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High-fidelity teleportation of entanglements of running-wave field states

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Abstract
We describe a scheme for the teleportation of entanglements of zero- and one-photon running-wave field states. In addition to linear optical elements, Kerr nonlinearity is also employed so as to achieve a 100% probability of success in the ideal case. A comprehensive phenomenological treatment of errors in the domain of running-wave physics, for linear and nonlinear optical elements, is also given, making it possible to calculate the fidelity of the teleportation process. A strategy for carrying out the Bell-type measurement which is able to probe the absorption of photons in the optical elements is adopted. Such a strategy, combined with usually small damping constants characterizing the optical devices, results in a high fidelity for the teleportation process. The feasibility of the proposed scheme relies on the fact that the Kerr nonlinearity it demands can be achieved through ultra-slow light propagation in cold atomic media as recently reported by Lukin and Imamoglu and by Petrosyan and Kurizki.

Keywords: Quantum noise, quantum teleportation, Bell-states measurement

1. Introduction

The property of nonlocality exhibited by entangled states, first pointed out by Einstein, Podolsky and Rosen (EPR) as the cornerstone for their argument against the uncertainty principle [1], has since then largely been invoked for investigating the foundations of quantum mechanics. The programme inaugurated by the confrontation of EPR with the standard Copenhagen interpretation of quantum mechanics, in keeping with the reinterpretation by Bohm [2] of the gedanken experiment designed by EPR, came to a head with the possibility of an empirical test of nonlocality formulated by Bell [3]. In the end, about two decades were devoted to the quest for an experimental demonstration of nonlocality through the violation of Bell's inequalities [4]. Despite the majority of the experiments having confirmed nonlocality, experimental loopholes have been pointed out which have to be circumvented for an impartial conclusion, which hopefully will be arrived at [5].

However, in the last decade a variety of potential applications of nonlocality have been devised which have definitely moved the focus of the nonlocality phenomenon. From its original purely theoretical role in the foundations of quantum mechanics, the nonlocality phenomenon together with other fundamental quantum processes seems to be about to inaugurate a novel technology for quantum communication [6] and computation [7]. Basically, such a possibility relies on the discovery made by Shor [8] that quantum information processing, involving two-state systems as quantum bits, provides a means of integer factorization much more efficiently than conventional computation. (Here we stress the recent remarkable experimental realization of Shor's quantum factoring algorithm using nuclear magnetic resonance, reported by the Solid State and Photonics Laboratory of IBM, California [9].) The information thus processed and transmitted in a quantum logical processor consists of arbitrary superpositions of quantum states instead of classical bits. The interference phenomena characteristic of quantum superposition states allows parallel computation

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paths which can reinforce or cancel one another, depending on their relative phase [10]. Besides being indispensable for correlating the input qubits in a quantum gate, the role of entanglement and nonlocality in a quantum processor provides a striking difference from any classical operation: states can be transmitted from one node of a network to another by quantum teleportation. Discovered by Bennett *et al* [11], teleportation is a process in which a superposition state, a qubit, is teleported from one quantum system to another, over arbitrary distances, via dual classical and EPR channels, and it furnishes a critical ingredient for the implementation of a quantum logical processor [10]. It is worth noting that the state to be teleported is destroyed during the required Bell state measurement; owing to the linearity of quantum mechanics, a quantum state cannot be cloned [12], preventing the nonlocality phenomenon from being employed for superluminal communication. However, quantum states can be transferred from one system to another and even interchanged between different quantum systems [13], despite the impossibility of cloning.

Since its proposition, the teleportation phenomenon has attracted great attention and a number of different protocols have been suggested for practical implementation of the process in the cavity QED [14], in trapped ions [15] and in running wave domains [16, 17]. Teleportation of entanglements [13, 18–20, 24–26] and $N(>2)$-dimensional states [27], of major interest for information processing, has also been addressed, in addition to the originally proposed protocol for teleporting qubits [11]. The process has also been demonstrated in experiments through photon polarized states [28, 29, 31], and also through a 'Schrödinger cat'-like state generated by parametric down-conversion as a running wave [32]. The Bell state measurement, performed on the Bell operator basis consisting of four states—a $2 \otimes 2$-dimensional basis for the particle whose state is to be teleported plus one of the two particles composing the quantum channel [11]—constitutes the main experimental challenge. In the Innsbruck experiment [28], designed to manipulate only the polarization state of single-photon pulses, only one of the four Bell states is discriminated, resulting in a success rate not larger than 25%. Employing the entanglement between the spacial and polarization degrees of freedom of a photon, the Rome experiment [29] was able to distinguish the four Bell states allowing, in the ideal case, a 100% success rate for teleportation. The experimental implementation reported in [29] was based on an approach suggested in [30], different from the original protocol in [11]: the polarization degree of freedom of one of the photons composing the EPR-quantum channel, a $\boldsymbol{k}$-vector entangled state, was used to prepare the unknown state to be teleported. Such a strategy avoids the difficulties associated with having three photons, making the Bell measurement more straightforward. By accomplishing the Bell state measurements on the basis of nonlinear interactions, the Baltimore experiment [31], which follows exactly the original protocol by Bennett *et al* [11], also achieves, in principle, a 100% success rate for teleporting a polarization state. In the Caltech experiment [32], the teleportation of a state of continuous quantum variables was achieved with a fidelity of 0.58 ± 0.02, demonstrating the nonclassical character of the process: the critical fidelity of 0.5 is the classical bound attainable in the absence of quantum correlation.

Here we introduce a high-fidelity technique for teleportation of entanglements of zero- and one-photon running-wave states. The teleportation apparatus is based on Mach–Zehnder (MZ) interferometry with a phase-sensitive element, a cross-Kerr medium, allowing a 100% probability of success in the ideal case. A scheme to teleport similar entanglements of running-wave states, employing linear optical elements at the expense of a 50% probability of success, was recently suggested [19, 20]. In this connection we note that the feasibility of our proposed scheme relies on a recent demonstration of ultra-slow light propagation in cold atomic media [23], which opened the way for the realization of significant conditional phase shifts between two travelling single-photon pulses [21, 22].

The teleportation and decoherence of entangled coherent running-wave states has also been addressed [24], the probability of success in this case being 50%. In the cavity QED domain, protocols have been reported that teleport two-particle entangled atomic states [13], multiparticle atomic states and entangled field states inside high-Q cavities [25]. We stress that experimental teleportation of an entangled qubit has recently been achieved [26]. Employing linear optics, the experimental accomplishment in [26], following lines suggested in [19], had a 25% probability of success. Efforts towards the achievement of 50% success rate (as described in [19]) are in progress [26].

Our experimental setup to teleport an entangled state consists basically of three stations, sketched in figure 1(*a*), employing 50/50 symmetric beam splitters (BSs), Kerr media (KMs) and photodetectors. In addition to the entanglement to be teleported, which we assume to be already prepared when injected through channels 1 and 2 ($|\Omega\rangle_{12}$), the quantum channel is engineered by the apparatus sketched in figure 1(*b*). This *quantum channel station* consists of a (BS_1), an MZ interferometer, composed of a pair of (BS_2, BS_3) and a (KM_1). The latter, used to entangle the output modes 4 and 5 of BS_1 and BS_2, respectively, is an ingredient crucial to the generation of the quantum channel (an entanglement of modes 3, 4, 5 and 6) required for our proposed teleportation protocol. A part from the photodetectors, *Alice's station*, shown in figure 1(*c*), consists of the same ingredients used to engineer the quantum channel. An MZ interferometer (BS_4, BS_5) is disposed so as to receive modes 3 and 4 of the quantum channel, which is coupled to the entanglement to be teleported through a (KM_2). Finally, BS_6 is employed to prepare the whole entanglement (involving the quantum channel and the state to be teleported) for the Bell-type measurement carried out with photodetectors D_i ($i = 1, 2, 3, 4$). In *Bob's station*, illustrated in figure 1(*d*), another MZ interferometer (BS_7, BS_8) and three phase plates are disposed so as to accomplish all the rotations required to convert the teleported state into a replica of the original entangled state $|\Omega\rangle_{12}$, allowing a 100% probability of success in the ideal case.

