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Generation of circular states and Fock states in a trapped ion

Wagner Duarte José and Salomon S Mizrahi

Departamento de Física, CCET, Universidade Federal de São Carlos, PO Box 676, São Carlos, 13565-905, SP, Brazil

E-mail: pwdj@iris.ufscar.br and salomon@power.ufscar.br

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Abstract. We propose three schemes to engineer a *circular state* (superposition of the harmonic oscillator coherent states on a circle in phase space) for the centre-of-mass motion of a trapped ion. We analyse the necessary duration of each laser pulse for constructing such states, and calculate the probability of obtaining the subtle superposition. We also show that it is possible to engineer Fock number states as a result of the interference effects in phase space of the coherent states superposition.

Keywords: Trapped ions, ionfluorescence, circular states, Fock states, quantum-states engineering

1. Introduction

One of the most exciting problems of quantum mechanics consists in generating experimentally states that do not exist in the natural world. Recent advances in quantum optics have allowed the realization of this challenge with much success. Many non-classical states have been proposed [1] and produced: for example, squeezed states, Schrödinger cat states and Fock states, in QED superconducting cavities [2] and in trapped ions [3–6]. From the theoretical point of view, Fock states can also be obtained by quantum nondemolition measurements [5, 7] and more subtly by a superposition of coherent states evenly distributed on a circle in the phase space [8–10], called *circular states*. Experimental schemes have been proposed to generate such superpositions as states of the electromagnetic (EM) field in superconducting cavities [11,12] and also as states of the harmonic oscillation of the centre of mass (CM) of a trapped ion [13]; however, the problem of producing *circular states* has not been treated more thoroughly. Unlike experiments with superconducting cavities, when decoherence of the states is an important effect to be considered, trapped ions are weakly coupled to the environment, so decoherence is less manifest, and they are therefore the best candidates to engineer states or to construct devices as logic gates for implementing quantum computation [14]; however, decoherence times must be of the order of 1 ms [15].

Regarding quantum states engineering, in [16] the authors propose a model with which an arbitrary field state in a cavity may be generated by manipulating an atom (source) inside the cavity during the atom–field interaction process, and thus all target states (superpositions of Fock number state) can be created from the same initial state. Regarding states engineering, in [7] the authors propose a quantum nondemolition measurement of the motional energy of a trapped atom confined in a harmonic potential: by monitoring the interaction times of the lasers interacting with electronic states of the atom, any initial vibrational state of the CM collapses into a Fock number state. Unlike the method proposed in this paper, the Fock state into which the atom collapses after a complete measurement sequence is not predictable. In [17] the authors present a method similar to the one reported in [16], where they generalize the method for trapped ions, producing the most general form of entangled quantum states between electronic and vibrational degrees of freedom in order to engineer a two-dimensional vibrational target state out from an initial vacuum one; they also analyse the sensitivity of the method due to errors in the amplitude and phase of the lasers. See also [18] for a universal algorithm for the synthesis of an arbitrary two-mode bosonic state.

Recently, in [19] the authors proposed a procedure to create as target state an arbitrary superposition of coherent states for the CM motion of a trapped ion, the coherent states being localized on a straight *line* in phase space. In this paper we propose and analyse three methods to generate *circular states* as target states for the CM of a trapped ion; we also estimate the total time of laser pulses and the probability to produce such a state. We also show that it is possible to engineer Fock number states due to the interference effects between the N coherent states superposition. We suppose that its production could be realized with the available technological facilities. Our treatment is different from the proposal of [13], for our method involves a reduced number of on–off operations and adjusting detunings of the laser beams, thus reducing the deleterious effects of the decoherence, in the same spirit of [19,20]. The paper is organized as follows. In section 2 we review properties of the circular states and analyse under what conditions such states reduce to Fock

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number states. In section 3 we review the dynamics of the ion–laser interaction and discuss some mechanisms that put the CM ion motion in a non-classical state; in section 4 we describe our schemes to engineer the circular states and also the Fock number states. Finally, we present a summary and comments in section 5.

2. Circular states and generation of Fock states

Circular states were proposed and studied by Janszky and co-workers [8, 10, 11] as a generalization of the Schrödinger cat state (superposition of two coherent states, $|\alpha\rangle$ and $|-\alpha\rangle$, on a circle of radius $r = |\alpha| \gg 1$ in phase space), being a superposition of N coherent states equally likely distributed on a circle of radius r. The same authors also showed that a particular superposition generates an arbitrary Fock number state when $r \ll 1$, and that such a state of the EM field can be obtained through a nonlinear interaction of the field with a Kerr medium [10]. Experimental realizations of superposition of coherent states with arbitrary coefficients have been proposed for EM cavities [11, 12] and in trapped ions [13, 20].

In this section we show under what conditions (specific relations between r and N) the circular states can reproduce a particular Fock number state. One kind of circular state was discussed in [21], where the analogy between its phase space properties and the n-slit interference pattern was pointed out; later this state was studied in [9], with emphasis on its non-classical properties, and the conditions for reproducing a Fock number state were found to be $1 \ll (r^2\mathrm{e}/N)^N \ll 4^N$. Another kind of circular state was introduced in [10], which becomes a Fock number state when the condition $(r^2\mathrm{e}/N)^N \ll 1$ is fulfilled.

2.1. The circular states

The circular states generalize the Schrödinger cat states: they constitute a superposition of the harmonic oscillator coherent states,

$$|\Psi_N(\alpha_0)\rangle = \lambda_N \sum_{k=1}^{N} C_k|\alpha_k\rangle \qquad \text{with} \quad \alpha_k = \alpha_0 \mathrm{e}^{\mathrm{i}\frac{2\pi k}{N}}, \quad (1)$$

where

$$|\alpha_k\rangle = \mathrm{e}^{-r^2/2} \sum_{n=0}^{\infty} \frac{\alpha_k^n}{\sqrt{n!}}|n\rangle$$

with the same modulus $|\alpha_k| = |\alpha_0| = r$ and λ^N is the normalization constant.

