong decay of the cavity field (i.e., the cavity decay must be much stronger than the effective atom-field coupling). With this scheme, some interesting effects in the dynamics of an atom in a squeezed reservoir could be tested.

DOI: [10.1103/PhysRevA.78.033820](https://doi.org/10.1103/PhysRevA.78.033820)

PACS number(s): 42.50.Pq, 42.50.Dv, 03.65.Yz

I. INTRODUCTION

The impressive experimental progress in manipulation of the interaction between light and atoms has led to a better understanding of several fundamentals of quantum theory and also the development of the area known as quantum-information theory [1]. The experimental verification of the granular nature of the radiation field [2] or of the motion of a single trapped ion [3], the study of the decoherence process of a Schrödinger cat superposition state [4], and the violation of the Bell inequalities [5], which reveals the nonlocal character of quantum phenomena, are some examples of fundamentals of physics recently investigated through the manipulation of radiation field states by atoms, or vice versa. On the other hand, the implementation of quantum logic gates in trapped ions [6] or in cavity quantum electrodynamics (QED) [7] and atomic teleportation [8] have contributed to the rapid development of the quantum-information area [1]. Through the precise manipulation of the atom-field interaction, many quantum states of light such as the Schrödinger cat states [9] and Fock states [10] have been generated in cavity QED. However, some nonclassical states, such as the squeezed states [11] and the two-mode squeezed state [12], have not been attained experimentally so far, either in the microwave or in the optical regimes in cavity QED, through the interaction of atoms and cavity modes. The experimental generation of these states is of great interest since they could be used to test the fundamentals of theoretical physics and to achieve quantum communication. For instance, the single-mode squeezed states could be used to verify the sub-Poissonian statistics of the radiation field [13], to measure gravitational waves [14], and for optical communication, through improvement of the signal-to-noise ratio [15]. The generation of multimode squeezed light would also be useful for manipulating the dynamics of two-level atoms. As

pointed out in [16,26], the interaction of a two-level atom with multimode squeezed light, which works as a squeezed reservoir, can produce some interesting effects in atomic dynamics such as suppression (enhancement) of decay of the in-phase (out-of-phase) components of atomic polarization and a line narrowing in resonance fluorescence and absorption spectra. Therefore, the generation of robust squeezed states of the radiation field and the generation of a squeezed reservoir for two-level atoms may help us to deepen our understanding of the quantum nature of the light, the properties of atoms, and the atom-field interaction itself.

Concerning radiation squeezed states, we find some theoretical schemes in the literature for the generation of these states in the cavity QED context employing the interaction of three-level atoms [17] or even a single two-level atom [18] with a trapped field inside a high-finesse cavity. There are some theoretical proposals, employing the manipulation of the interaction between a three-level atom in a Λ configuration and a single cavity mode, to generate arbitrary single-mode cavity field states [19] and Fock states with a large number of photons, through selective interactions [20]. However, none of these schemes take into account the system-environment interaction, which degrades the quantum states so that, in general, the fidelity of the generated states decays quickly in time. To circumvent this problem and to generate robust nonclassical states of the radiation field or of ionic motion, an approach based on *reservoir engineering* [21,22] has been proposed. This technique has been employed in the trapped ion domain to protect, against decoherence and relaxation, any superposition of Fock states [23] in one-dimensional motion of a single ion. Using similar procedures, there are also proposals for the generation of trapping states (Fock states) of a single cavity mode [24] and entangled states of two cavity modes, i.e., the two-mode squeezed vacuum state [25].

In addition, schemes have been proposed to implement the interaction between an atomic system and a squeezed reservoir, the simplest of which consists in considering a two-level atom immersed in a squeezed multimode radiation field [26]. However, the scheme described does not represent the interaction of an atomic system with an ideal squeezed reservoir, since the action of the usual vacuum reservoir, due to the other modes of the electromagnetic field, cannot be turned off and it is difficult to embed the atom in a squeezed vacuum in a complete 4π solid angle. Parkins *et al.* [27] have shown how a two-level system can be coupled to an almost ideal squeezed vacuum by assuming an atom strongly interacting with a cavity field which is illuminated by finite-bandwidth squeezed light. In Ref. [28], the authors show how to mimic the interaction of a two-level system with a squeezed reservoir through quantum reservoir engineering. In this scheme, a four-level atom interacts with circularly polarized fields. Then, assuming a strong decay of the two most excited levels, it can be shown that the dynamics of the two ground atomic states is effectively similar to that of a two-level system interacting with a squeezed reservoir. We have also found a few experimental studies of the dynamics of an atomic system in a squeezed reservoir [29], but these could not verify some predicted phenomena, such as the phase-sensitive decay of the atomic polarization, mainly due to the difficulty of embedding atoms in a squeezed vacuum in a complete 4π solid angle [28].

In the present paper we make a theoretical study of the manipulation of an atom-field interaction and how to use engineered Hamiltonians to generate both robust displaced squeezed states of a cavity field mode and a squeezed reservoir for a two-level atomic system. To this end we employ the interaction of three-level atoms in a Λ configuration with a single cavity mode and classical fields. Adjusting the intensity and the detuning of the classical field, we derive an effective Hamiltonian which involves a Jaynes-Cummings (JC), an anti-JC, and a rotation interaction of a two-level atom with a cavity mode. With this kind of interaction, we can generate (i) a robust displaced squeezed state in a single cavity mode and (ii) an ideal squeezed reservoir for atoms. In the next section, we present the model used to obtain the desired Hamiltonian interaction. In Sec. III, we show how to use this interaction to generate a displaced squeezed state in a single cavity mode and present a numerical analysis of this system. In Sec. IV, assuming the atoms to be trapped in a bad cavity, we use the same effective Hamiltonian to simulate an ideal squeezed reservoir for a two-level system. We also carry out a numerical analysis to verify the validity of approximations employed to simulate a squeezed reservoir for atoms. Finally, in Sec. V, we present some concluding remarks.