It should be stressed that the fidelity of the scheme proposed for the teleportation of an entangled state is estimated by taking into account the noise introduced by dissipation in both the optical systems, BSs and KMs. We present here a phenomenological approach that allows for the

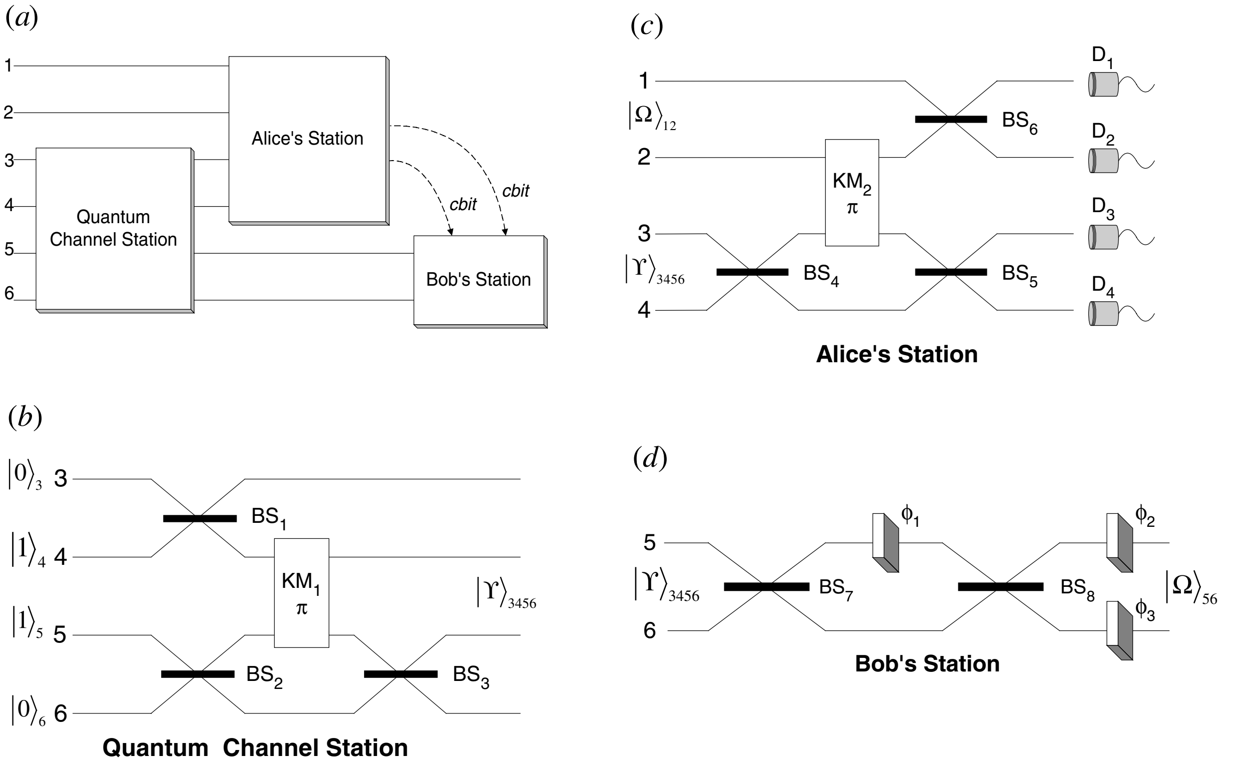

Figure 1. (*a*) Sketch of the experimental setup for quantum teleportation of entanglements of zero- and one-photon running-wave states. The apparatus is composed of a station to prepare the quantum channel, Alice's station for carrying out the Bell-type measurement, and Bob's station to accomplish all the rotations required to convert the teleported state into a replica of the original entangled state. The dashed curves indicate the classical channel composed of two classical bits (cbit). (*b*) Sketch of the station used to engineer the quantum channel, an entanglement of modes 3–6, which is shared by Alice and Bob. The output modes 3 and 4 enter Alice's station to be entangled with modes 1 and 2 (encapsulating the state to be teleported). The output modes 5 and 6, which receive the teleported state after Alice's Bell-type measurement, enter Bob's station for the accomplishment, if needed, of the rotation required to convert the teleported state into a replica of the original entanglement $|\Omega\rangle_{12}$. The *quantum channel station* consists of a (BS_1) and an MZ interferometer, composed of a pair of (BS_2, BS_3), and a (KM_1). (*c*) Sketch of *Alice's station* which consists of the same ingredients as the station used to engineer the quantum channel plus photodetectors D_i (i = 1–4). The lack of a single photodetection event in couples D_1–D_2 or D_3–D_4 would indicate that a photon, either from the state to be teleported or the quantum channel, has been absorbed (between the input and output channels 1–4). The events where a single photon or both of them have been absorbed or scattered by the optical elements are disregarded and the teleportation process must be restarted. This strategy contributes to the higher fidelity of the present teleportation scheme, since we do not compute the terms in equation (28) for the nonideal quantum channel (associated with ket$|\vartheta\rangle_{3456}$), which involve only zero-photon states in modes 3 and 4.
(d) Sketch of *Bob's station,* where an MZ interferometer (BS_7, BS_8) and three phase plates are employed to accomplish all the rotations required to convert the teleported state into a replica of the original entangled state $|\Omega\rangle_{12}$, allowing a 100% probability of success in the ideal case.

influence of damping in KMs, reasoning by analogy with the treatment of lossy BSs given in [33]. The efficiency of the photodetectors has also been included, using the relations previously established in [17]. We have pursued a strategy of carrying out the Bell measurement in such a way as to probe the absorption of photons in the optical elements placed between channels 1–4. This strategy, combined with the usually small damping constants characterizing BSs and KMs and the high efficiency of the photodetectors, results in a high-fidelity teleportation process.

2. Teleportation of entangled states: ideal process

Let us first introduce the situation where losses are disregarded and describe the physical operations taking place in the optical elements. The general relationships between the input and output operators $\hat{\alpha}$, $\hat{\beta}$ (see figure 2(*a*)), arising from the unitary operator $\hat{U}_{BS} = \exp[\mathrm{i}\theta(\hat{\alpha}_{in}^{\dagger}\hat{\beta}_{in} + \hat{\beta}_{in}^{\dagger}\hat{\alpha}_{in})]$ describing the action of an ideal symmetric BS, are written as

$$\hat{\alpha}_{out} = t\hat{\alpha}_{in} + r\hat{\beta}_{in}, \tag{1a}$$

$$\hat{\beta}_{out} = t\hat{\beta}_{in} + r\hat{\alpha}_{in}, \tag{1b}$$

where $t = \cos(\theta)$ and $r = \mathrm{i}\sin(\theta)$ are the beam-splitter transmission and reflection coefficients satisfying $|t|^2 + |r|^2 = 1$. For a 50/50 symmetric beam splitter, where $\theta = \pi/4$, $t = |r| = 1/\sqrt{2}$. These coefficients, and thus the operators, depend on the frequency of the fields and here a monochromatic source is assumed.

The coupling between the input and output operators $\hat{\alpha}$, $\hat{\beta}$ (see figure 2(*b*)), representing the fields crossing KM, follows from the action of the unitary operator $\hat{U}_{Kerr} = \exp(-\mathrm{i}\chi\tau\hat{\alpha}_{in}^{\dagger}\hat{\alpha}_{in}\hat{\beta}_{in}^{\dagger}\hat{\beta}_{in})$, and is given by

$$\hat{\alpha}_{out} = \exp(\mathrm{i}\chi\tau\hat{\beta}_{in}^{\dagger}\hat{\beta}_{in})\hat{\alpha}_{in}, \tag{2a}$$

$$\hat{\beta}_{out} = \exp(\mathrm{i}\chi\tau\hat{\alpha}_{in}^{\dagger}\hat{\alpha}_{in})\hat{\beta}_{in}. \tag{2b}$$

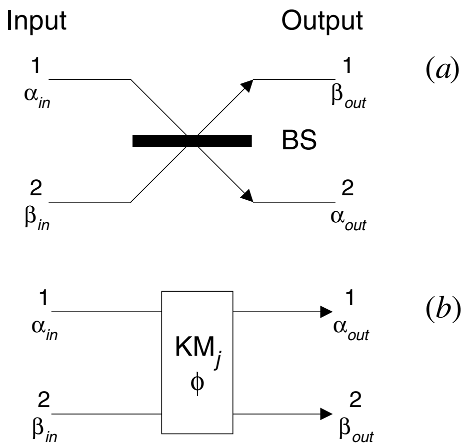

Figure 2. Schematic representation of input and output modes for (*a*) the BS and (*b*) the KM.

The conditional phase shift $\phi = \chi\tau$ depends upon the third-order nonlinear susceptibility χ for the optical Kerr effect and the interaction time τ within the KM. Finally, phase plates are used to introduce an adjustable phase shift in the output field states and photodetectors are employed for the projective Bell measurement.

The entangled state to be teleported can easily be prepared from a single-photon field incident on a BS with arbitrary unknown transmission and reflection coefficients $\mathcal{C}_1$ and $\mathcal{C}_2$, satisfying $|\mathcal{C}_1|^2 + |\mathcal{C}_2|^2 = 1$. Such a state, injected through the input modes 1 and 2, can be written

$$|\Omega\rangle_{12} = \mathcal{C}_1|0\rangle_1|1\rangle_2 + \mathcal{C}_2|1\rangle_1|0\rangle_2. \tag{3}$$

Simultaneously with the preparation of the entanglement to be teleported, the quantum channel is prepared from single-photon fields $|1\rangle_4$ and $|1\rangle_5$ incident on BS_1 and BS_2 respectively. It is easily verified from equations (1*a*), (1*b*) and (2*a*), (2*b*) that the entangled state resulting in the output modes of the quantum channel station (figure 1(*b*)) is written, apart from an irrelevant phase factor, as

$$|\Upsilon\rangle_{3456} = \frac{1}{\sqrt{2}}(|1\rangle_3|0\rangle_4|0\rangle_5|1\rangle_6 + |0\rangle_3|1\rangle_4|1\rangle_5|0\rangle_6), \tag{4}$$

where the interaction parameter in the KM has been adjusted so that $\chi\tau = \pi$ [21, 22]. It must be stressed that the KM is indispensable for engineering the correlated channels in equation (4). The product of the entanglement to be teleported and the quantum channel, $|\Omega\rangle_{12} \otimes |\Upsilon\rangle_{3456}$, can be expanded as

$$\begin{aligned}|\Xi\rangle_{123456} = \tfrac{1}{2}\{&|\Psi^+\rangle_{1234}(\mathcal{C}_1|0\rangle_5|1\rangle_6 + \mathcal{C}_2|1\rangle_5|0\rangle_6)\\ &+ |\Psi^-\rangle_{1234}(\mathcal{C}_1|0\rangle_5|1\rangle_6 - \mathcal{C}_2|1\rangle_5|0\rangle_6)\\ &+ |\Phi^+\rangle_{1234}(\mathcal{C}_2|0\rangle_5|1\rangle_6 + \mathcal{C}_1|1\rangle_5|0\rangle_6)\\ &- |\Phi^-\rangle_{1234}(\mathcal{C}_2|0\rangle_5|1\rangle_6 - \mathcal{C}_1|1\rangle_5|0\rangle_6)\}.\end{aligned} \tag{5}$$