For N even and $C_{k+N/2} = \pm C_k$ the parity of the state may be even or odd, i.e. $|\Psi_N(-\alpha_0)\rangle = \pm|\Psi_N(\alpha_0)\rangle$; the circular state is also an eigenstate of the Nth power of the annihilation operator, $\hat{a}^N|\Psi_N(\alpha_0)\rangle = \alpha_0^N|\Psi_N(\alpha_0)\rangle$ [12]. In [9], the particular case $C_k = 1$,

$$|\Psi_N\rangle = \lambda_N \sum_{k=1}^{N} |\alpha_k\rangle, \quad (2)$$

$$\lambda_N = \left[N + 2\sum_{k=1}^{N-1} k\mathrm{e}^{-2r^2\sin^2(\frac{\pi}{N}k)} \cos\left[r^2 \sin\left(\frac{2\pi}{N}k\right)\right]\right]^{-1/2}, \quad (3)$$

was considered: the authors verified that the interference between the coherent states approximately produces a Fock number state when the condition $1 \ll (\mathrm{e}r^2/N)^N \ll 4^N$ is satisfied, while for $(\mathrm{e}r^2/N)^N \ll 1$ an almost vacuum state is produced. This happens because, due to interference effects, the probability for a Fock number state $|n\rangle$ to be singled out from the superposition (2) is

$$P_n \equiv |\langle n|\Psi_N\rangle|^2 = \lambda_N^2 \frac{r^{2n}}{n!}\delta_{n,Nk} \qquad k = 0, 1, 2, \ldots, \qquad N = 2, 3, 4, \ldots. \quad (4)$$

For instance, for $N = 4$, P_n is non-zero only for $n = 0, 4, 8, \ldots$.

We plotted P_N against r and N for $k = 1$ (figure 1(*a*) and $k = 0$ (figure 1(*b*)); in figure 1(*a*) we see a *plateau* with $P_N \approx 1$ and a *plain* with $P_N \approx 0$, this same region assuming value $P_0 \approx 1$ in figure 1(*b*). Therefore, from figure 1(*a*) we see that the superposition (2) becomes practically a Fock number state $|\tilde{N}\rangle$ in the *plateau* region, while in the *plain* region we get close to a vacuum state $|\tilde{0}\rangle$.

The Wigner function of the state (1) is given by

$$W_{cs}(x, y, r) = 2\lambda_N^2 \mathrm{e}^{-r^2-2(x^2+y^2)} \sum_{k,l=1}^{N} |C_k||C_l| \times \exp[U_{k,l}(x, y, r)] \cos[V_{k,l}(x, y, r)] \quad (5)$$

where $C_k = |C_k| \exp(\mathrm{i}\phi_k)$ and

$$\begin{aligned} U_{k,l}(x, y, r) &= -r^2 \cos\left(\frac{2\pi}{N}(k-l)\right) \\ &\quad +2xr\left[\cos\left(\frac{2\pi k}{N} + \theta_0\right) + \cos\left(\frac{2\pi l}{N} + \theta_0\right)\right] \\ &\quad +2yr\left[\sin\left(\frac{2\pi k}{N} + \theta_0\right) + \sin\left(\frac{2\pi l}{N} + \theta_0\right)\right], \\ V_{k,l}(x, y, r) &= \phi_k - \phi_l \\ &\quad -2yr\left[\cos\left(\frac{2\pi k}{N} + \theta_0\right) - \cos\left(\frac{2\pi l}{N} + \theta_0\right)\right] \\ &\quad -r^2 \sin\left(\frac{2\pi}{N}(k-l)\right) \\ &\quad +2xr\left[\sin\left(\frac{2\pi k}{N} + \theta_0\right) - \sin\left(\frac{2\pi l}{N} + \theta_0\right)\right], \end{aligned} \quad (6)$$

with properties $U_{k,l} = U_{l,k}$ and $V_{k,l} = -V_{l,k}$. The even circular state, $C_k = 1$, is plotted in figure 2(*a*) for $N = 16$, $\theta_0 = 0$ and $r = 4$, and can be compared with the Wigner function of the Fock number state $|N\rangle$, given by

$$W_N(x, y, N) = 2(-1)^N L_N((x^2 + y^2))\mathrm{e}^{-2(x^2+y^2)}, \quad (7)$$

plotted in figure 2(*b*): one sees that the superposition (2) reproduces satisfactorily only the central peak since the Wigner function is very sensible to interference effects.

Another kind of circular state is defined by the coefficients $C_k = \mathrm{e}^{\mathrm{i}2\pi k/N}$,

$$|\tilde{\Psi}_N\rangle = \lambda_N \sum_{k=1}^{N} \mathrm{e}^{\frac{2\pi \mathrm{i}k}{N}} |\alpha_k\rangle, \quad (8)$$

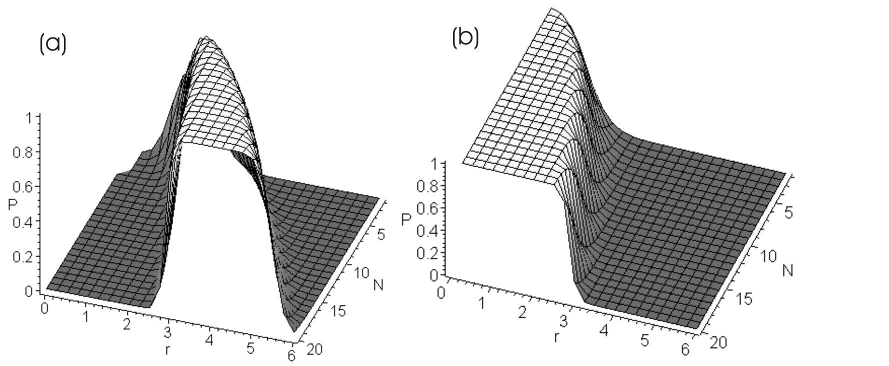

Figure 1. Number states distribution P_N (equation (4)) as a function of r and N for (a) $k = 1$, (b) $k = 0$.

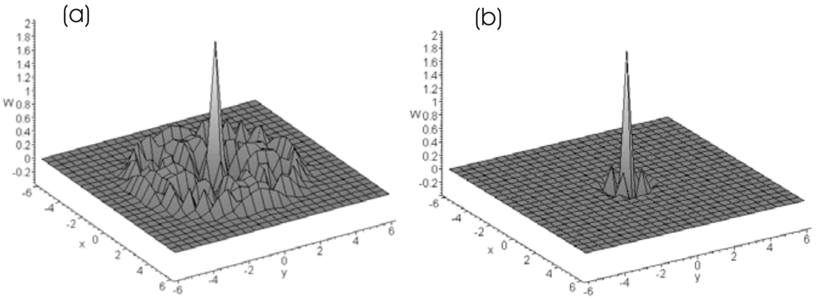

Figure 2. Wigner function $W(x, y)$ for the parameters $N = 16$, $\theta_0 = 0$ and $r = 4$ for (a) even circular state, (b) Fock number state.