II. THE MODEL

To generate the desired effective interaction we will employ the interaction of a three-level atom in a Λ configuration with a single cavity mode and classical fields. As depicted in Fig. 1, the ground $|g\rangle$ and excited $|e\rangle$ states are coupled to an auxiliary state $|i\rangle$ through classical fields, with

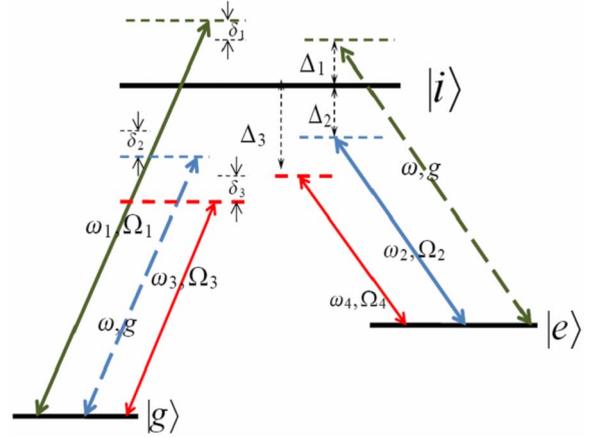


FIG. 1. (Color online) Atomic and field configuration employed in the interaction engineering process. The ground $|g\rangle$ and excited $|e\rangle$ states are coupled to the auxiliary state $|i\rangle$ through laser fields and the cavity mode.

coupling Ω_i and frequency ω_i ($i=1-4$), and a cavity mode, with coupling g and frequency ω . For this system, the total Hamiltonian is $H=H_0+V(t)$, with

$$H_0 = \hbar\omega_g\sigma_{gg} + \hbar\omega_e\sigma_{ee} + \hbar\omega_i\sigma_{ii} + \hbar\omega a^\dagger a, \quad (1a)$$

$$V(t) = \hbar(ga + \Omega_1 e^{-i\omega_1 t} + \Omega_3 e^{-i\omega_3 t})\sigma_{ig} + \hbar(ga + \Omega_2 e^{-i\omega_2 t} + \Omega_4 e^{-i\omega_4 t})\sigma_{ie} + \text{H.c.}, \quad (1b)$$

where $\hbar\omega_\alpha$, $\alpha=g,e,i$, are the energies of the atomic levels, $\sigma_{lm}=|l\rangle\langle m|$, $l,m=g,e,i$, are the atomic operators, a and a^\dagger are the annihilation and creation operators for the cavity field, respectively, and H.c. stands for the Hermitian conjugate. Using the unitary transformation $U_0=e^{-iH_0 t/\hbar}$ we can rewrite the Hamiltonian in the interaction picture

$$H_I(t) = \hbar(gae^{i(\Delta_2+\delta_2)t} + \Omega_1 e^{-i(\Delta_1+\delta_1)t} + \Omega_3 e^{i(\Delta_3+\delta_3)t})\sigma_{ig} + \hbar(gae^{-i\Delta_1 t} + \Omega_2 e^{i\Delta_2 t} + \Omega_4 e^{i\Delta_3 t})\sigma_{ie} + \text{H.c.}, \quad (2)$$

where we have defined $\Delta_1 \equiv \omega - (\omega_i - \omega_e) = \omega_1 - (\omega_i - \omega_g) - \delta_1$, $\Delta_2 \equiv (\omega_i - \omega_g) - \omega - \delta_2 = (\omega_i - \omega_e) - \omega_2$, and $\Delta_3 \equiv (\omega_i - \omega_g) - \omega_3 - \delta_3 = (\omega_i - \omega_e) - \omega_4$. Considering the nonresonant regime $|\Delta_k| \sim (|\Delta_k| - |\Delta_l|) \gg |g|\sqrt{\bar{n}}, |\Omega_k|$, $k \neq l=1,2,3$, \bar{n} being the mean number of photons in the cavity mode, we can adiabatically eliminate the transitions between the ground and excited states and the auxiliary state, for example by the methods described in [30]. Thus, the effective dynamics, considering only the atomic subspace $\{|g\rangle, |e\rangle\}$, is governed by the effective Hamiltonian

$$H_{\text{eff}} = +\hbar \left[\left(-\frac{|g|^2}{\Delta_2} a^\dagger a + \varpi_g \right) \sigma_{gg} + \left(\frac{|g|^2}{\Delta_1} a^\dagger a + \varpi_e \right) \sigma_{ee} \right] + \hbar [(\lambda_1 a e^{i\delta_1 t} + \lambda_2 a^\dagger e^{-i\delta_2 t} + \beta e^{-i\delta_3 t}) \sigma_{ge} + \text{H.c.}], \quad (3)$$

where $\varpi_g = |\Omega_1|^2/\Delta_1 - |\Omega_3|^2/\Delta_3$, $\varpi_e = -|\Omega_2|^2/\Delta_2 - |\Omega_4|^2/\Delta_3$, $\lambda_1 = g\Omega_1^*/\Delta_1$, $\lambda_2 = -g^*\Omega_2/\Delta_2$, $\beta = -\Omega_3^*\Omega_4/\Delta_3$. For $|\Omega_k| \gg |g|$, the dispersive atom-quantum-field interactions are much smaller than the other terms in the effective Hamiltonian. Therefore, under these conditions, we make a new approxi-

mation, so that the effective Hamiltonian may be rewritten as

$$H_{\text{eff}} \approx +\hbar(\varpi_g \sigma_{gg} + \varpi_e \sigma_{ee}) + \hbar[(\lambda_1 a e^{i\delta_1 t} + \lambda_2 a^\dagger e^{-i\delta_2 t} + \beta e^{-i\delta_3 t}) \sigma_{ge} + \text{H.c.}]. \quad (4)$$