We have introduced the complete set of 4-particle eigenstates of the Bell operators $\hat{\mathcal{O}}_k = |\Theta_k^{\pm}\rangle_{1234}\langle\Theta_k^{\pm}|$ ($\sum_k \hat{\mathcal{O}}_k = 1$), defined by

$$|\Theta_k^{\pm}\rangle_{1234} = \frac{1}{\sqrt{2}}(|\mathrm{bin}(k)\rangle_{1234} \pm |\mathrm{bin}(15-k)\rangle_{1234}),\quad k = 0, 1, \ldots, 15, \tag{6}$$

where $\mathrm{bin}(k)$ refers to the 4-bit binary representation of the integer k. For the present teleportation protocol we are concerned only with four states out of the complete Bell basis $|\Theta_k^{\pm}\rangle_{1234}$, those employed for the expansion in (5):

$$|\Psi^{\pm}\rangle_{1234} = |\Theta_6^{\pm}\rangle_{1234} = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4 \pm |1\rangle_1|0\rangle_2|0\rangle_3|1\rangle_4), \tag{7a}$$

$$|\Phi^{\pm}\rangle_{1234} = |\Theta_5^{\pm}\rangle_{1234} = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2|0\rangle_3|1\rangle_4 \pm |1\rangle_1|0\rangle_2|1\rangle_3|0\rangle_4). \tag{7b}$$

Therefore, measurements on the output fields 1–4, yielding the equally likely Bell state outcomes in equations (7*a*) and (7*b*), project the output modes 5 and 6 on the entangled states described in equation (5). This required joint measurement can be accomplished by the Bell-state analyser comprised by Alice's station.

From equations (1*a*), (1*b*) and (2*a*), (2*b*), it follows that a measurement through detectors D_i ($i = 1$–4), of the output state $|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4$, which requires the incoming Bell state $|\Psi^+\rangle_{1234}$, projects the modes 5 and 6 entering Bob's station exactly on to the original entangled state injected through channels 1 and 2. After knowing the result of this measurement, communicated by Alice via the classical channel depicted in figure 1(*a*), Bob does not need to do anything further to produce a replica of the state $|\Omega\rangle_{12}$. On the other hand, a joint measurement of the Bell state $|\Psi^-\rangle_{1234}$ is achieved by measuring the output state $|1\rangle_1|0\rangle_2|1\rangle_3|0\rangle_4$, leaving the input modes at Bob's laboratory in the entanglement $\mathcal{C}_1|0\rangle_5|1\rangle_6 - \mathcal{C}_2|1\rangle_5|0\rangle_6$. In this case, an appropriate unitary transformation has to be performed by Bob in order to convert this entanglement into a replica of the original state $|\Omega\rangle_{12}$. Such a unitary transformation is accomplished through the application of the operator $\hat{\sigma}_z$, which refers to the Pauli matrix in the basis $\{|0,1\rangle_{5,6}, |1,0\rangle_{5,6}\}$.

Regarding the remaining Bell-type measurements, the result $|\Phi^+\rangle_{1234}$ ($|\Phi^-\rangle_{1234}$) is achieved by measuring the output state $|1\rangle_1|0\rangle_2|0\rangle_3|1\rangle_4$ ($|0\rangle_1|1\rangle_2|0\rangle_3|1\rangle_4$), leaving the input modes at Bob's laboratory in the entanglement $\mathcal{C}_2|0\rangle_5|1\rangle_6 + \mathcal{C}_1|1\rangle_5|0\rangle_6$ ($\mathcal{C}_2|0\rangle_5|1\rangle_6 - \mathcal{C}_1|1\rangle_5|0\rangle_6$). Here, the required unitary transformation corresponds to applying the Pauli matrix $\hat{\sigma}_x$ ($\hat{\sigma}_y$) in the basis $\{|0,1\rangle_{5,6}, |1,0\rangle_{5,6}\}$. The unitary operations $\hat{\sigma}_k$ ($k = z, x, y$), can be implemented through the quantum resources in Bob's station, sketched in figure 1(*d*), via appropriate choices of the phase shifts ϕ_1, ϕ_2 and ϕ_3 introduced by the phase plates. The outcomes of the photodetections in Alice's station associated with the four Bell states in equations (7*a*) and (7*b*) are summarized in table 1, along with the values of the phase shifts required to achieve the rotations $\hat{\sigma}_k$ ($k = z, x, y$) in Bob's station. As in the preparation of the quantum channel, the interaction parameter in the KM in Alice's station has been adjusted so that $\chi\tau = \pi$ [21, 22]. We finally note, as far as the Bell-type measurements are concerned, that in a quantum circuit Bob's rotations must be automatically implemented after the information on the photodetections, received through classical bits.

In table 1 we summarize the Bell states, in the first column, projected by the four possible photodetections arranged in the second column. The 180° rotations around the z, x and y axes, required to convert the resulting entangled modes 5 and 6 into a

Table 1. In this table we present the Bell states, the corresponding photodetections and the rotations required to convert the teleported entanglement into a replica of the original state.

Bell state	Photodetection	Rotations	Phase shifts
$\|\Psi^+\rangle_{1234}$	$\|0\rangle_1\|1\rangle_2\|1\rangle_3\|0\rangle_4$	**1**	—
$\|\Psi^-\rangle_{1234}$	$\|1\rangle_1\|0\rangle_2\|1\rangle_3\|0\rangle_4$	$\hat{\sigma}_z$	$\phi_1 = \phi_2 = \phi_3 = \pi$
$\|\Phi^+\rangle_{1234}$	$\|1\rangle_1\|0\rangle_2\|0\rangle_3\|1\rangle_4$	$\hat{\sigma}_x$	$\phi_1 = 0,\ \phi_2 = \phi_3 = 3\pi/2$
$\|\Phi^-\rangle_{1234}$	$\|0\rangle_1\|1\rangle_2\|0\rangle_3\|1\rangle_4$	$\hat{\sigma}_y$	$\phi_1 = \phi_3 = 0, \phi_2 = \pi$

replica of the original state $|\Omega\rangle_{12}$, are given in the third column. The Pauli matrices $\hat{\sigma}_k$ ($k = z, x, y$) are expressed in the basis $\{|0,1\rangle_{5,6}, |1,0\rangle_{5,6}\}$. Finally, in the fourth column we show the phase shifts ϕ_1, ϕ_2 and ϕ_3 introduced by the phase plates so as to achieve the required rotations.

Therefore, we have demonstrated that the proposed scheme for teleporting an entangled state allows a 100% probability of success in the ideal case, where only the four Bell states presented in equations (7*a*) and (7*b*) occur, out of the sixteen composing the Bell basis described in equation (6).

3. Losses in the optical elements

3.1. Absorptive beam splitters

When the errors due to photoabsorption in the BSs are taken into account, the relationships between the input and output operators $\hat{\alpha}$, $\hat{\beta}$ (see figure 2(*a*)), described in equations (1*a*) and (1*b*) are generalized to [33]

$$\hat{\alpha}_{out} = T\hat{\alpha}_{in} + R\hat{\beta}_{in} + \hat{\mathcal{L}}_\alpha, \tag{8a}$$

$$\hat{\beta}_{out} = T\hat{\beta}_{in} + R\hat{\alpha}_{in} + \hat{\mathcal{L}}_\beta, \tag{8b}$$

in order to account for the Langevin noise operator $\hat{\mathcal{L}}$ associated with fluctuating currents within the medium composing the BS. The transmission and reflection coefficients for an absorptive BS generalize those introduced in equations (1*a*) and (1*b*) as $T = \sqrt{\kappa}t$, $R = \sqrt{\kappa}r$, such that $|T|^2 + |R|^2 = \kappa$, a constant indicating the probability of nonabsorption, a kind of quality factor for a BS. The input fields and the noise sources are required to be independent, so the input operators must commute with the Langevin operators

$$[\hat{\alpha}_{in}, \hat{\mathcal{L}}_\alpha] = [\hat{\alpha}_{in}, \hat{\mathcal{L}}_\beta] = [\hat{\alpha}_{in}, \hat{\mathcal{L}}_\alpha^\dagger] = [\hat{\alpha}_{in}, \hat{\mathcal{L}}_\beta^\dagger] = 0, \tag{9}$$

with similar relations for the operators $\hat{\beta}$. Imposition of the bosonic commutation relations on the output mode operators leads to the requirements on the commutation relations for the Langevin operators

$$[\hat{\mathcal{L}}_\alpha, \hat{\mathcal{L}}_\alpha^\dagger] = [\hat{\mathcal{L}}_\beta, \hat{\mathcal{L}}_\beta^\dagger] = \Gamma, \tag{10a}$$