with normalization factor

$$\lambda_N = \left[N + 2 \sum_{k=1}^{N-1} k \mathrm{e}^{-2r^2 \sin^2(\frac{\pi}{N}k)} \times \cos\left[\frac{2\pi k}{N} + r^2 \sin\left(\frac{2\pi}{N} k \right) \right] \right]^{-1/2} \tag{9}$$

being an odd state when N is even. In the Fock number state basis it can be written as

$$|\tilde{\Psi}_N\rangle = \lambda_N \sum_{k=1}^{\infty} \frac{r^{(kN-1)}}{\sqrt{(kN-1)!}} |kN-1\rangle, \tag{10}$$

and λ_N can still be defined as

$$\lambda_N^{-2} = \sum_{k=1}^{\infty} \frac{r^{2(kN-1)}}{(kN-1)!}$$

which was identified as the *partition function* associated to the state $|\tilde{\Psi}_N\rangle$ [9], since it is the fundamental quantity that gives the statistical properties of the quanta. The probability for each Fock state $|n\rangle$ to be present in the superposition is

$$P_n \equiv |\langle n|\tilde{\Psi}_N\rangle|^2 = \lambda_N^2 \frac{r^{2(kN-1)}}{(kN-1)!} \delta_{n,kN-1}, \tag{11}$$

so the only Fock states present in (10) are $N-1$, $2N-1$, $3N-1, \ldots, kN-1 (k = 1, 2, \ldots; N = 2, 3, \ldots,)$. Thus, the superposition (10) can be written as

$$|\tilde{\Psi}_N\rangle = \frac{|N-1\rangle + r^N (\frac{(N-1)!}{(2N-1)!})^{1/2} |2N-1\rangle + \cdots}{\sqrt{1 + r^{2N} \frac{(N-1)!}{(2N-1)!} + \cdots}}, \tag{12}$$

reproducing very closely the Fock number state $|N-1\rangle$ under the condition $(r^2\mathrm{e}/4N)^N \ll 1$. In fact, this is shown in figure 3 where we plotted P_N, using the same parameters as in figure 1(a). The Wigner function is calculated from (5): in figure 4(a) we show its plot for $N = 16$, $\theta_0 = 0$ and $r = 4$. As expected, it reproduces closely the central peak of the Wigner function for the Fock number state $|15\rangle$; the small wiggles occurring in figures 2(a) and 4(a) happen because the Wigner function is very sensible to the phases of the c-numbers α_k. This sensibility is credited to the nondiagonal terms of the Wigner function which are responsible for all the characteristics of interference present in the Wigner function [22].

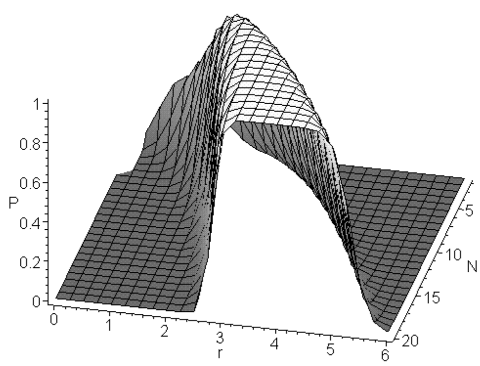

Figure 3. Number states distribution P_{N-1} (equation (11)) as function of r and N for $k = 1$.

In summary, whereas the superposition (2) leads to a state close to the Fock number state $|N\rangle$ under the conditions $1 \ll (r^2\mathrm{e}/N)^N \ll 4^N$, the superposition (8) also approximately gives the number state $|N-1\rangle$ under the condition $(r^2\mathrm{e}/4N)^N \ll 1$.

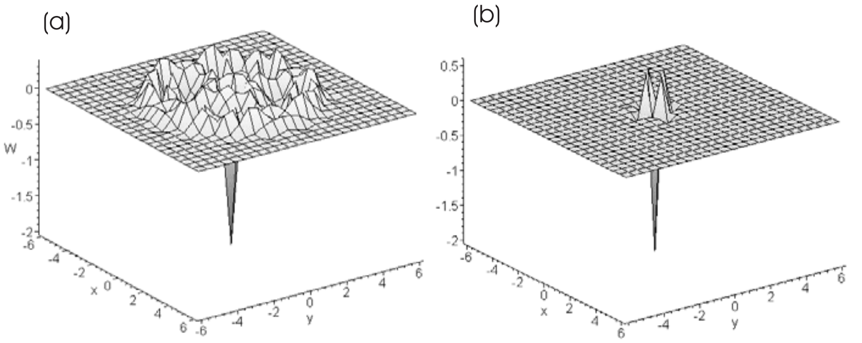

Figure 4. Wigner function $W(x, y)$ for the parameters $N = 16$, $\theta_0 = 0$ and $r = 4$ for (*a*) odd circular state, (*b*) Fock number state.

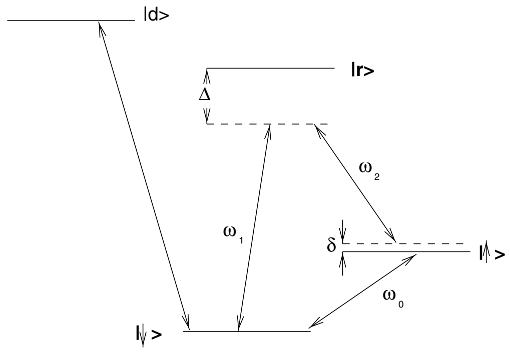

Figure 5. Energy levels of a trapped ion interacting with the laser beams of frequency ω_1 and ω_2: ω_0 is transition frequency between the ion forbidden electronic levels $|\uparrow\rangle$ (excited) and $|\downarrow\rangle$ (ground), $\delta = \omega_1 - \omega_2 - \omega_0$ ($\delta \ll \Delta$), $|r\rangle$ (adiabatically eliminated) and $|d\rangle$ (measurement) are auxiliary electronic levels.

3. Ion–laser interaction

In this section we briefly review the dynamical processes that use trapped ions and laser beams (conveniently tuned) for generating specific states of the ion CM vibrational motion. One single trapped ion is considered moving in a one-dimensional harmonic effective pseudopotential [3], interacting with two laser beams (with frequencies ω_1 and ω_2) in a Raman-type configuration, which is responsible for a forbidden transition between two metastable internal electronic states, $|\uparrow\rangle$ and $|\downarrow\rangle$, separated by frequency ω_0 and called excited and ground, respectively. In this configuration, the two levels are indirectly coupled via a third one, $|r\rangle$, which is adiabatically eliminated when one takes the detuning Δ (see figure 5) much larger than the following three quantities: the linewidth of the level $|r\rangle$, the coupling between $|\uparrow\rangle$ and $|r\rangle$, and $|\downarrow\rangle$ and $|r\rangle$, and the frequency defined as $\delta \equiv \omega_l - \omega_0$ ($\omega_l = \omega_1 - \omega_2$) [5, 23]. The level $|d\rangle$ is used to cool the ion and to measure its internal state by fluorescence emission [3, 4, 14, 23]; moreover, it can still be used to generate the ion motional states [4, 14].