By numerical analysis, we have verified the validity of this approximation. We found that the bigger the ratio $|\Omega/g|$, the better were the results, as expected. Applying a new unitary transformation, defined by the operator $U = e^{-i(\varpi_g \sigma_{gg} + \varpi_e \sigma_{ee})t}$, with the assumption $\delta_1 = -\delta_2 = -\delta_3 = \varpi_e - \varpi_g$, we can finally write the effective Hamiltonian as

$$H_{\text{eff}} \approx \hbar[(\lambda_1 a + \lambda_2 a^\dagger + \beta) \sigma_- + \text{H.c.}], \quad (5)$$

with $\sigma_- = \sigma_{ge}$ and $\sigma_+ = (\sigma_-)^\dagger = \sigma_{eg}$. This effective Hamiltonian, which represents a Jaynes-Cummings ($\lambda_1 a \sigma_+ + \text{H.c.}$) and an anti-Jaynes-Cummings ($\lambda_2 a^\dagger \sigma_+ + \text{H.c.}$) interaction, besides a rotation of the electronic states ($\beta \sigma_+ + \text{H.c.}$), can be used to carry out two distinct processes to generate (i) a robust displaced squeezed state for the radiation field and (ii) a squeezed reservoir for an atom or an atomic sample. (A similar interaction was employed in Ref. [20] for the generation of large Fock states through selective interactions.)

III. DISPLACED SQUEEZED STATE IN A CAVITY MODE

In this section, by a method similar to that used to generate a displaced squeezed state in the trapped ion domain [21], we analyze the generation of the same state for the radiation field trapped inside a high- Q cavity, $|\alpha, \varepsilon\rangle = D(\alpha)S(\xi)|0\rangle$, where $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ is the displacement operator, α being the amplitude of displacement, and $S(\xi) = \exp[(\xi^* a^2 - \xi a^{\dagger 2})/2]$ is the squeezing operator, with $\xi = r e^{i\phi}$, r and ϕ being the squeezing factor and squeezing angle, respectively. To implement our proposal, an atomic beam should cross the cavity under the action of classical fields in such a way that the effective interaction between each atom and the cavity mode is given by the effective Hamiltonian (5). The atoms prepared in the ground state $|g\rangle$ are made to interact, one by one, with the cavity mode during a short time interval τ ($\lambda_l \tau \ll 1$, $l=1, 2$), so that the atomic beam acts as a reservoir at absolute zero ($T=0$ K) for the cavity mode, as described in various papers [25]. Under these conditions, the steady state of the cavity field is exactly the displaced squeezed state. To demonstrate this, we first apply a time-independent unitary transformation to the effective Hamiltonian, as in Ref. [21]: $\tilde{H}_{\text{eff}} = S(\xi)D(\alpha)H_{\text{eff}}D^\dagger(\alpha)S^\dagger(\xi)$. In this transformed representation, the Hamiltonian reads

$$\tilde{H} = \hbar \lambda a^\dagger \sigma_- + \text{H.c.}, \quad (6)$$

with the following adjustments: $\alpha \lambda_1 + \alpha^* \lambda_2 = -\beta$, $\tanh(r) = (\lambda_1/\lambda_2) e^{-i\phi}$, and $\lambda \equiv \cosh(r) \lambda_2 - e^{-i\phi} \sinh(r) \lambda_1 = \lambda_2 / \cosh(r)$. Here we can see that the squeezing factor r is determined by the ratio $|\lambda_1/\lambda_2|$, and the amplitude of the coherent displacement, α , by the parameters β , λ_1 , and λ_2 . In this new picture, the transformed Hamiltonian (6) represents a Jaynes-Cummings interaction between a cavity mode and a single two-level atom

As we can see from the diagram of levels in Fig. 1, the transitions $|g\rangle \leftrightarrow |i\rangle$ and $|e\rangle \leftrightarrow |i\rangle$ are dipole allowed and, by

the selection rules, the transition $|g\rangle \leftrightarrow |e\rangle$ is not. In this way, it would be very hard for the decay rate Γ from the excited state $|e\rangle$ to the ground state $|g\rangle$ to be stronger than the effective atom-field coupling λ , so that we cannot use this channel of dissipation to engineer our reservoir for the cavity mode, as in Ref. [21]. To get round this difficulty we can employ the scheme presented in Refs. [31,25] to simulate an atomic reservoir for the cavity mode. To this end, we first assume that the atoms are initially prepared in the ground state $|g\rangle$ and that the atoms arrive in the cavity, one by one, at the rate r_{at} . Next, we assume that each atom interacts with the cavity field during a short time interval τ , so that $\lambda \tau \ll 1$. In this transformed picture the atom-field interaction is governed by the transformed Hamiltonian (6). Tracing on the atomic variables, the effective master equation for the transformed cavity mode is given by [25,31]

$$\frac{\partial \tilde{\rho}}{\partial t} = \frac{\gamma_{\text{eng}}}{2} (2a \tilde{\rho} a^\dagger - a^\dagger a \tilde{\rho} - \tilde{\rho} a^\dagger a), \quad (7)$$

where $\gamma_{\text{eng}} = r_{\text{at}} \lambda^2 \tau^2$ is the engineered cavity field decay rate. It is known that the vacuum state $|0\rangle$ is the steady state of Eq. (7) for the cavity mode. Then, applying the reverse unitary transformation, we can easily see that the steady state (for time $t \gg 1/\gamma_{\text{eng}}$) of this system in the interaction picture is

$$\begin{aligned} \rho(t \rightarrow \infty) &= D(\alpha) S(\xi) \tilde{\rho} S^\dagger(\xi) D^\dagger(\alpha) \\ &= D(\alpha) S(\xi) |0\rangle \langle 0| S^\dagger(\xi) D^\dagger(\alpha), \end{aligned}$$