$$[\hat{\mathcal{L}}_\alpha, \hat{\mathcal{L}}_\beta^\dagger] = [\hat{\mathcal{L}}_\beta, \hat{\mathcal{L}}_\alpha^\dagger] = -\Delta, \tag{10b}$$

where $\Gamma = 1 - \kappa$ is the damping constant and $\Delta = TR^* + RT^* = 0$. We note that Δ can assume nonzero values only for an asymmetric BS. At optical frequencies the state of the environment can be very well approximated by the vacuum state even at room temperature, so that

$$\hat{\mathcal{L}}_\alpha|0\rangle = \hat{\mathcal{L}}_\beta|0\rangle = \hat{\alpha}_{in}|0\rangle = \hat{\beta}_{in}|0\rangle = 0, \tag{11}$$

and, from the input–output relations (8*a*) and (8*b*), it also follows that

$$\hat{\alpha}_{out}|0\rangle = \hat{\beta}_{out}|0\rangle = 0. \tag{12}$$

We note that in the above relations $|0\rangle$ represents the vacuum state for modes α, β and their respective environments. Finally, the quantum averages of the Langevin operators vanish,

$$\langle\hat{\mathcal{L}}_\alpha\rangle = \langle\hat{\mathcal{L}}_\beta\rangle = \langle\hat{\mathcal{L}}_\alpha^\dagger\rangle = \langle\hat{\mathcal{L}}_\beta^\dagger\rangle = 0, \tag{13}$$

and the ground-state expectation values for the products of pairs of noise operators are

$$\langle\hat{\mathcal{L}}_\alpha\hat{\mathcal{L}}_\alpha^\dagger\rangle = \langle\hat{\mathcal{L}}_\beta\hat{\mathcal{L}}_\beta^\dagger\rangle = \Gamma, \tag{14a}$$

$$\langle\hat{\mathcal{L}}_\alpha\hat{\mathcal{L}}_\beta^\dagger\rangle = \langle\hat{\mathcal{L}}_\beta\hat{\mathcal{L}}_\alpha^\dagger\rangle = 0. \tag{14b}$$

As noted in [33], the above relations for the averages of the Langevin operators may also be derived from a canonical one-dimensional theory applied to a dielectric slab.

Next, it is readily shown that, similarly to the relations (8*a*) and (8*b*), the transformations relating the output to the input operators, preserving the above-mentioned properties for the Langevin operators, are

$$\hat{\alpha}_{in} = T^*\hat{\alpha}_{out} + R^*\hat{\beta}_{out} + \hat{\mathcal{L}}_\alpha, \tag{15a}$$

$$\hat{\beta}_{in} = T^*\hat{\beta}_{out} + R^*\hat{\alpha}_{out} + \hat{\mathcal{L}}_\beta, \tag{15b}$$

where the bosonic commutation relations are satisfied by the input mode operators. From equations (15*a*) and (15*b*), the output state arising from the injection of a photon through mode α of a beam splitter is given by

$$|1,0\rangle_{\alpha\beta}^{in} = \hat{\alpha}_{in}^\dagger|0,0\rangle_{\alpha\beta}^{in}$$
$$= [T|1,0\rangle_{\alpha\beta}^{out} + R|0,1\rangle_{\alpha\beta}^{out} + \hat{\mathcal{L}}_\alpha^\dagger|0,0\rangle_{\alpha\beta}^{out}]|\mathbf{0}\rangle_{\boldsymbol{E}}, \tag{16}$$

where $|\mathbf{0}\rangle_{\boldsymbol{E}} = \prod_k |0\rangle_k = |\{0_k\}\rangle$ stands for the state of the environment composed of a huge number of vacuum-field states $|0\rangle_k$.

3.2. Absorptive Kerr medium

To deal with photoabsorption in a KM we again take advantage of the Langevin operators. Similarly to the procedure adopted above, to introduce photoabsorption into a BS, the coupling between the input and output operators $\hat{\alpha}$ and $\hat{\beta}$ (see figure 2(b)), described in an ideal KM by equations (2*a*) and (2*b*), is generalized in a lossy KM to

$$\hat{\alpha}_{out} = \sqrt{\eta}\exp(\mathrm{i}\chi\tau\hat{\beta}_{in}^\dagger\hat{\beta}_{in})\hat{\alpha}_{in} + \hat{\mathfrak{L}}_\alpha, \tag{17a}$$

$$\hat{\beta}_{out} = \sqrt{\eta}\exp(\mathrm{i}\chi\tau\hat{\alpha}_{in}^\dagger\hat{\alpha}_{in})\hat{\beta}_{in} + \hat{\mathfrak{L}}_\beta, \tag{17b}$$

where $\Lambda = 1 - \eta$ is the probability of photoabsorption, the damping constant for the KM. (We use different characters for the Langevin operators, to differentiate the particular optical element responsible for the photoabsorption.)

The algebraic rules satisfied by the input and output operators $\hat{\alpha}$, $\hat{\beta}$ and the Langevin operators are similar to those for the BS. Assuming the commutation of the Langevin operators with the input operators $\hat{\alpha}_{in}$, $\hat{\beta}_{in}$, and imposing bosonic commutation rules on the output mode operators $\hat{\alpha}_{out}$, $\hat{\beta}_{out}$, it follows that

$$[\hat{\mathfrak{L}}_\alpha, \hat{\mathfrak{L}}_\alpha^\dagger] = [\hat{\mathfrak{L}}_\beta, \hat{\mathfrak{L}}_\beta^\dagger] = \Lambda, \tag{18a}$$

$$[\hat{\mathfrak{L}}_\alpha, \hat{\mathfrak{L}}_\beta^\dagger] = [\hat{\mathfrak{L}}_\beta, \hat{\mathfrak{L}}_\alpha^\dagger] = 0. \tag{18b}$$

Approximating the state of the environment by the vacuum state, we obtain relations analogous to those in equations (11)–(13) for the KM operators. The ground-state expectation values for the products of pairs of noise operators are thus

$$\langle \hat{\mathfrak{L}}_\alpha \hat{\mathfrak{L}}_\alpha^\dagger \rangle = \langle \hat{\mathfrak{L}}_\beta \hat{\mathfrak{L}}_\beta^\dagger \rangle = \Lambda, \tag{19a}$$

$$\langle \hat{\mathfrak{L}}_\alpha \hat{\mathfrak{L}}_\beta^\dagger \rangle = \langle \hat{\mathfrak{L}}_\beta \hat{\mathfrak{L}}_\alpha^\dagger \rangle = 0. \tag{19b}$$

The transformations relating the output to the input operators, preserving all the above-mentioned properties, are

$$\hat{\alpha}_{in} = \sqrt{\eta}\exp(-\mathrm{i}\chi\tau\hat{\beta}_{out}^\dagger\hat{\beta}_{out})\hat{\alpha}_{out} + \hat{\mathfrak{L}}_\alpha, \tag{20a}$$

$$\hat{\beta}_{in} = \sqrt{\eta}\exp(-\mathrm{i}\chi\tau\hat{\alpha}_{out}^\dagger\hat{\alpha}_{out})\hat{\beta}_{out} + \hat{\mathfrak{L}}_\beta. \tag{20b}$$

When computing the output state arising from the injection of two photons through modes α and β of a KM, we obtain from equations (20a) and (20b)

$$\begin{aligned} |1,1\rangle_{\alpha\beta}^{in} &= \hat{\alpha}_{in}^\dagger\hat{\beta}_{in}^\dagger|0,0\rangle_{\alpha\beta}^{in} \\ &= [\eta\mathrm{e}^{-\mathrm{i}\chi\tau}|1,1\rangle_{\alpha\beta}^{out} + \sqrt{\eta}|0,1\rangle_{\alpha\beta}^{out}\hat{\mathfrak{L}}_\alpha^\dagger \\ &\quad + \sqrt{\eta}|1,0\rangle_{\alpha\beta}^{out}\hat{\mathfrak{L}}_\beta^\dagger + |0,0\rangle_{\alpha\beta}^{out}\hat{\mathfrak{L}}_\alpha^\dagger\hat{\mathfrak{L}}_\beta^\dagger]|0\rangle_{\boldsymbol{E}}, \end{aligned} \tag{21}$$

where $|0\rangle_{\boldsymbol{E}}$ denotes the initial state of the environment. In this equation, note that when considering the ideal case, $\eta = 1$, we correctly obtain $|1,1\rangle_{\alpha\beta}^{in} = \mathrm{e}^{-\mathrm{i}\chi\tau}|1,1\rangle_{\alpha\beta}^{out}$. In an absorptive KM, we find that the expected value for the states $|0,1\rangle_{\alpha\beta}^{out}$ or $|1,0\rangle_{\alpha\beta}^{out}$ is correctly given by $\eta(1-\eta)$, while the probability for the absorption of both photons is $(1-\eta)^2$. We stress that a detailed treatment of dissipation in a KM should consider the time intervals, between zero and τ (the interaction time within an ideal KM), for the absorption of photons in modes α and β; as a consequence, additional phase factors would appear in equation (21). However, the above treatment is sufficient for providing a good estimate about the fidelity of the proposed teleportation process.