The Hamiltonian describing the effective interaction between the two-level electronic states and the quantized motion of the ion CM is written as [4,5] (we consider $\hbar = 1$)

$$H = \omega a^\dagger a + \frac{\delta}{2}\sigma_z - \Omega\,(\sigma_- \mathrm{e}^{-\mathrm{i}\eta(a+a^\dagger)+\mathrm{i}\phi} + \sigma_+ \mathrm{e}^{\mathrm{i}\eta(a+a^\dagger)-\mathrm{i}\phi}), \quad (13)$$

where $\sigma_+(\sigma_-) = |\uparrow\rangle\langle\downarrow|$ ($|\downarrow\rangle\langle\uparrow|$) and σ_z are the usual Pauli pseudo-spin operators, $a^\dagger(a)$ is the creation (annihilation) operator of vibrational quanta, Ω is the effective Rabi frequency of the transition $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$ and η is the Lamb–Dicke parameter defined as [4, 23]

$$\eta = \frac{\Delta k}{\sqrt{2m\omega}}, \quad (14)$$

where $\Delta k = (\vec{k}_1 - \vec{k}_2)\cdot\hat{x}$ and $|\vec{k}_{1(2)}| = \omega_{1(2)}/c$, being $\vec{k}_1(\vec{k}_2)$ the wave vector of laser 1(2), $\hat{x}$ is the operator referring to the CM position of the ion, m is its mass and ω is the angular frequency of its harmonic oscillation in the trap. Thus, the strength of η can be arbitrarily selected by changing the relative direction of the laser beams.

Writing first H in the interaction picture and then expanding the resulting Hamiltonian in terms of the Lamb–Dicke parameter we get

$$H_I = -\hbar\Omega \mathrm{e}^{-\eta^2/2} \times \left[\sum_{m,l=0}^{\infty} \frac{(\mathrm{i}\eta)^{m+l}}{m!\,l!} a^{\dagger m} a^l \mathrm{e}^{\mathrm{i}[(m-l)\omega+\delta]t+\mathrm{i}\phi}\sigma_- + \mathrm{h.c.}\right]. \quad (15)$$

Since the vibrational frequency ω is large in comparison with Ω, resonance conditions result for $\delta = -k\omega$ ($k = m - l$). The choice $k > 0$ selects the kth red frequency and $k < 0$ the kth blue frequency of the resolved vibrational sideband of the Raman spectrum. So, the engineering of a particular quantum state becomes possible by choosing a particular term from the summation in equation (15) [7, 24, 25]. Considering the Lamb–Dicke limit, where $\eta^2 \ll 1$, some specific Hamiltonians have been proposed and investigated, namely the Jaynes–Cummings Hamiltonian ($k = 1$), the anti-Jaynes–Cummings Hamiltonian ($k = -1$) and the carrier Hamiltonian ($k = 0$), which couple, respectively, the levels $|m+1\downarrow\rangle \longleftrightarrow |m\uparrow\rangle$, $|m\downarrow\rangle \longleftrightarrow |m+1\uparrow\rangle$ and $|m\downarrow\rangle \longleftrightarrow |m\uparrow\rangle$.

Fock states of ion vibrational mode can be created by a sequence of laser pulses δ-tuned in some resolved sideband frequency. Since the ion is initially cooled in the ground state $|0\downarrow\rangle$, we can get the ion in the upper electronic level with three phonons in the vibrational mode $|3\downarrow\rangle$ applying four laser pulses of duration $t = \pi/(2\Omega)$ each, tuned in the blue, red, blue-sideband and, finally, carrier frequency [4]. On the other hand, vibrational coherent states can be easily created by either displacing the trap or by applying a spatially uniform classical driving field; laser pulses and driving fields must be combined to generate a cat state [4].

The measurement of the ion vibrational state is achieved indirectly through the measurement of the ion electronic state, which is realized by collecting the emitted resonance fluorescence signal from the transition $|d\rangle \leftrightarrow |\downarrow\rangle$ (see figure 5) through another laser strongly coupled to the electronic ground state [4]. The measured signal is the probability of finding the ion in the internal state $|\downarrow\rangle$. Since the fluorescence emission disturbs the ion CM motion, for each new measurement the ion should be cooled back to its ground state. In section 4 we comment on the duration of the measurement.

4. Generation of circular states in a trapped ion

Here we propose and analyse three different schemes to generate circular states of the kind of equations (2) and (8). For this purpose we consider a Kerr-type interaction between the ion and the effective laser which is realized by tuning it resonantly ($\delta = 0$) with the electronic transition frequency ω_0. In the Lamb–Dicke limit, the carrier Hamiltonian becomes

$$\begin{aligned} H &= \Omega\sigma_x - \eta^2\Omega \\ &\quad\times\left[\left(1+\frac{\eta^2}{4}\right)a^\dagger a - \frac{\eta^2}{4}\left(a^\dagger a\right)^2 + \mathrm{O}(\eta^4)\right]\sigma_x, \\ &\approx \Omega\sigma_x - \eta^2\Omega a^\dagger a\sigma_x, \end{aligned} \tag{16}$$

describing the dynamics of the effective interaction (we choose $\phi = \pi$ without loss of generality). The approximation in (16) is valid under the condition

$$\frac{\eta^2}{4}(Q + \bar{n} + 1) \ll 1, \tag{17}$$

where Q is the Mandel parameter, $Q \equiv (\mathrm{Var}(\hat{n}) - \bar{n})/\bar{n}$, $\bar{n}$ is the mean number of phonons and $\mathrm{Var}(\hat{n})$ is the variance. For instance, if the ion is prepared initially to be in a coherent state $|\alpha_0\rangle$, then (17) becomes $(|\alpha_0|^2 + 1)\eta^2/4 \ll 1$, which must be satisfied in order to consider the final approximation in (16). Thus, assuming condition (17) (which allows the use of (16)), the evolution operator is

$$U(t) \equiv \mathrm{e}^{-\mathrm{i}Ht} = \mathrm{e}^{-\mathrm{i}\Omega t\sigma_x}\mathrm{e}^{\mathrm{i}\bar{\Omega}t\sigma_x a^\dagger a}, \tag{18}$$

where $\bar{\Omega} \equiv \eta^2\Omega$. The eigenvalues and eigenstates of σ_x are $\sigma_x|\uparrow_x\rangle = |\uparrow_x\rangle$, $\sigma_x|\downarrow_x\rangle = -|\downarrow_x\rangle$; therefore it follows that

$$\mathrm{e}^{-\mathrm{i}\Omega t\sigma_x}|\uparrow_x\rangle = \mathrm{e}^{-\mathrm{i}\Omega t}|\uparrow_x\rangle,$$

$$\mathrm{e}^{\mathrm{i}\bar{\Omega}t\sigma_x a^\dagger a}|\uparrow_x\rangle = \mathrm{e}^{\mathrm{i}\bar{\Omega}t a^\dagger a}|\uparrow_x\rangle,$$

$$\mathrm{e}^{-\mathrm{i}\Omega t\sigma_x}|\downarrow_x\rangle = \mathrm{e}^{\mathrm{i}\Omega t}|\downarrow_x\rangle,$$

$$\mathrm{e}^{\mathrm{i}\bar{\Omega}t\sigma_x a^\dagger a}|\downarrow_x\rangle = \mathrm{e}^{-\mathrm{i}\bar{\Omega}t a^\dagger a}|\downarrow_x\rangle.$$