which is a pure state for the cavity mode, i.e., exactly the displaced squeezed state $|\Psi\rangle = D(\alpha)S(\xi)|0\rangle$. The degree of squeezing r is determined by the amplitudes of the classical fields Ω_j , since $\tanh(r) = |\lambda_1/\lambda_2|$ and $\lambda_1 = g\Omega_1^*/\Delta_1$ and $\lambda_2 = -g^*\Omega_2/\Delta_2$. This steady state does not depend on the initial cavity mode state: the generation of the displaced squeezed state in the present scheme is achieved when the system reaches the steady state. Here, the initial cavity field state only influences the time needed for the system to achieve the steady state, as discussed in [25]. As pointed out in Ref. [31], the effective master equation (7) for the cavity mode can be built even for an atomic beam with random arrival times and without the need for atomic detection nor precise interaction times between the atoms and the radiation field. Hence, as in Ref. [25], our scheme is robust against stochastic fluctuations in the atomic beam and does not require precise interaction times (velocity selection) or atomic detection. In this scheme we have assumed that the atoms arrive one by one in the cavity. If at any time two or more atoms enter the cavity simultaneously, the effective Hamiltonian (5) will be modified, appearing terms of effective interaction between the atoms mediated by the field even when it is in the vacuum, as can be seen in [32] where the authors showed how to generate entangled states between two atoms that interact nonresonantly with a cavity mode. Thus, when two or more atoms come together in the cavity, the dynamics of the whole system will modify the field state, introducing errors. However, with the passage of additional atoms, one by one, the field state can be brought back to the desired state. Also, as it is required a short interaction time between each atom and the cavity field for the dynamics be that governed by Eq. (7), the

probability of having two or more atoms simultaneously inside the cavity is very small. Therefore, those errors due to the simultaneous interaction of two or more atoms with the cavity mode may be neglected.

To test the validity of the scheme we have performed a numerical solution of the system. Starting with the cavity mode in the vacuum $|0\rangle$ state and all the atoms in the ground state $|g\rangle$, we carried out a numerical evolution of the system based on Hamiltonian (3). For simplicity, we fixed $\Omega_3=\Omega_4=0$, which implies $\beta=0$ and a null displacement ($\alpha=0$). We also chose the amplitudes $\Omega_1=\Omega_2=10g$ and the detunings $\Delta_1=100g$ and $\Delta_2=131g$ of the classical fields such that $\lambda_1=0.1g$ and $\lambda_2\approx 0.076g$, giving a squeezing factor $r=1.0$, squeezing angle $\phi=0$, and $\lambda\approx 0.065g$. The interaction parameter was fixed at $\lambda\tau=0.2$, implying an interaction time $\tau=0.2/\lambda\approx 3.1/g$. We have obtained the numerical evolution of the full master equation of the system $\dot{\rho}=-i/\hbar[H_{\text{eff}},\rho]+\mathcal{L}_f(\rho)$, with H_{eff} given by Eq. (3) and

$$\begin{aligned}\mathcal{L}_f(\rho) &= \frac{\kappa}{2}(n_t+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \\ &+ \frac{\kappa}{2}n_t(2a^\dagger \rho a - a a^\dagger \rho - \rho a a^\dagger),\end{aligned}$$

which describes the interaction of the cavity field with a thermal reservoir. (Here we have neglected the spontaneous atomic decay since the effective levels $|g\rangle$ and $|e\rangle$ in our atomic system can be metastable ones.) At $T=0.8$ K, $n_t=0.05$ and $\kappa=2.5\times 10^{-5}$ g [33]. Considering these parameters, we have plotted (solid lines) the evolution of the mean number of photons, $\langle n\rangle=\langle a^\dagger a\rangle$ [Fig. 2(a)], and that of the variance of the cavity field quadratures $(\Delta X_l)^2=\langle X_l^2\rangle-\langle X_l\rangle^2$, $l=1,2$, $X_1=(1/2)(a+a^\dagger)$, and $X_2=-(i/2)(a-a^\dagger)$ [Fig. 2(b)], both against the number of atoms that cross the cavity. In Fig. 2(a), for $r=1$, the expected value for the mean number of photons of an ideal squeezed state, $\langle n\rangle=\sinh^2(r)=\sinh^2(1)\approx 1.38$, is almost reached asymptotically. In Fig. 2(b), the expected values for the variance in the quadratures of the cavity field $(\Delta X_1)^2=\exp(2r)/4=\exp(2)/4\approx 1.85$ and $(\Delta X_2)^2=\exp(-2r)/4=\exp(-2)/4\approx 0.034$, which corresponds to 86% of squeezing, are also almost approached asymptotically. In this case, the fidelity (computed numerically) $F=\langle\Psi|\rho|\Psi\rangle=\langle 0|S^\dagger(\xi)\rho S(\xi)|0\rangle$ of the generated state is 0.95, which is slightly different from the value 0.98 obtained disregarding the cavity decay, i.e., for $\kappa=0$. In Fig. 3 we have plotted the Wigner function $\mathcal{W}=\mathcal{W}(x,y)=(2/\pi^2)e^{2|\alpha|^2}\int\langle-\beta|\rho|\beta\rangle e^{-2(\beta\alpha^*-\beta^*\alpha)}d^2\beta$ [26], with $\alpha=x+iy$, of the cavity field state: (a) for the initial state (vacuum state) and then after the passage of (b) 50, (c) 100, and (d) 200 atoms (steady state). Considering present-day technology [9,33], the cavity coupling strength $g\approx 3\times 10^5$ Hz implies an interaction time per atom $\tau\approx 3.1/g\approx 10^{-5}$ s and a total interaction time to reach the steady state around $200\times\tau\approx 10^{-3}$ s, which is almost three orders of magnitude smaller than the current lifetime of a photon inside a cavity ($\sim 10^{-1}$ s) [10,33].