3.3. Efficiency of the detectors

Introducing output operators to account for the detection of a given input field α reaching the detectors, we have

$$\hat{\alpha}_{out} = \sqrt{\varepsilon}\hat{\alpha}_{in} + \hat{\mathsf{L}}_\alpha, \tag{22}$$

where ε stands for the efficiency of the detector. Obviously, differently from the case of the BSs and KMs, the detectors do not couple different modes. The Langevin operators $\hat{\mathsf{L}}_\alpha$, besides satisfying all the properties of those introduced above, obey the commutation relations

$$[\hat{\mathsf{L}}_\alpha, \hat{\mathsf{L}}_\alpha^\dagger] = 1 - \varepsilon, \tag{23a}$$

$$[\hat{\mathsf{L}}_\alpha, \hat{\mathsf{L}}_\beta^\dagger] = 0, \tag{23b}$$

and the ground-state expectation values for the products of pairs are

$$\langle \hat{\mathsf{L}}_\alpha \hat{\mathsf{L}}_\alpha^\dagger \rangle = 1 - \varepsilon, \tag{24a}$$

$$\langle \hat{\mathsf{L}}_\alpha \hat{\mathsf{L}}_\beta^\dagger \rangle = 0. \tag{24b}$$

3.4. Absorptive phase plates

The photoabsorption in the phase plates is modelled by analogy with the above treatment for the efficiency of the detectors. Introducing an output operator for the α mode of a phase plate with damping constant $1 - \varkappa$, it follows that

$$\hat{\alpha}_{out} = \sqrt{\varkappa}\hat{\alpha}_{in} + \hat{\mathbb{L}}_\alpha, \tag{25}$$

with the Langevin operator $\hat{\mathbb{L}}_\alpha$ obeying similar relations to those in equations (23a), (23b) and (24a), (24b).

3.5. General relations for the errors due to beam splitters and detectors

For the sake of generality, we next introduce relations accounting for both sources of error: photoabsorption in the BSs (equations (8a) and (8b)) and the efficiency of detectors (equations (22)). One can prove that in this formulation the output operators $\hat{\alpha}$, $\hat{\beta}$, which describe the output fields from BS_5 and BS_6 reaching the detectors, are

$$\hat{\alpha}_{out} = \boldsymbol{T}\hat{\alpha}_{in} + \boldsymbol{R}\hat{\beta}_{in} + \hat{\boldsymbol{L}}_\alpha, \tag{26a}$$

$$\hat{\beta}_{out} = \boldsymbol{T}\hat{\beta}_{in} + \boldsymbol{R}\hat{\alpha}_{in} + \hat{\boldsymbol{L}}_\beta, \tag{26b}$$

where $\boldsymbol{T} = \sqrt{\varepsilon}T$, $\boldsymbol{R} = \sqrt{\varepsilon}R$, and $\hat{\boldsymbol{L}}_\alpha = \hat{\mathcal{L}}_\alpha + \hat{\mathfrak{L}}_\alpha$. In fact, combining all the above-mentioned properties of the operators in relations (26a) and (26b), we obtain

$$[\hat{\boldsymbol{L}}_\alpha, \hat{\boldsymbol{L}}_\alpha^\dagger] = [\hat{\boldsymbol{L}}_\beta, \hat{\boldsymbol{L}}_\beta^\dagger] = \varepsilon\Gamma + (1-\varepsilon), \tag{27a}$$

$$[\hat{\boldsymbol{L}}_\alpha, \hat{\boldsymbol{L}}_\beta^\dagger] = [\hat{\boldsymbol{L}}_\beta, \hat{\boldsymbol{L}}_\alpha^\dagger] = 0. \tag{27b}$$

When substituting $\eta = 1$ in (27a) and (27b), we recover the relations (10a) and (10b), while for $\Gamma = 0$, we recover the relations (23a) and (23b) respectively.

4. Teleportation of an entangled state: noise effects

In calculating the fidelity of the teleportation process, we assume that the state to be teleported, $|\Omega\rangle_{12}$, is prepared ideally, with fidelity equal to unity. However, by taking into account the damping constants of the three BSs and the KM involved in preparing the quantum channel, as depicted in figure 1(b), we obtain the nonideal entanglement

$$\begin{aligned} |\tilde{\Upsilon}\rangle_{3456} = \{&\xi^{3/2}[(1-\eta^{1/2})|1\rangle_3|0\rangle_4 \\ &+ \mathrm{i}\eta^{1/2}(1+\eta^{1/2})|0\rangle_3|1\rangle_4]|1\rangle_5|0\rangle_6 \\ &+ \xi^{3/2}[\mathrm{i}(1+\eta^{1/2})|1\rangle_3|0\rangle_4 \\ &- \eta^{1/2}(1-\eta^{1/2})|0\rangle_3|1\rangle_4]|0\rangle_5|1\rangle_6 \\ &+ \xi^{1/2}[(\hat{\mathcal{L}}_5^{(2)\dagger} + \mathrm{i}\xi^{1/2}\eta^{1/2}\hat{\mathcal{L}}_5^{(3)\dagger} \\ &+ \mathrm{i}\xi^{1/2}\hat{\mathfrak{L}}_5^\dagger + \xi^{1/2}\hat{\mathcal{L}}_6^{(3)\dagger})|1\rangle_3|0\rangle_4 \\ &+ \mathrm{i}\eta^{1/2}(\hat{\mathcal{L}}_5^{(2)\dagger} - \mathrm{i}\xi^{1/2}\eta^{1/2}\hat{\mathcal{L}}_5^{(3)\dagger} \\ &+ \mathrm{i}\xi^{1/2}\hat{\mathfrak{L}}_5^\dagger + \xi^{1/2}\hat{\mathcal{L}}_6^{(3)\dagger})|0\rangle_3|1\rangle_4]|0\rangle_5|0\rangle_6 \\ &+ |\vartheta\rangle_{3456}\}|\mathbf{0}\rangle_{\boldsymbol{E}}, \end{aligned} \tag{28}$$

where $\xi = \kappa/2$ and the superscript ℓ of the Langevin operators $\hat{\mathcal{L}}^{(\ell)}$ refers to the ℓth beam splitter, and thus the ℓth environment where the photon has been absorbed, so that $[\hat{\mathcal{L}}_{\alpha}^{(\ell)}, \hat{\mathcal{L}}_{\alpha}^{(\ell')\dagger}] = \delta_{\ell\ell'}\Gamma$. Note that $|\mathbf{0}\rangle_{\boldsymbol{E}}$ is the product of all the environments, referred to each BS and KM_1, i.e. $|\mathbf{0}\rangle_{\boldsymbol{E}} = |\{0_k\}\rangle_{BS_1}|\{0_k\}\rangle_{BS_2}|\{0_k\}\rangle_{BS_3}|\{0_k\}\rangle_{KM_1}$. There is no need to introduce a superscript to label the Langevin operator accounting for the absorptive KM, since they can be differentiated only through their respective modes. The ket $|\vartheta\rangle_{3456}$, which involves only zero-photon states in modes 3 and 4, is expanded in appendix A.

The use of four photodetectors to achieve the Bell measurement enables us to probe the occurrence of photoabsorptions in the optical elements placed between the input and output channels 1–4. Since one photon is injected through mode 1 or 2, composing the state to be teleported, and another is injected through mode 3 or 4, composing part of the quantum channel, Alice communicates to Bob only the successful events, where the former photon is detected through D_1 or D_2 and the latter through D_3 or D_4. As far as the photon injected through modes 3 or 4 is concerned, these successful events include only the states explicitly shown in superposition (28), since the remaining ket $|\vartheta\rangle_{3456}$ contains only zero-photon states in modes 3 and 4. Evidently, when one of the photons (from the channel couples 1–2 or 3–4) or both of them have been absorbed or scattered by the optical elements, that event is disregarded and the teleportation process must be restarted. This strategy, in association with the usually small damping constants characterizing BSs and KMs, results in a high-fidelity teleportation process. Such fidelity is not affected by the efficiency of the photodetectors, which plays a role only in the probability of the Bell state measurements: in the real situation, each of the four measurement outcomes occurs with probability smaller then 1/4, as discussed below. We note that the above strategy does not rule out the error coming from dark counts in the detectors, which can be treated according to [34]. However, the error due to dark counts will be effective only in two cases: (i) when a photon from the channel couples 1–2 or 3–4 is absorbed and a dark count occurs simultaneously with the detection of the remaining nonabsorbed photon and (ii) when both photons from the channel couples 1–2 and 3–4 are absorbed and two dark counts occur simultaneously in the detector couples D_1–D_2 and D_3–D_4. All other possibilities for dark counts are excluded by the above strategy, resulting in an rather small error due to false signals.

Starting with the fidelity of the nonideal quantum channel in equation (28), i.e. the fidelity of the reduced density operator $\mathrm{Tr}_{\boldsymbol{E}}(|\tilde{\Upsilon}\rangle\langle\tilde{\Upsilon}|)$ relative to the ideal quantum channel $|\Upsilon\rangle$, we obtain the expression

$$\mathcal{F} = \langle\Upsilon|\,\mathrm{Tr}_{\boldsymbol{E}}(|\tilde{\Upsilon}\rangle\langle\tilde{\Upsilon}|)|\Upsilon\rangle = \frac{\xi^3}{2}[1 + 4\eta^{1/2}(1+\eta) + \eta(6+\eta)]. \tag{29}$$

As expected, when disregarding the losses in BSs 1, 2 and 3 ($\kappa = 2\xi = 1$) and in the KM_1 ($\eta = 1$) we find $\mathcal{F} = 1$.