In all three schemes the ion is prepared initially in the upper electronic state $|\uparrow\rangle$ and its CM is in a coherent state, according to [4],

$$|\Psi(0)\rangle \equiv |\alpha_0\rangle \otimes |\downarrow\rangle = |\alpha_0\rangle \otimes \left(\frac{|\uparrow_x\rangle + |\downarrow_x\rangle}{\sqrt{2}}\right). \tag{19}$$

The second equality follows because in terms of the eigenvectors $|\uparrow\rangle$, $|\downarrow\rangle$ of σ_z one has

$$|\uparrow_x\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}, \qquad |\downarrow_x\rangle = \frac{|\uparrow\rangle - |\downarrow\rangle}{\sqrt{2}}. \tag{20}$$

The general scheme is the following: one applies N laser pulses, the duration t_k of each is chosen according to the target state one wants to generate. One triggers the N pulses sequentially (each with its specific duration) until one 'measures' N sequentially no-fluorescence events on measuring the $|d\rangle \leftrightarrow |\downarrow\rangle$ transition; if in a specific sequence of measurements a fluorescence emission is detected, it should mean that the ion is in the electronic ground state $|\downarrow\rangle$. Thus, the sequence of measurement should be stopped and we must repeat the process. The no-fluorescence event is a necessary condition since it assures no-recoil of the ion vibrational CM motion, thus maximizing the probability to realize successfully the target state. In [13] the authors introduce a different method to generate circular states on the circle in a trapped ion; however, they do not consider in much detail the probability and necessary duration of pulses to produce such a state.

4.1. First scheme—2^N even circular states

The goal is to generate a circular state (2) with superposition of 2^N coherent states. We have to perform, repeatedly, two operations on the ion with the pulsed lasers: the first is a unitary evolution with $U(t)$ (18) and the second consists in measuring the probability for the upper electronic state by projecting the ion state vector on $|\uparrow\rangle$—this constitutes a *cycle*. So, in this scheme each cycle involves the application of one pulse and one measurement. The first pulse having duration t_1, the ion state becomes

$$\begin{aligned} |\Psi(t_1)\rangle &= U(t_1)|\Psi(0)\rangle \\ &= \tfrac{1}{2}(\mathrm{e}^{-\mathrm{i}\Omega t_1}|\alpha(t_1)\rangle + \mathrm{e}^{\mathrm{i}\Omega t_1}|\alpha(-t_1)\rangle) \otimes |\uparrow\rangle \\ &\quad+ \tfrac{1}{2}(\mathrm{e}^{-\mathrm{i}\Omega t_1}|\alpha(t_1)\rangle - \mathrm{e}^{\mathrm{i}\Omega t_1}|\alpha(-t_1)\rangle) \otimes |\downarrow\rangle, \end{aligned} \tag{21}$$

where $\alpha(t)\rangle \equiv |\alpha_0\mathrm{e}^{\mathrm{i}\bar{\Omega}t}\rangle$. In the second operation if the ion is found in state $|\uparrow\rangle$ (no fluorescence), we consider that it was *successful*, and state (21) reduces to

$$|\Psi'(t_1)\rangle = \frac{\mathcal{N}_1}{2}(\mathrm{e}^{-\mathrm{i}\Omega t_1}|\alpha(t_1)\rangle + \mathrm{e}^{\mathrm{i}\Omega t_1}|\alpha(-t_1)\rangle) \otimes |\uparrow\rangle, \tag{22}$$

where $\mathcal{N}_1$ is the normalization factor. After the first cycle, the probability to find the ion in state $|\uparrow\rangle$ is

$$P_\uparrow(t_1) = \tfrac{1}{2}[1 + \mathrm{e}^{-r^2(1-\cos(2\bar{\Omega}t_1))}\cos(r^2\sin(2\bar{\Omega}t_1) - 2\Omega t_1)]. \tag{23}$$

Supposing the ion was measured in this state, the second cycle begins with a second laser pulse driving the ion into state $|\Psi(t_1 + t_2)\rangle = U(t_2)|\Psi'(t_1)\rangle$ and the second successful measurement reduces this state to

$$\begin{aligned} |\Psi'(t_1 + t_2)\rangle &= \frac{\mathcal{N}_2}{4}(\mathrm{e}^{-\mathrm{i}\Omega(t_1+t_2)}|\alpha(t_1 + t_2)\rangle \\ &\quad+ \mathrm{e}^{-\mathrm{i}\Omega(t_1-t_2)}|\alpha(t_1 - t_2)\rangle \\ &\quad+ \mathrm{e}^{\mathrm{i}\Omega(t_1-t_2)}|\alpha(t_2 - t_1)\rangle \\ &\quad+ \mathrm{e}^{-\mathrm{i}\Omega(t_1+t_2)}|\alpha(-t_1 - t_2)\rangle) \otimes |\uparrow\rangle, \end{aligned} \tag{24}$$

with probability

$$
\begin{aligned}
&P_{\uparrow}(t_1+t_2)\\
&\quad=\tfrac{1}{4}\{1+\mathrm{e}^{-r^2(1-\cos(2\bar{\Omega}t_1))}\cos[r^2\sin(2\bar{\Omega}t_1)-2\Omega t_1]\\
&\qquad+\mathrm{e}^{-r^2(1-\cos(2\bar{\Omega}t_2))}\cos[r^2\sin(2\bar{\Omega}t_2)-2\Omega t_2]\\
&\qquad+\mathrm{e}^{-r^2(1-\cos(2\bar{\Omega}(t_1+t_2)))}\cos[r^2\sin(2\bar{\Omega}(t_1+t_2))\\
&\qquad-2\Omega(t_1+t_2)]\\
&\qquad+\mathrm{e}^{-r^2(1-\cos(2\bar{\Omega}(t_1-t_2)))}\cos[r^2\sin(2\bar{\Omega}(t_1-t_2))\\
&\qquad-2\Omega(t_1-t_2)]\}.
\end{aligned}
\tag{25}
$$