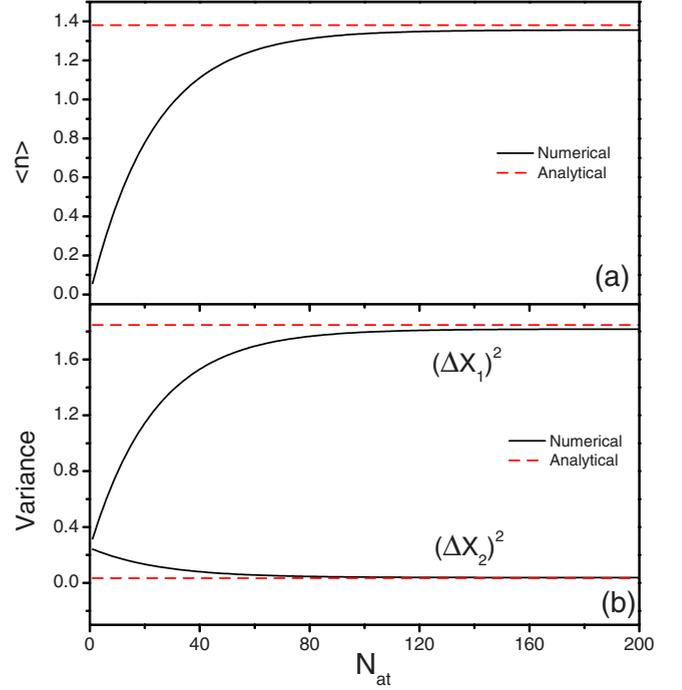


FIG. 2. (Color online) (a) Mean number of photons $\langle n\rangle=\langle a^\dagger a\rangle$ and (b) variance of the cavity field quadratures $(\Delta X_l)^2=\langle X_l^2\rangle-\langle X_l\rangle^2$, $l=1,2$, versus the number of atoms N_{at} that cross the cavity, each of them interacting during a time $\tau\approx 3.1/g$. Squeezing factor $r=1.0$. Solid line is obtained from the exact (numerical) solution of Eq. (3) for a sequence of N_{at} atoms. The dashed line represents the expected (analytical) value.

IV. SQUEEZED VACUUM RESERVOIR FOR ATOMS

Our purpose in this section is to show how to simulate an ideal squeezed vacuum for an atom or an atomic sample, trapped inside a bad cavity, whose effective atom-cavity-mode interaction is given by the same effective Hamiltonian (5) employed in the last section for the generation of a displaced squeezed cavity field state. In this case, the strong cavity decay (Γ), compared to the other system parameters ($\lambda_1, \lambda_2, \beta$), enables the atomic dynamics to be governed by an effective Liouvillian identical to the squeezed vacuum reservoir for atoms. Below we explain how this can be achieved. First, turn off the classical fields that generate rotations in the electronic states, i.e., $\Omega_3=\Omega_4=\beta=0$, and rewrite the effective Hamiltonian, Eq. (5), as

$$H_{\text{eff}}\approx\hbar(\lambda R a^\dagger + \lambda^* R^\dagger a), \quad (8)$$

with $\lambda=\lambda_2/\cosh(r)$, as defined above, and $R=\cosh(r)\sigma_- - \sinh(r)e^{i\phi}\sigma_+$. When the cavity decay is taken into account, the master equation that governs the dynamics of the system, in the interaction picture, is given by

$$\dot{\rho}=-i/\hbar[H_{\text{eff}},\rho]+\frac{\Gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \mathcal{L}_{\text{at}}\rho, \quad (9)$$

where $\mathcal{L}_{\text{at}}\rho=(\gamma/2)(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)$ stands for the usual (weak) decay of a two-level system, γ being the decay rate of the atomic system, and H_{eff} is given by Eq. (8). To obtain the engineered reservoir, we take the decay constant