We now consider, for simplicity, the realistic situation where the photon states $|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4$ are detected, corresponding to Alice's measurement, in the ideal case, of the Bell state $|\Psi^+\rangle_{1234}$. As mentioned above, the measurement of the output state $|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4$ projects the modes 5 and 6 entering Bob's station exactly on to the original entangled state $|\Omega\rangle_{12}$, so that Bob need not do anything further to obtain a replica of this state in the ideal case. Assuming realistic nonideal optical elements, the output modes 5 and 6 in Alice's station, before the photodetection, read

$$\begin{aligned}|\psi\rangle_{56} &= \mathcal{N}[\boldsymbol{a}|0\rangle_5|1\rangle_6 + \boldsymbol{b}|1\rangle_5|0\rangle_6 \\ &\quad + (\boldsymbol{c}\hat{\mathcal{L}}_5^{(2)\dagger} + \boldsymbol{d}\hat{\mathcal{L}}_5^{(3)\dagger} + \boldsymbol{e}\hat{\mathfrak{L}}_5^{\dagger} + \boldsymbol{f}\hat{\mathcal{L}}_6^{(3)\dagger})|0\rangle_5|0\rangle_6]|\mathbf{0}\rangle_{\boldsymbol{E}},\end{aligned} \tag{30}$$

where the normalization constant is

$$\begin{aligned}\mathcal{N} &= [|\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 + (1-2\xi)(|\boldsymbol{c}|^2 + |\boldsymbol{d}|^2 \\ &\quad + |\boldsymbol{f}|^2) + |\boldsymbol{e}|^2(1-\eta)]^{-1/2}.\end{aligned} \tag{31}$$

Therefore, the probability of detecting the state $|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4$, given by $\mathsf{P}_{0110} = \mathcal{N}^{-2}\varepsilon^2$ (as can be obtained from the evolution of the state $|\Omega\rangle_{12} \otimes |\tilde{\Upsilon}\rangle_{3456}$ through Alice's station), depends on the efficiency of the photodetectors and turns out to be 1/4 when assuming ideal BSs ($\kappa = 2\xi = 1$), KMs ($\eta = 1$) and photodetectors ($\varepsilon = 1$). In fact, for the coefficients in appendix B, we obtain $\boldsymbol{a} = -\mathcal{C}_1/2$ and $\boldsymbol{b} = -\mathcal{C}_2/2$, so that $\mathcal{N} = [|\boldsymbol{a}|^2 + |\boldsymbol{b}|^2]^{-1/2} = 2$.

The coefficients $\boldsymbol{a}$ to $\boldsymbol{f}$ are displayed in appendix B. From equation (30), the reduced density operator can be obtained:

$$\begin{aligned}\hat{\rho}_{56} &= \mathrm{Tr}_{\boldsymbol{E}}\,|\psi\rangle_{56}\langle\psi| \\ &= \mathcal{N}\{|\boldsymbol{a}|^2|0,1\rangle_{56}\langle 0,1| \\ &\quad + \boldsymbol{a}\boldsymbol{b}^*|0,1\rangle_{56}\langle 1,0| + \boldsymbol{a}^*\boldsymbol{b}|1,0\rangle_{56}\langle 0,1| \\ &\quad + |\boldsymbol{b}|^2|1,0\rangle_{56}\langle 1,0| + [(1-2\xi)(|\boldsymbol{c}|^2 + |\boldsymbol{d}|^2 + |\boldsymbol{f}|^2) \\ &\quad + |\boldsymbol{e}|^2(1-\eta)]|0,0\rangle_{56}\langle 0,0|\}.\end{aligned} \tag{32}$$

Finally, the fidelity of the teleportation process, the overlap between the ideal state $|\Omega\rangle_{56} = \mathcal{C}_1|0\rangle_5|1\rangle_6 + \mathcal{C}_2|1\rangle_5|0\rangle_6$ and the nonideal teleported state in equation (30), is given by

$$\begin{aligned}\mathsf{F} &= {}_{56}\langle\Omega|\hat{\rho}_{56}|\Omega\rangle_{56} \\ &= \mathcal{N}^2(|\boldsymbol{a}|^2|\mathcal{C}_1|^2 + \boldsymbol{a}\boldsymbol{b}^*\mathcal{C}_1^*\mathcal{C}_2 + \boldsymbol{a}^*\boldsymbol{b}\mathcal{C}_1\mathcal{C}_2^* + |\boldsymbol{b}|^2|\mathcal{C}_2|^2).\end{aligned} \tag{33}$$

It follows immediately that, for the ideal case where $\kappa = \eta = \varepsilon = 2\xi = 1$, so that $\boldsymbol{a} = -\mathcal{C}_1/2$, $\boldsymbol{b} = -\mathcal{C}_2/2$ and $\mathcal{N} = 2$, we obtain $\mathsf{F} = (|\mathcal{C}_1|^2 + |\mathcal{C}_2|^2)^2 = 1$. Remember that the function F given here is the fidelity of the teleported state associated with Alice's measurement of the Bell state $|\Psi^+\rangle_{1234}$. We note that the normalization factor $\mathcal{N}$, and so the fidelity F, would be considerably smaller when considering events where no photon is detected in either of the channel couples 1–2 or 3–4.

Evidently, different expressions for the fidelity of the teleported state follow from different results of the Bell measurement. In fact, Bob's intervention on the state entering his station, after Alice's measurement, must be specific according to table 1, and the more the optical elements used to accomplish the rotation required to convert the teleported state into a replica of the original entanglement $|\Omega\rangle_{12}$, the more the errors introduced in the teleported state. In this connection, we expect an increasing fidelity for the teleported entanglement, when detecting the Bell states $|\Psi^-\rangle_{1234}$, $|\Phi^+\rangle_{1234}$, $|\Phi^-\rangle_{1234}$, $|\Psi^+\rangle_{1234}$, in that order.

An important point to be stressed is that the strategy for probing the absorption of photons between the inputs and outputs of channels 1–4 is insufficient to eliminate the introduction of errors by the optical elements between these channels. In fact, even when both photons injected through modes 1 or 2 and 3 or 4 are not absorbed, the errors introduced by the optical elements will invalidate table 1, in the sense that the measurement of the state $|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4$, for example, turns out to be associated not only with the Bell state $|\Psi^+\rangle_{1234}$ (as in the ideal case), but also to the three other Bell states, although with smaller probabilities. In appendix C we present the evolution of the Bell states $|\Psi^\pm\rangle_{1234}$ (appearing in the expansion of the product $|\Omega\rangle_{12} \otimes |\tilde{\Upsilon}\rangle_{3456}$) through Alice's station, showing the mixing of all the possibilities of photodetections in table 1.

5. Comments and conclusion

In this paper we have presented a scheme for the teleportation of an EPR-type entanglement of zero- and one-photon running-wave states. Besides employing linear optical elements, such as BSs and phase plates, our teleportation apparatus also incorporates KMs to allow a 100% probability of success in the ideal case. A scheme for teleporting similar entanglements of running-wave states, employing only linear optical elements, was recently suggested [19, 20], but the theoretical probability of success was only 50%. Following the lines suggested in [19], the experimental teleportation of an entangled qubit was recently carried out [26]. In this connection, we point out that the Kerr nonlinearity required in our proposal can be achieved with presently available technology, given the recently reported ultra-slow light propagation in cold atomic media [21–23]. Lukin and Imamoglu [21] have demonstrated that a conditional phase shift of the order of π could be achieved if both light pulses, propagating with slow but equal group velocity in a coherently prepared atomic gas, were submitted to electromagnetically induced transparency. It is worth mentioning the increasing research interest in achieving giant crossed-Kerr nonlinearity through ultra-slow light propagation in a cold gas of atoms [21, 23]. Such activity encourages theoretical propositions such as the present one and the recently reported scheme for complete quantum teleportation of the polarization state of a photon, employing Kerr nonlinearity, which also requires a conditional phase shift π [35]. In [22] a scheme is suggested that allows equal, slow group velocities of the interacting photons and therefore a cross-phase shift of π, which cannot be achieved using the scheme in [21] owing to the group velocity mismatch of the two photons.

We have also provided a phenomenological approach to compute the influence of damping in KMs, reasoning by analogy with the treatment of lossy BSs given in [33]. The efficiency of the photodetectors has also been introduced, as well as the influence of damping in phase plates, by making use of the relations previously developed in [17]. Therefore, a comprehensive treatment of errors in the domain of running-wave physics, for linear and nonlinear optical elements, has been presented which allows the fidelity of the teleportation process to be computed (in the particular situation where the result of Alice's Bell-type measurement prevents Bob from the necessity to perform any appropriate unitary transformation on the received state). The strategy employed to carry out the Bell measurement is able to probe the occurrence of photoabsorptions in the optical elements. This strategy, combined with the usually small damping constants of the BSs and KMs and the high efficiency of the photodetectors, results in a high-fidelity teleportation process (as analysed below).