After N cycles of successful measurements the state of the ion becomes

$$\left|\Psi'\left(\sum_{k=1}^{N}t_k\right)\right\rangle=\mathcal{N}_N\left[\prod_{k=1}^{N}\langle\uparrow|U(t_k)|\uparrow\rangle\right]|\alpha\rangle\otimes|\uparrow\rangle. \tag{26}$$

In order to engineer the ion vibrational state as equation (2) we need to adjust the phases in (26) of the coherent states putting them evenly distributed around the circle: this is only possible when the duration of the kth evolution pulse is adjusted to be

$$t_k=\frac{\pi}{2^k\bar{\Omega}}. \tag{27}$$

Recalling that all the coefficients of superposition (26) should be equal to 1, and choosing the Lamb–Dicke parameter as $\eta^2=2^{-(N+1)}$, where N refers to the last Nth pulse, it becomes possible to generate the wanted superposition state. Thus, we can write (26) simply as

$$\left|\Psi'\left(\sum_{k=1}^{N}t_k\right)\right\rangle=\frac{\mathcal{N}_N}{2^N}|\Psi_{2^N}\rangle\otimes|\uparrow\rangle. \tag{28}$$

The probability of getting the ion in the upper electronic state during the N cycles is given by

$$P_{\uparrow}\left(\sum_{k=1}^{N}t_k\right)=\frac{1}{\mathcal{N}_N{}^2}=\frac{1}{2^{2N}\lambda_{2^N}^2}, \tag{29}$$

with λ_{2^N} defined in equation (4). In the coherent states $|\alpha_k\rangle$ the c-numbers α_k have the following phases distributed on the circle of radius r:

$$\theta_k=\frac{2\pi k}{2^N}-\frac{\pi}{2^N}. \tag{30}$$

As an illustration let us consider the following example, if one wants to engineer a superposition with 16 coherent states ($N=4$ is the number of cycles) we need $\eta\approx 0.18$. Proceeding along the lines outlined above we get the following states after each successful measurement:

$$
\begin{aligned}
&t_1=\frac{\pi}{2\bar{\Omega}}\Rightarrow|\Psi'(t_1)\rangle=\mathcal{N}_1(|\alpha_0\mathrm{e}^{\mathrm{i}\pi/2}\rangle+|\alpha_0\mathrm{e}^{-\mathrm{i}\pi/2}\rangle)/2\\
&t_2=\frac{\pi}{4\bar{\Omega}}\Rightarrow|\Psi'(t_{\bar{1}}+t_2)\rangle=\mathcal{N}_2(|\alpha_0\mathrm{e}^{3\mathrm{i}\pi/4}\rangle+|\alpha_0\mathrm{e}^{\mathrm{i}\pi/4}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{-\mathrm{i}\pi/4}\rangle|\alpha_0\mathrm{e}^{-3\mathrm{i}\pi/4}\rangle)/4\\
&t_3=\frac{\pi}{8\bar{\Omega}}\Rightarrow|\Psi'(t_{\bar{1}}+t_2+t_3)\rangle=\mathcal{N}_3[|\alpha_0\mathrm{e}^{7\mathrm{i}\pi/8}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{5\mathrm{i}\pi/8}\rangle+|\alpha_0\mathrm{e}^{3\mathrm{i}\pi/8}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{\mathrm{i}\pi/8}\rangle+|\alpha_0\mathrm{e}^{-\mathrm{i}\pi/8}\rangle+|\alpha_0\mathrm{e}^{-3\mathrm{i}\pi/8}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{-5\mathrm{i}\pi/8}\rangle+|\alpha_0\mathrm{e}^{-7\mathrm{i}\pi/8}\rangle]/8\\
&t_4=\frac{\pi}{16\bar{\Omega}}\Rightarrow|\Psi'(t_{\bar{1}}+t_2+t_3+t_4)\rangle=\mathcal{N}_4(|\alpha_0\mathrm{e}^{15\mathrm{i}\pi/16}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{13\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{11\mathrm{i}\pi/16}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{9\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{7\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{5\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{3\mathrm{i}\pi/16}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{-\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{-3\mathrm{i}\pi/16}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{-5\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{-7\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{-9\mathrm{i}\pi/16}\rangle\\
&\qquad+|\alpha_0\mathrm{e}^{-11\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{-13\mathrm{i}\pi/16}\rangle+|\alpha_0\mathrm{e}^{-15\mathrm{i}\pi/16}\rangle)/16.
\end{aligned}
$$

After the first cycle the ion CM state is a cat state and N: 2^N is the ratio of the number of pulses to the number of states in the superposition. A similar superposition was proposed in [12] for the state of the field in a QED cavity, by making a beam of atoms crossing one Ramsey zone, then entering in a superconducting cavity, interacting and correlating with the field resonance mode, then again crossing a second Ramsey zone and thereafter being detected in an ionization chamber. The authors report that this scheme is very difficult to realize due to the great amount of experimental detail. In our case the scheme is simpler: it suffices to control the on–off switchings of the laser beams, and the total time of the pulses is still less than that proposed in [5], where Fock number states can be generated through a sequence of quantum nondemolition measurements on an initial coherent state.

The total duration of the N pulses is

$$T=\sum_{k=1}^{N}t_k=\frac{\pi}{\bar{\Omega}}\left(1-\frac{1}{2^N}\right)=\frac{2\pi}{\Omega}(2^N-1)\approx(2^N-1)\ \mu\text{s}, \tag{31}$$

since $2\pi/\Omega\simeq 1\ \mu$s, so about 15 μs is the duration of the pulses in order to produce the desired superposition (16 coherent states); this time is much less than the required time for doing the experiment proposed in [5].

Now we have to estimate the required time for the measurement process, which is done by a laser coupled to the $|d\rangle\leftrightarrow|\downarrow\rangle$ transition, where the width of the $|d\rangle$ level is Γ and $\Gamma/2\pi\approx 20$ MHz ($2\pi/\Gamma\approx 0.05\ \mu$s). The duration of the pulse t' must be sufficiently long to allow the emission, with high probability, of at least one photon if $|d\rangle$ is populated. The non-observation of a fluorescence photon during a time t' after the pulse is considered to be a successful measurement, meaning that the internal electronic state is in level $|\uparrow\rangle$, thus the ion wavefunction collapses to the desired state. This also allows the experimenter to proceed with the subsequent cycle of laser pulses. According to [5] $t'\ll 2\ \mu$s, and is taken as $t'\approx 0.2\ \mu$s. The time Nt', where N is the number of cycles, is to be added to the total time of the other pulses. The time Nt' represents no more than 10% of the total time; for $N=4$ it represents about 1 μs, compared with the necessary pulse time, 15 μs, to generate the Fock state.