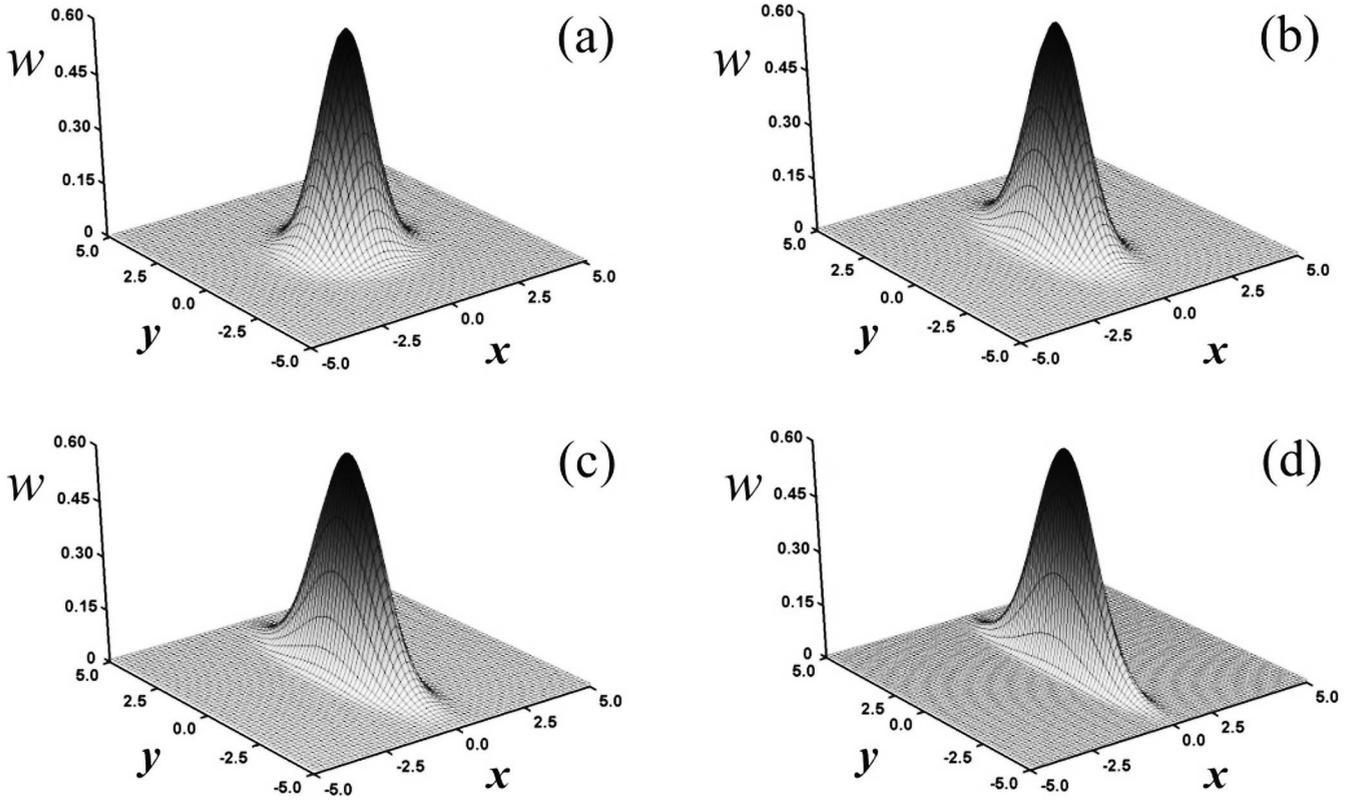


FIG. 3. Wigner function $\mathcal{W}=\mathcal{W}(x,y)$ of the cavity field state: (a) the initial (vacuum) state and after the passage of (b) 50, (c) 100, and (d) 200 atoms (steady state).

of the harmonic field to be significantly larger than the effective couplings λ_1 and λ_2 , and the decay constant γ of the two-level system. In our “cavity QED+atom” system, the regime $\Gamma \gg g$, γ is easily achieved with a cavity of low quality factor Q . Together with the good approximation of a reservoir at absolute zero, the regime $\Gamma \gg g, \gamma$ enables us to consider only the matrix elements $\rho_{mn}=\langle m|\rho|n\rangle$ inside the subspace $\{|0\rangle, |1\rangle\}$ of Fock states. The equations of motion for the elements $\rho_{mn}=\langle m|\rho|n\rangle$ thus read

$$\dot{\rho}_{00} = -i(\lambda^* R^\dagger \rho_{10} - \lambda \rho_{01} R) + \Gamma \rho_{11} + \mathcal{L}_{\text{at}} \rho_{00},$$

$$\dot{\rho}_{10} = -i(\lambda R \rho_{00} - \lambda \rho_{11} R) - \Gamma/2 \rho_{10} + \mathcal{L}_{\text{at}} \rho_{10},$$

$$\dot{\rho}_{11} = -i(\lambda R \rho_{01} - \lambda^* \rho_{10} R^\dagger) - \Gamma \rho_{11} + \mathcal{L}_{\text{at}} \rho_{11},$$

with $\dot{\rho}_{01}=(\dot{\rho}_{10})^\dagger$. Following the reasoning in Ref. [23], the strong decay rate Γ allows the adiabatic elimination of the elements ρ_{01} and ρ_{10} . Tracing over the cavity field variables, the atomic master equation reduces to

$$\begin{aligned} \dot{\rho}_{\text{at}} &= \frac{\Gamma_{\text{eng}}}{2} (2R\rho_{\text{at}}S^\dagger - R^\dagger S\rho_{\text{at}} - \rho_{\text{at}}R^\dagger S) + \mathcal{L}_{\text{at}}\rho_{\text{at}} \\ &= \frac{\Gamma_{\text{eng}}}{2} [(N+1)(2\sigma_- \rho_{\text{at}} \sigma_+ - \rho_{\text{at}} \sigma_+ \sigma_- - \sigma_+ \sigma_- \rho_{\text{at}}) \\ &\quad + N(2\sigma_+ \rho_{\text{at}} \sigma_- - \rho_{\text{at}} \sigma_- \sigma_+ - \sigma_- \sigma_+ \rho_{\text{at}}) \\ &\quad - 2M\sigma_+ \rho_{\text{at}} \sigma_+ - 2M^* \sigma_- \rho_{\text{at}} \sigma_-] + \mathcal{L}_{\text{at}}\rho_{\text{at}}, \end{aligned} \quad (10)$$

where $\Gamma_{\text{eng}}=4|\lambda|^2/\Gamma$ stands for the coupling strength of the engineered reservoir, $N=\sinh(r)^2$ and $M=e^{i\phi}\sinh(r)\cosh(r)$. The inevitable and undesired action of the multimode vacuum $\mathcal{L}_{\text{at}}\rho_{\text{at}}$ thus works against the engineered reservoir for the two-level system, leading to a nonideal squeezed vacuum reservoir for the atoms. However, in our proposal, as the transition $|g\rangle \leftrightarrow |e\rangle$ is dipole forbidden, these levels can be chosen in such a way that the decay rate γ can be very weak, so that $\Gamma_{\text{eng}} \gg \gamma$, making it possible to neglect the term $\mathcal{L}_{\text{at}}\rho_{\text{at}}$ in Eq. (10). Therefore, with the present scheme we have achieved the interaction of an ideal squeezed reservoir and a two-level atomic system. As pointed out in Ref. [28], this kind of interaction can produce some interesting effects in atomic dynamics, such as suppression (enhancement) of decay of the in-phase (out-of-phase) components of atomic polarization, and line narrowing in resonance fluorescence and absorption spectra. Hence, the present scheme could enable the observation of the effects predicted in the context of squeezed bath-atom interactions, the properties of the squeezing parameters, such as the (effective) photon-number expectation N , and the squeezing phase ϕ , being manipulated by the amplitude and phase of the pumping fields that act on the atomic system. The equations of motion for the expectation values of the operators $\sigma_x=(\sigma_-+\sigma_+)$ and $\sigma_y=-i(\sigma_- - \sigma_+)$ are

$$\langle \dot{\sigma}_x \rangle = -\frac{\Gamma_{\text{eng}}}{2} \{ [2N+2|M|\cos(\phi)+1] \langle \sigma_x \rangle + 2|M|\sin(\phi) \langle \sigma_y \rangle \}, \quad (11)$$