To estimate the fidelities of the prepared quantum channel (equation (29)) and the teleported state (equation (33)), we note that the efficiency of single-photon detectors is about 70%, yielding $\varepsilon \simeq 0.7$, while the damping constant for a BS is rather small, less than 2% in BK7 crystals, given $\kappa \simeq 0.98$. Considering light pulses of tiny energy crossing the KM, the methods proposed in [21, 22] to achieve a nonlinear phase shift of the order of π permits the interaction to be maintained for a very long time without dissipation. Hence, for the KM composed of a cold gas of atoms, the damping constant is also very small, and for the purpose of this example we assume the same value as for the BS, so that $\eta \simeq 0.98$. As stressed above, we have computed in equation (33) the fidelity of the teleported state corresponding to Alice's measurement, in the ideal case, of the Bell state $|\Psi^+\rangle_{1234}$. This choice was motivated only to simplify the calculation of the fidelity since the three other Bell states require appropriate rotations accomplished in *Bob's station* through phase plates and additional BSs. With these considerations we obtain for the fidelity of the nonideal quantum channel $\mathcal{F} = 0.92$. The fidelity F of the teleported state, given by equation (33), is plotted in figure 3 as a function of parameters γ and λ defining the coefficients $\mathcal{C}_1 = \cos(\gamma)$ and $\mathcal{C}_2 = \sin(\gamma)\mathrm{e}^{\mathrm{i}\lambda}$. As is evident from figure 3, the fidelity of the teleported state for $\gamma = 0(\pi)$, when $|\Omega\rangle_{12} = |0\rangle_1|1\rangle_2(-|0\rangle_1|1\rangle_2)$, is smaller than that for $\gamma = \pi/2$, when $|\Omega\rangle_{12} = |1\rangle_1|0\rangle_2$. In fact, when the photon goes through mode 2 ($\gamma = 0, \pi$), it crosses KM_2, introducing errors into the process which do not occur when the photon travels through mode $1(\gamma = \pi/2)$. On the other hand, for a superposition state, i.e. $\gamma \neq 0, \pi$, we note that the phase factor $\mathrm{e}^{\mathrm{i}\lambda}$ plays an important role in the fidelity. Even for $\gamma = \pi/4$, when the photon in the state to be teleported has equal probabilities of travelling through modes 1 or 2, the interference process occurring in BS_6, depending on the phase factor $\mathrm{e}^{\mathrm{i}\lambda}$, ensures different values for the fidelity F. The reason is that the function F gives the fidelity of the teleported state associated with Alice's measurement of the Bell state $|\Psi^+\rangle_{1234}$ and the phase factor $\mathrm{e}^{\mathrm{i}\lambda}$ leads to different probabilities for the output photodetection $|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4$ associated with $|\Psi^+\rangle_{1234}$. As shown in appendix C, the errors introduced by the optical elements mix together the Bell states associated with a given output photodetection and the probabilities of measuring each of the Bell states will depend on λ.

From the values fixed above for the efficiency of the detectors and the probability of nonabsorption of the BSs and the KMs, we can also estimate the probability $\mathsf{P}_{0110} = \mathcal{N}^{-2}\varepsilon^2$ of detecting the output state $|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4$ in Alice's station. When considering $\mathcal{C}_1 = \mathcal{C}_2 = 1/\sqrt{2}$, the probability P_{0110}, which in the ideal case is 0.25, turns to be about 0.11. For the efficiency of the detectors equal to unity, we obtain $\mathsf{P}_{0110} = \mathcal{N}^{-2} \approx 0.22$. Note that the factor ε^2, which reduces the probability P_{0110} from 0.22 to 0.11, follows from the necessity of detecting both photons in modes 2 and 3.

In addition to the above-mentioned errors arising from the absorptive BSs, KMs and phase plates, there are other experimental nonidealities that seem to be important. In fact, the two modes interfering at the BSs are never matched perfectly and the effects of the mode mismatch can be discussed within the multimode theory [36]. Moreover, we have to account for the fact that we do not have perfect single-photon sources. The commonly cited method of parametric

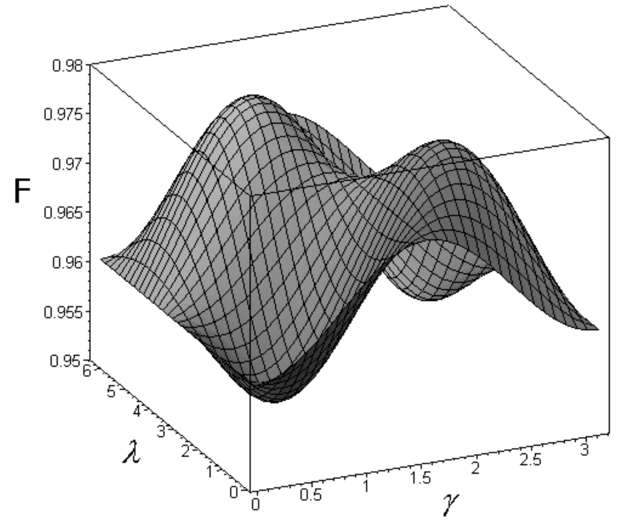

Figure 3. Fidelity F of the teleported state (associated with Alice's measurement of the Bell state $|\Psi^+\rangle_{1234}$), expressed in equation (33), as a function of parameters γ and λ defining the coefficients $\mathcal{C}_1 = \cos(\gamma)$ and $\mathcal{C}_2 = \sin(\gamma)e^{i\lambda}$.

fluorescence only approximates a single-photon source, and this approximation must be evaluated for its effect on the fidelity of the teleported state. The interaction between the two light pulses crossing the KM is also subject to errors due to fluctuations of the associated physical parameters. Finally, it is worth mentioning that the errors arising from fluctuating parameters could be taken into account through the recently proposed phenomenological-operator technique [37].

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Appendix A

The ket $|\vartheta\rangle_{3456}$, in equation (28), involving only zero-photon states in modes 3 and 4 reads

$$\begin{aligned}
|\vartheta\rangle_{3456} = \{&\xi^2[(1-\eta^{1/2}) + i\xi - i\xi\eta^{1/2}\hat{\mathfrak{L}}_4^\dagger]|1\rangle_5|0\rangle_6 \\
&+ i\xi^2[(1+\eta^{1/2}) + \xi + \xi\eta^{1/2}\hat{\mathfrak{L}}_4^\dagger]|0\rangle_5|1\rangle_6 \\
&+ [(\hat{\mathcal{L}}_5^{(2)\dagger} + i\xi\eta^{1/2}\hat{\mathcal{L}}_5^{(3)\dagger} + i\xi\hat{\mathfrak{L}}_5^\dagger + \xi\hat{\mathcal{L}}_6^{(3)\dagger}) \\
&+ i\xi(\hat{\mathcal{L}}_5^{(2)\dagger} + i\xi\eta^{1/2}\hat{\mathcal{L}}_5^{(3)\dagger} + i\xi\hat{\mathfrak{L}}_5^\dagger + \xi\hat{\mathcal{L}}_6^{(3)\dagger})\hat{\mathfrak{L}}_4^\dagger] \\
&\times |0\rangle_5|0\rangle_6\}|0\rangle_3|0\rangle_4|\mathbf{0}\rangle_{\boldsymbol{E}}.
\end{aligned}$$

Appendix B

The coefficients related to the teleported state in equation (30) are given by

$$\begin{aligned}
\boldsymbol{a} &= \xi^3[-\eta^{1/2}(1+\eta^{1/2}+3\eta-\eta^{3/2})\mathcal{C}_1 \\
&\quad + i(1-\eta^{1/2}-\eta+\eta^{3/2})\mathcal{C}_2], \\
\boldsymbol{b} &= \xi^3[i\eta^{1/2}(1-\eta^{1/2}-\eta+\eta^{3/2})\mathcal{C}_1 \\
&\quad + (1-3\eta^{1/2}-\eta-\eta^{3/2})\mathcal{C}_2], \\
\boldsymbol{c} &= \xi^2[i\eta^{1/2}(1+\eta)\mathcal{C}_1 + (1-2\eta^{1/2}-\eta)\mathcal{C}_2], \\
\boldsymbol{d} &= \xi^{5/2}\{-\eta[1+2\eta^{1/2}-\eta]\mathcal{C}_1 + i\eta^{1/2}(1+\eta)\mathcal{C}_2\}, \\
\boldsymbol{e} &= \xi^{5/2}\{-\eta^{1/2}[1+\tfrac{1}{2}\eta^{1/2}-\eta^{1/2}(1-\eta^{1/2})+\eta]\mathcal{C}_1 \\
&\quad + i[1-\tfrac{3}{2}\eta^{1/2}-\eta^{1/2}(1+\eta^{1/2})-\tfrac{1}{2}\eta]\mathcal{C}_2\}, \\
\boldsymbol{f} &= \xi^{5/2}[i\eta^{1/2}(1-\eta)\mathcal{C}_1 + (1-2\eta^{1/2}-\eta)\mathcal{C}_2].
\end{aligned}$$