The probability for the first successful cycle is $P_{\uparrow}(t_1)=(1+\mathrm{e}^{-2r^2})/2$ while for the first and second successful cycles it becomes $P_{\uparrow}(t_1+t_2)=(1+\mathrm{e}^{-2r^2}+2\mathrm{e}^{-r^2}\cos r^2)/4$ and for the four it is

$$
\begin{aligned}
P_{\uparrow}(t_1+\cdots+t_4)=\tfrac{1}{16}\Bigg(1+\tfrac{1}{8}\sum_{k=1}^{15}k\mathrm{e}^{-2r^2\sin^2(\pi k/16)}\\
\times\cos(r^2\sin(\pi k/8))\Bigg).
\end{aligned}
\tag{32}
$$

Since the times t_k are already fixed we only have the freedom to choose the 'radius' r, i.e. the intensity of the initial coherent

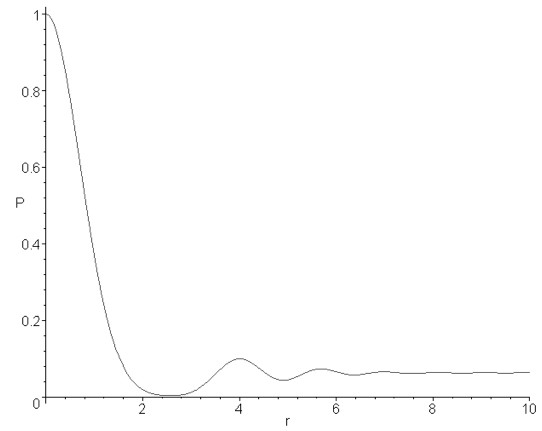

Figure 6. Probability (equation (32)) for producing a Fock number state $|2^4\rangle$ after four successful cycles as a function of $r = |\alpha_0|$, out from a initial coherent state $|\alpha_0\rangle$.

state, in order to maximize $P_\uparrow(t_1 + \cdots + t_4)$. This probability is plotted in figure 6 as a function of r for $2^4 = 16$ superposed coherent states: we see that $r = 4$ maximizes (32), $P_\uparrow \approx 0.1$, which is not a bad figure since it means that an average of one in ten runs should produce an approximate Fock number state $|\tilde{16}\rangle$ whenever the condition $1 \ll (\mathrm{e}r^2/2^4)^{2^4} \ll 4^{2^4}$ is satisfied.

4.2. Second scheme—$N + 1$ even circular states

Now, we will describe a scheme that generates a superposition of $N + 1$ arbitrary coherent states on the circle as a result of selecting all pulses having the same time duration τ. In [13] a similar superposition is obtained for arbitrary coefficients C_k considering a different arrangement of the laser beams.

If we consider equal times τ in equations (21) and (24), after two successful cycles we get

$$\begin{aligned} |\Psi'(\tau)\rangle &= \frac{\mathcal{N}_1}{2}(|\alpha(\tau)\rangle + |\alpha(-\tau)\rangle) \otimes |\uparrow\rangle, \\ |\Psi'(2\tau)\rangle &= \frac{\mathcal{N}_2}{4}(|\alpha(2\tau)\rangle \\ &\quad +2|\alpha(0)\rangle + |\alpha(-2\tau)\rangle) \otimes |\uparrow\rangle, \end{aligned} \tag{33}$$

which is not what we need because the second term has a factor 2 and we want all the coefficients in (33) to be equal to 1. Thus, we have to use a second pair of lasers (also in the Lamb–Dicke limit) with another Lamb–Dicke parameter, η_r, satisfying the condition $\eta_r^2\Lambda \ll \eta^2\Omega$ in order to rotate the electronic levels according to the evolution ruled by

$$H_r = \Lambda\sigma_x \Rightarrow U_r(t) = \mathrm{e}^{-\mathrm{i}\Lambda t\sigma_x}. \tag{34}$$

Now, each cycle consists of one rotating pulse, one evolution pulse and one measurement pulse. The action of the lth rotating pulse on $|\uparrow\rangle$ with duration t_l' is

$$\begin{aligned} U_r(t_l')|\uparrow\rangle &= \frac{\mathrm{e}^{-\mathrm{i}\Lambda t_l'}}{\sqrt{2}}|\uparrow_x\rangle + \frac{\mathrm{e}^{\mathrm{i}\Lambda t_l'}}{\sqrt{2}}|\downarrow_x\rangle \\ &= \frac{a_l}{\sqrt{2}}|\uparrow_x\rangle + \frac{b_l}{\sqrt{2}}|\downarrow_x\rangle. \end{aligned} \tag{35}$$

If each cycle ends with a successful no-fluorescence measurement, the evolution of the ion vibrational state after three cycles, for instance, is as follows:

$$\begin{aligned} |\alpha\rangle &\to a_1|\alpha(\tau)\rangle + b_1|\alpha(-\tau)\rangle \to a_1a_2|\alpha(2\tau)\rangle \\ &\quad +(a_1b_2 + b_1a_2)|\alpha(0)\rangle \\ &\quad +b_1b_2|\alpha(-2\tau)\rangle \to a_1a_2a_3|\alpha(3\tau)\rangle \\ &\quad +(a_1a_2b_3 + a_1b_2a_3 + b_1a_2a_3)|\alpha(\tau)\rangle \\ &\quad +(a_1b_2b_3 + b_1a_2b_3 + b_1b_2a_3)|\alpha(-\tau)\rangle \\ &\quad +b_1b_2b_3|\alpha(-3\tau)\rangle. \end{aligned} \tag{36}$$

In order to adjust the coefficients in (36) to reproduce the phases of α_k in equation (2), we need first to assign values to the duration of each evolution pulse and to the Lamb–Dicke parameter too:

$$\tau = \frac{\pi}{(N+1)\bar{\Omega}}, \qquad \eta = \frac{1}{\sqrt{2(N+1)}}. \tag{37}$$