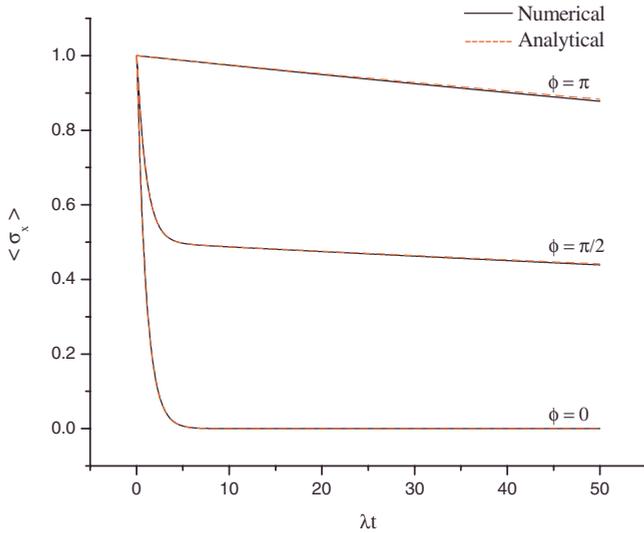


FIG. 4. (Color online) Evolution of $\langle \sigma_x(t) \rangle$ for a squeezing factor $r=1.5$, $\Gamma=40g$, $\gamma=0$, and three different values of the squeezing angle $\phi=0, \pi/2$, and π . Solid line is obtained from the exact (numerical) solution of Eq. (9) with H_{eff} given by Eq. (3). The dashed line represents the expected (analytical) value, for an ideal squeezed reservoir.

$$\langle \dot{\sigma}_y \rangle = -\frac{\Gamma_{\text{eng}}}{2} \{ [2N - 2|M|\cos(\phi) + 1] \langle \sigma_y \rangle + 2|M|\sin(\phi) \langle \sigma_x \rangle \}. \quad (12)$$

from which it can easily be shown that the atom has a phase-sensitive decay when interacting with a squeezed vacuum reservoir [26,28]. Therefore, the in-phase and out-of-phase components $\langle \sigma_x \rangle$ and $\langle \sigma_y \rangle$ of the atomic polarization decay at different rates, depending on its initial phase relative to the phase ϕ of the engineered reservoir. For an atom initially prepared in the eigenstate of the operator σ_x , i.e., $|\Psi\rangle = (1/\sqrt{2})(|g\rangle + |e\rangle)$, the mean value $\langle \sigma_x \rangle$ evolves as

$$\langle \sigma_x(t) \rangle = \frac{1}{2} \exp(-\Gamma_{\text{eng}} e^{2r} t/2) [1 + \cos(\phi)] + \frac{1}{2} \exp(-\Gamma_{\text{eng}} e^{-2r} t/2) [1 - \cos(\phi)]. \quad (13)$$

We show the validity of our approximations leading to the dynamics of the squeezed reservoir for atoms, by solving Eq. (9) numerically with H_{eff} given by Eq. (3). In Fig. 4 we have plotted the evolution of $\langle \sigma_x(t) \rangle$, obtained from the approximate solution, i.e., Eq. (13), which is based on an ideal

squeezed reservoir for atoms, and its exact (numerical) solution. We have fixed $r=1.5$, so that $|\lambda_1/\lambda_2| = \tanh(1.5) \approx 0.90$, $\lambda = \lambda_2/\cosh(1.5) \approx 0.4\lambda_2$. In the exact solution we have also assumed $|\Omega_{1,2}| \sim 10|g|$ and $|\Delta_{1,2}| \sim 100g$ so that $\lambda \approx 0.4g/10 = 0.04g$. We have assumed $\Gamma=40g$ and $\gamma=0$, and found the evolution of $\langle \sigma_x(t) \rangle$ for three different values of the squeezing angle, $\phi=0, \pi/2$, and π . As we see in Fig. 4, the evolution of $\langle \sigma_x(t) \rangle$ is phase dependent, as expected for an ideal squeezed reservoir for atoms.

V. CONCLUDING REMARKS

We have presented a theoretical study of the manipulation of the atom-field interaction and its use in reservoir engineering. To build the desired effective Hamiltonian we considered the interaction between an atomic system in a Λ configuration, driven by classical fields, and a single cavity mode. With the engineered interaction, composed of interactions such as Jaynes-Cummings, anti-Jaynes-Cummings, and a rotation in the electronic states, we first showed how to generate a decoherence-free displaced squeezed state for the cavity field based on an atomic reservoir. In our scheme an atomic beam works as a reservoir for the radiation field trapped inside the cavity, as recently employed in Ref. [25] to generate an Einstein-Podolsky-Rosen entangled radiation state in high- Q resonators. Our scheme, as in Ref. [25], is robust against stochastic fluctuations in the atomic beam and does not require precise interaction times (velocity selection) or atomic detection. Using this system, we believe that a displaced squeezed cavity field state could be experimentally generated with present-day technology. In addition, with small changes, we were also able to generate an ideal squeezed reservoir for two-level atomic systems [26,28]. For this purpose, we had to assume, besides the engineered atom-field interaction, that the decay of the cavity field was much stronger than the effective atom-field couplings. With this proposal some interesting effects in the dynamics of an atom or an atomic sample in a squeezed reservoir can be experimentally investigated. All the approximate theoretical results presented in this work were checked by numerical analysis and all of them showed excellent agreement with the exact (numerical) solutions.

ACKNOWLEDGMENTS

We wish to acknowledge the support of the Brazilian agencies CNPq, CAPES, FAPESP (Process No. 2005/04105-5), and Brazilian Millennium Institute for Quantum Information. R.G. wishes to acknowledge the support of FOND-ECYT (Project No. 11060477).

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000); M. Fleischhauer and W. P. Schleich, Phys. Rev. A **47**, 4258 (1993).
 [2] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987).

[3] D. M. Meekhof, C. Monroe, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **76**, 1796 (1996).
 [4] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996).
 [5] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1965); M.

