JOURNAL OF MODERN OPTICS, 15 APRIL–10 MAY 2004
VOL. 51, NO. 6–7, 1091–1092

Taylor & Francis
Taylor & Francis Group

Short communication
The harmonic oscillator interacting with a heat bath

L. M. ARÉVALO AGUILAR†, N. G. DE ALMEIDA‡ and C. J. VILLAS-BOAS

†Loma del Bosque No 115, CP 37150. León, Gto. México
‡Departamento de Matemáticas e Física, Universidade Católica de Goiás, Av. Universitária 1440, St. Universitário, CP 86, CEP 74605-010, Goiâna, Brazil
§Departamento de Física, Universidade Federal de São Carlos, Via Washington Luis, Km 235, São Carlos 13565-905, SP, Brazil

(*Received 30 October 2003*)

Abstract. We show that the harmonic oscillator linearly coupled with a heat bath has a serious defect, even in the case of conserving the rotating terms in the interaction Hamiltonian, i.e. without the rotating wave approximation.

1. Introduction

The most used model in quantum optics employed to investigate the transition from the quantum world to the classical regime is the harmonic oscillator linearly coupled with a reservoir, the latter being modelled as an infinity number of the harmonic oscillator [1]. From this model was deduced a master equation which produced the evolution of the single harmonic oscillator [1]. Traditionally this master equation is obtained by tracing over the reservoir, i.e. it focuses on the single harmonic oscillator. The solution to this master equation shows that a coherent state evolves towards a thermal state [2, 3]. A few years ago, Ford and O'Connell showed that the Hamiltonian used to model this system, and used to obtain the master equation, has a serious defect in that it lacks a lower bound energy [4]. The rotating wave approximation is made in this Hamiltonian. However, in this contribution, we show that even with the counter-rotating terms this Hamiltonian has the same defect. For a complete treatment of this problem see [5].

2. The Hamiltonian without the rotating wave approximation

In the absence of the rotating wave approximation (RWA), the Hamiltonian describing the harmonic oscillator and reservoir systems reads as [1]

$$\hat{H} = \hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \sum_j \hbar\omega_j\left(\hat{b}_j^\dagger\hat{b}_j + \frac{1}{2}\right) + \hbar\sum_j\left(\lambda_j\hat{b}_j + \lambda_j^*\hat{b}_j^\dagger\right)\left(\hat{a} + \hat{a}^\dagger\right), \qquad (1)$$

Journal of Modern Optics ISSN 0950–0340 print/ISSN 1362–3044 online © 2004 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/09500340410001664025

where $\hat{a}^{\dagger}$ and $\hat{a}$ are the creation and annihilation operators for the field mode, respectively, obeying $[\hat{a}, \hat{a}^{\dagger}] = 1$, and $\hat{b}_j$ and $\hat{b}_j^{\dagger}$ are the creation and annihilation operators of the jth oscillator of the reservoir having frequency ω_j and obeying the commutation relation $[\hat{b}_j, \hat{b}_k^{\dagger}] = \delta_{jk}$, ω_0 is the frequency of the single mode and λ_j is the coupling constant field reservoir. In order to survey the lower bound energy in the Hamiltonian equation (1), we assume an initial normalized state consisting of a product of coherent sates of the form

$$|\Psi\rangle = |\alpha\rangle \prod_j |\beta_j\rangle. \tag{2}$$

Taking the expectation value of the Hamiltonian $\langle\Psi|\hat{H}|\Psi\rangle$ in equation (6), and if we take β_j as $\beta_j = -\lambda_j^*(\alpha + \alpha^*)/\omega_j$ (see [5]), the minimun of the expectation value of the Hamiltonian is given by

$$\langle\Psi|\hat{H}|\Psi\rangle\Big|_{\min} = \left(\hbar\omega_0 - 2\hbar\sum_j \frac{|\lambda_j|^2}{\omega_j}\right)|\alpha|^2 - \hbar\sum_j(\alpha^2 + \alpha^{*2})\frac{|\lambda_j|^2}{\omega_j} + \sum_j \hbar\omega_j. \tag{3}$$

In conclusion, from equation (3) we can deduce that the Hamiltonian equation (1) does not have a lower bound [5].

Acknowledgments

L. M. Arévalo Aguilar would like to acknowledge support from Consejo de Ciencia y Tecnología del Estado de Guanajuato (CONCYTEG) and CONACYT. N. G. de Almeida and C. J. Villas-Bôas thank CNPQ and FAPESP, Brazilian agencies.

References

[1] Louisell, W. H., 1973, *Quantum Statistical Properties of Radiation* (New York: Wiley), particularly see equations 6.1.8 and 7.1.1; Carmichael, H., 1993, *An Open Systems Approach to Quantum Optics, Lectures Notes in Physics* (Berlin: Springer-Verlag); Walls, D. F., and Milburn, G. J., 1994, *Quantum Optics* (Berlin: Springer-Verlag).

[2] Arévalo-Aguilar L. M., and Moya-Cessa, H., 1998, *Quant. Semiclass. Optics*, **10**, 671.

[3] Lu, H.-X., Yang, J., Zhang, Y.-D., and Chen, Z.-B., 2003, *Phys. Rev.* A, **67**, 024101.

[4] Ford, G. W., and O'Connell, R. F., 1997, *Physica* A, **243**, 377.

[5] Arévalo Aguilar, L. M., De Almeida, N. G., and Villas-Boas, C. J., 2003, *Phys. Lett.* A, **313**, 257.