For determining the duration t_l' of each rotating pulse we need each coefficient in (36) to be equal to 1; thus it becomes necessary to solve a system of algebraic equations. Calling $z_l = b_l/a_l$, the system of equations is

$$\begin{aligned} z_1z_2z_3 &= 1, \\ z_1 + z_2 + z_3 &= 1, \\ z_1z_2 + z_2z_3 + z_1z_3 &= 1, \end{aligned} \tag{38}$$

whose solution is the roots of the polynomial equation $z^3 - z^2 + z - 1 = 0$. In general, after N successful cycles the solution of the equation

$$\sum_{k=0}^{N}(-1)^{N-k}z^k = 0, \tag{39}$$

is given by the roots $z_l = \mathrm{e}^{\mathrm{i}\pi + 2\pi\mathrm{i}l/(N+1)}$, $l = 1, 2, \ldots, N$. But since $z_l = \mathrm{e}^{2\mathrm{i}\Lambda t_l'}$, then the lth pulse duration must be

$$t_l' = \left(\frac{l}{N+1} + \frac{1}{2}\right)\frac{\pi}{\Lambda}. \tag{40}$$

After N successful cycles the ion state becomes

$$\left|\Psi'\left(N\tau + \sum_{l=1}^{N}t_l'\right)\right\rangle = \frac{\mathcal{N}_N}{2^N}|\tilde{\Psi}_{N+1}\rangle \otimes |\uparrow\rangle. \tag{41}$$

The probability to produce such a state is

$$P_\uparrow\left(N\tau + \sum_{l=1}^{N}t_l'\right) = \frac{1}{\mathcal{N}_N{}^2} = \frac{1}{2^{2N}\lambda_{N+1}^2}, \tag{42}$$

and the total duration of pulses is

$$T = N\tau + \sum_{l=1}^{N}\left(\frac{l}{N+1} + \frac{1}{2}\right)\frac{\pi}{\Lambda} = N\left(\frac{2\pi}{\Omega} + \frac{\pi}{\Lambda}\right). \tag{43}$$

One verifies that for a large N, $P_\uparrow(N\tau + \sum_{l=1}^{N}t_l')$ is very small and T is very large. As an example, for engineering $N + 1 = 16$ superposed coherent states one needs to carry out 15 cycles, the same number as required in [4],

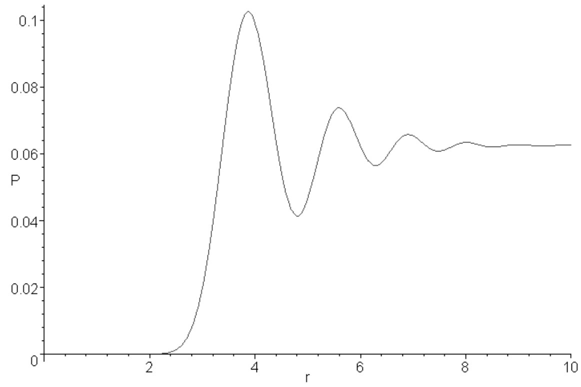

Figure 7. Probability (equation (46)) for producing a Fock number state $|2^4 - 1\rangle$ after four successful cycles as a function of $r = |\alpha_0|$, out of an initial coherent state $|\alpha_0\rangle$.

with $P_\uparrow(t_1 + \cdots + t_{15}) \approx 1/2^{15}$; the total pulse duration is $T = 15(2\pi/\Omega + \pi/\Lambda)$ which exceeds the total time of the previous scheme by $15\pi/\Lambda$ which is large because experimental conditions imply $\Lambda \ll \Omega$ since $\eta_r \lesssim \eta$. Thus it looks unlikely to generate a Fock state for $N > 4$ due to the very low probability [13], whereas for a small N we recall that the condition $(\mathrm{e}r^2/(N+1))^{N+1} \gg 1$ must be fulfilled.

4.3. Third scheme—2^N odd circular states

To construct the superposition (8) we have to combine the previous schemes: we now need one rotating pulse, one evolution pulse and one measurement per cycle. A pair of in-phase lasers is used to produce a rotating pulse, such that $H_r = -\Lambda\sigma_x$.

The evolution and rotating pulses have duration $t_k = \pi/(2^k\bar{\Omega})$ and $t'_l = \pi/(2^l\Lambda)$, respectively, and $\eta_r^2\Lambda \ll \eta^2\Omega$. After N successful cycles the ion state becomes

$$\left|\tilde{\Psi}'\left(\sum_{k=1}^{N} t_k + \sum_{l=1}^{N} t'_l\right)\right\rangle = \frac{\mathcal{N}_N}{2^N}|\tilde{\Psi}_{2^N}\rangle \otimes |\uparrow\rangle, \qquad (44)$$

with probability

$$P_\uparrow\left(\sum_{k=1}^{N} t_k + \sum_{l=1}^{N} t'_l\right) = \frac{1}{\mathcal{N}_N{}^2} = \frac{1}{2^{2N}\lambda_{2^N}^2}. \qquad (45)$$

Recall that the condition to generate an approximate Fock number state $|2^N - 1\rangle$ is $(r^2\mathrm{e}/2^{N+2})^{2^N} \ll 1$. As an example, for $N = 4$ one generates approximately the Fock number state $|\tilde{15}\rangle$ with probability

$$P_\uparrow\left(\sum_{k=1}^{4} t_k + \sum_{l=1}^{4} t'_l\right) = \frac{1}{16}\left(1 + \frac{1}{8}\sum_{k=1}^{15} k\mathrm{e}^{-2r^2\sin^2(\pi k/16)} \times \cos\left(\frac{\pi k}{8} + r^2\sin\left(\frac{\pi k}{8}\right)\right)\right) \qquad (46)$$

and figure 7 shows that the maximum of (46) occurs for $r = 4$. The total time involved in the engineering of the state $|\tilde{\Psi}'(\sum_{k=1}^{4} t_k + \sum_{l=1}^{4} t'_l)\rangle$ is $T = 15(2\pi/\Omega + \pi/16\Lambda)$, which is larger than in the first scheme by $15\pi/16\Lambda$, showing that it takes longer (in pulse time) to generate an odd Fock number state than an even one.

5. Summary and comments

We have analysed three schemes to engineer particular circular states that may lead (under specific conditions) to Fock number states of the vibrational mode of a trapped ion. For this purpose, we have considered a single trapped ion interacting with two laser beams in a stimulated Raman configuration. We selected the effective laser frequency in resonance with the ion transition electronic frequency, resulting in an effective interaction of Kerr type for the ion–laser system. After preparing the system in a particular initial state $|\alpha_0\uparrow\rangle$ we considered N-cycle operations where each successful cycle results in a new extended circular state for the vibrational motion. We have calculated the total time of pulses necessary for each kind of circular state and the probability to produce it. We have shown the possibility of constructing Fock number states out of even and odd circular states and that it takes more pulse time to generate an odd circular state $|2^N - 1\rangle$ than an even $|2^N\rangle$, as the latter feels the effects of the decoherence less.

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