- A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, *Nature (London)* **409**, 791 (2001); B. B. Blinov, D. L. Moehring, L. M. Duan, and C. Monroe, *ibid.* **428**, 153 (2004).
- [6] J. I. Cirac and P. Zoller, *Phys. Rev. Lett.* **74**, 4091 (1995); *Nature (London)* **404**, 579 (2000); D. Kielpinski, C. Monroe, and D. J. Wineland, *ibid.* **417**, 709 (2002); Stephan Gulde, Mark Riebe, Gavin P. T. Lancaster, Christoph Becher, Jürgen Eschner, Hartmut Häffner, Ferdinand Schmidt-Kaler, Isaac L. Chuang, and Rainer Blatt, *ibid.* **421**, 48 (2003).
- [7] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche, *Phys. Rev. Lett.* **83**, 5166 (1999).
- [8] M. Riebe, H. Häffner, C. F. Roos, W. Hansel, J. Benhelm, G. P. T. Lancaster, T. W. Korber, C. Becher, F. Schmidt-Kaler, D. F. V. James, and R. Blatt, *Nature (London)* **429**, 734 (2004); M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri, and D. J. Wineland, *ibid.* **429**, 737 (2004).
- [9] J. M. Raimond, M. Brune, and S. Haroche, *Rev. Mod. Phys.* **73**, 565 (2001).
- [10] B. T. H. Varcoe, S. Brattke, M. Weidinger, and H. Walther, *Nature (London)* **403**, 743 (2000).
- [11] For a review of squeezed states, see V. V. Dodonov, *J. Opt. B: Quantum Semiclassical Opt.* **4**, R1 (2002).
- [12] M. D. Reid, *Phys. Rev. A* **40**, 913 (1989); A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
- [13] D. Stoler, *Phys. Rev. Lett.* **33**, 1397 (1974).
- [14] J. N. Hollenhorst, *Phys. Rev. D* **19**, 1669 (1979); Carlton M. Caves, Kip S. Thorne, Ronald W. P. Dreve, Vernon D. Sandberg, and Mark Zimmermann, *Rev. Mod. Phys.* **52**, 341 (1980).
- [15] H. P. Yuen and J. H. Shapiro, *IEEE Trans. Inf. Theory* **24**, 657 (1978); D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, *Phys. Rev. A* **50**, 67 (1994).
- [16] C. W. Gardiner, *Phys. Rev. Lett.* **56**, 1917 (1986).
- [17] C. J. Villas-Bôas, N. G. de Almeida, R. M. Serra, and M. H. Y. Moussa, *Phys. Rev. A* **68**, 061801(R) (2003); R. M. Serra, C. J. Villas-Bôas, N. G. de Almeida, and M. H. Y. Moussa, *ibid.* **71**, 045802 (2005); C. J. Villas-Boas and M. H. Y. Moussa, *Eur. Phys. J. D* **32**, 147 (2005); R. Guzmán, J. C. Retamal, E. Solano, and N. Zagury, *Phys. Rev. Lett.* **96**, 010502 (2006).
- [18] F. O. Prado, N. G. de Almeida, M. H. Y. Moussa, and C. J. Villas-Boas, *Phys. Rev. A* **73**, 043803 (2006).
- [19] C. K. Law and J. H. Eberly, *Phys. Rev. Lett.* **76**, 1055 (1996).
- [20] M. F. Santos, *Phys. Rev. Lett.* **95**, 010504 (2005).
- [21] J. I. Cirac, A. S. Parkins, R. Blatt, and P. Zoller, *Phys. Rev. Lett.* **70**, 556 (1993).
- [22] J. F. Poyatos, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **77**, 4728 (1996).
- [23] A. R. R. Carvalho, P. Milman, R. L. de Matos Filho, and L. Davidovich, *Phys. Rev. Lett.* **86**, 4988 (2001).
- [24] A. Z. Khoury and T. B. L. Kist, *Phys. Rev. A* **55**, 2304 (1997).
- [25] R. R. Puri, C. K. Law, and J. H. Eberly, *Phys. Rev. A* **50**, 4212 (1994); S. Pielawa, G. Morigi, D. Vitali, and L. Davidovich, *Phys. Rev. Lett.* **98**, 240401 (2007); P. Li, *Phys. Rev. A* **77**, 015809 (2008).
- [26] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, UK, 1997).
- [27] A. S. Parkins, P. Zoller, and H. J. Carmichael, *Phys. Rev. A* **48**, 758 (1993).
- [28] N. Lutkenhaus, J. I. Cirac, and P. Zoller, *Phys. Rev. A* **57**, 548 (1998).
- [29] N. P. Georgiades, E. S. Polzik, K. Edamatsu, H. J. Kimble, and A. S. Parkins, *Phys. Rev. Lett.* **75**, 3426 (1995).
- [30] D. F. V. James, *Fortschr. Phys.* **48**, 823 (2002); A. B. Klimov, L. L. Sánchez-Soto, A. Navarro, and E. C. Yustas, *J. Mod. Opt.* **49**, 2211 (2002); A. B. Klimov, J. L. Romero, J. Delgado, and L. L. Sánchez-Soto, *J. Opt. B: Quantum Semiclassical Opt.* **5**, 34 (2003).
- [31] B. G. Englert and G. Morigi, *Coherent Evolution in Noisy Environments* (Springer, Berlin, 2002).
- [32] Shi-Biao Zheng and Guang-Can Guo, *Phys. Rev. Lett.* **85**, 2392 (2000); S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, *ibid.* **87**, 037902 (2001).
- [33] S. Gleyzes, S. Kuhr, C. Guerlin, J. Bernu, S. Deleglise, U. B. Hoff, M. Brune, J. M. Raimond, and S. Haroche, *Nature (London)* **446**, 297 (2007); C. Guerlin, J. Bernu, S. Deleglise, C. Sayrin, S. Gleyzes, S. Kuhr, M. Brune, J. M. Raimond, and S. Haroche, *ibid.* **448**, 889 (2007).