Appendix C

The evolution of the Bell states $|\Psi^\pm\rangle_{1234}$ (appearing in the expansion of the product $|\Omega\rangle_{12} \otimes |\widetilde{\Upsilon}\rangle_{3456}$) through Alice's station (figure 1(*c*)) is given by

$$\begin{aligned}
|\Psi^\pm\rangle_{1234} &= \frac{1}{\sqrt{2}}(|0,1\rangle_{12}|1,0\rangle_{34} \pm |1,0\rangle_{12}|0,1\rangle_{34}) \\
&\overset{Alice's.station}{\longrightarrow} \frac{1}{\sqrt{2}}\{\xi^{3/2}[(\eta^{1/2}+\eta) \\
&\quad \mp (\eta^{1/2}+1)]|1,0\rangle_{12}|1,0\rangle_{34} + i\xi^{3/2}[(\eta^{1/2}+\eta) \\
&\quad \pm (\eta^{1/2}+1)]|0,1\rangle_{12}|1,0\rangle_{34} + \xi^{3/2}[-(\eta^{1/2}-\eta) \\
&\quad \pm (\eta^{1/2}-1)]|0,1\rangle_{12}|0,1\rangle_{34} + i\xi^{3/2}[(\eta^{1/2}-\eta) \\
&\quad \pm (\eta^{1/2}-1)]|1,0\rangle_{12}|0,1\rangle_{34} + |\vartheta_{\Psi^\pm}\rangle_{1234}\},
\end{aligned}$$

where the ket $|\vartheta_{\Psi^\pm}\rangle_{1234}$, presenting zero-photon states in at least one of the channel couples 1–2 or 3–4, reads

$$\begin{aligned}
|\vartheta_{\Psi^\pm}\rangle_{1234} = \{&\xi[(\eta^{1/2}+\eta)\hat{\mathcal{L}}_2^{(6)\dagger}|1,0\rangle_{34} \\
&+ i(\eta^{1/2}-\eta)\hat{\mathcal{L}}_2^{(6)\dagger}|0,1\rangle_{34} \\
&+ (\hat{\mathfrak{L}}_2^\dagger - \eta^{1/2}\hat{\mathfrak{L}}_2^\dagger)|1,0\rangle_{34} + i(\hat{\mathfrak{L}}_2^\dagger + \eta^{1/2}\hat{\mathfrak{L}}_2^\dagger)|0,1\rangle_{34} \\
&\pm (\eta^{1/2}-1)\hat{\mathcal{L}}_1^{(6)\dagger}|0,1\rangle_{34} \pm i(\eta^{1/2}+1)\hat{\mathcal{L}}_1^{(6)\dagger} \\
&\times |1,0\rangle_{34}]|0,0\rangle_{12} + [(\xi\eta)^{1/2}(\hat{\mathcal{L}}_3^{(4)\dagger} - i(\xi\eta)^{1/2}\hat{\mathcal{L}}_3^{(5)\dagger} \\
&+ i\xi^{1/2}\hat{\mathfrak{L}}_3^\dagger + \xi^{1/2}\hat{\mathcal{L}}_4^{(5)\dagger})(|1,0\rangle_{12} + i|0,1\rangle_{12}) \\
&\pm \xi^{1/2}((\xi\eta)^{1/2}\hat{\mathcal{L}}_3^{(5)\dagger} + \xi^{1/2}\hat{\mathfrak{L}}_3^\dagger + \hat{\mathcal{L}}_4^{(4)\dagger} \\
&+ i\xi^{1/2}\hat{\mathcal{L}}_4^{(5)\dagger})(|0,1\rangle_{12} + i|1,0\rangle_{12})]|0,0\rangle_{34} \\
&+ [\eta^{1/2}(\hat{\mathcal{L}}_3^{(4)\dagger} - i(\xi\eta)^{1/2}\hat{\mathcal{L}}_3^{(5)\dagger} + i\xi^{1/2}\hat{\mathfrak{L}}_3^\dagger \\
&+ \xi^{1/2}\hat{\mathcal{L}}_4^{(5)\dagger})\hat{\mathcal{L}}_2^{(6)\dagger} + (\hat{\mathcal{L}}_3^{(4)\dagger} + i(\xi\eta)^{1/2}\hat{\mathcal{L}}_3^{(5)\dagger} \\
&+ i\xi^{1/2}\hat{\mathfrak{L}}_3^\dagger + \xi^{1/2}\hat{\mathcal{L}}_4^{(5)\dagger})\hat{\mathfrak{L}}_2^\dagger \\
&\pm ((\xi\eta)^{1/2}\hat{\mathcal{L}}_3^{(5)\dagger} + \xi^{1/2}\hat{\mathfrak{L}}_3^\dagger + \hat{\mathcal{L}}_4^{(4)\dagger} \\
&+ i\xi^{1/2}\hat{\mathcal{L}}_4^{(5)\dagger})\hat{\mathcal{L}}_1^{(6)\dagger}]|0,0\rangle_{12}|0,0\rangle_{34}\}|\mathbf{0}\rangle_{\boldsymbol{E}}.
\end{aligned}$$

For the realistic values $\kappa = \eta = 0.98$ we have

$$\begin{aligned}
|\Psi^+\rangle_{1234} &\overset{Alice's.station}{\longrightarrow} i(0.9604)|0,1\rangle_{12}|1,0\rangle_{34} \\
&\quad - (0.0049)|0,1\rangle_{12}|0,1\rangle_{34} \\
&\quad - (0.0049)|1,0\rangle_{12}|1,0\rangle_{34} + |\vartheta_{\Psi^+}\rangle_{1234}, \\
|\Psi^-\rangle_{1234} &\overset{Alice's.station}{\longrightarrow} (0.9604)|1,0\rangle_{12}|1,0\rangle_{34} \\
&\quad - i(0.0049)|0,1\rangle_{12}|1,0\rangle_{34} \\
&\quad + (0.0049)|1,0\rangle_{12}|0,1\rangle_{34} + |\vartheta_{\Psi^-}\rangle_{1234},
\end{aligned}$$

showing the breakdown of the one-to-one relation between the photodetections and Bell states presented in table 1 for the ideal case. A similar analysis applies to Bell states $|\Phi^\pm\rangle_{1234}$.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] Bohm D and Aharonov Y 1957 *Phys. Rev.* **108** 1070

[3] Bell J S 1964 *Physics* vol 1 (New York: Long Island City) p 195
[4] Aspect A, Grangier P and Roger G 1982 *Phys. Rev. Lett.* **49** 91
Aspect A, Dalibard J and Roger G 1982 *Phys. Rev. Lett.* **49** 1804
Kwiat P G, Mattle K, Weinfurter H, Zeilinger A, Sergienko A and Shih Y 1995 *Phys. Rev. Lett.* **75** 4337
Kuzmich A, Walmsley I A and Mandel L 2000 *Phys. Rev. Lett.* **85** 1349
[5] Kwiat P G, Eberhard P H, Steinberg A M and Chiao R 1994 *Phys. Rev.* A **49** 3209
Fry E S, Walther T and Li S 1995 *Phys. Rev.* A **52** 4381
Monken C H, Souto Ribeiro P H and Padua S 1998 *Phys. Rev.* A **57** R2267
[6] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 *Phys. Rev. Lett.* **78** 3221
Pellizzari T 1997 *Phys. Rev. Lett.* **79** 5242
Briegel H J, Dür W, Cirac J I and Zoller P 1998 *Phys. Rev. Lett.* **81** 5932
van Enk S J, Kimble H J, Cirac J I and Zoller P 1999 *Phys. Rev.* A **59** 2659
[7] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091
Turchette Q A, Hood C J, Lange W, Mabuchi H and Kimble H J 1995 *Phys. Rev. Lett.* **75** 4710
Chuang I L, Vandersypen L M K, Zhou X, Leung D W and Lloyd S 1998 *Nature* **393** 143
Kane B E 1998 *Nature* **393** 133
[8] Shor P 1994 *Proc. 35th Annual Symp. on the Theory of Computer Science* ed S Goldwasser (Los Alamitos, CA: IEEE Computer Society Press) p 124
Shor P 1995 *Preprint* quant-ph/9508027
[9] Vandersypen L M K, Steffen M, Breyta G, Yannoni C S, Sherwood M H and Chuang I L 2001 *Nature* **414** 883
[10] Bennett C H 1995 *Phys. Today* **48** 24
Bennett C H and Divincenzo D P 1995 *Nature* **377** 389
[11] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W 1993 *Phys. Rev. Lett.* **70** 1895
[12] Wootters W K and Zurek W H 1982 *Nature* **299** 802
Dieks D 1982 *Phys. Lett.* A **92** 271
[13] Moussa M Y M 1997 *Phys. Rev.* A **55** R3287
[14] Davidovich L, Zagury N, Brune M, Raimond J M and Haroche S 1994 *Phys. Rev.* A **50** R895
Cirac J I and Parkins A S 1994 *Phys. Rev.* A **50** R4441
de Almeida N G, Maia L P, Villas-Bôas C J and Moussa M H Y 1998 *Phys. Lett.* A **241** 213
de Almeida N G, Napolitano R and Moussa M H Y 2000 *Phys. Rev.* A **62** 010101(R)
[15] Solano E, Cesar C L, de Matos Filho R L and Zagury N 2001 *Eur. Phys. J.* D **13** 121
[16] Braunstein S and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869
[17] Villas-Bôas C J, de Almeida N G and Moussa M H Y 1999 *Phys. Rev.* A **60** 2759
[18] van Loock P and Braunstein S L 2000 *Phys. Rev.* A **61** 010302(R)
[19] Lee H W and Kim J 2000 *Phys. Rev.* A **63** 012305
[20] Lee H W 2001 *Phys. Rev.* A **64** 014302
[21] Lukin M D and Imamoglu A 2000 *Phys. Rev. Lett.* **84** 1419
[22] Petrosyan D and Kurizki G 2002 *Phys. Rev.* A **65** 033833
[23] Arimondo E 1996 *Prog. Opt.* vol 35, ed E Wolf (Amsterdan: Elsevier)
Schmidt H and Imamoglu A 1996 *Opt. Lett.* **21** 1936
Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 *Nature* **397** 594
Kash M M, Sautenkov V A, Zibrov A S, Hollberg L, Welch G R, Lukin M D, Rostovtsev Y, Fry E S and Scully M O 1999 *Phys. Rev. Lett.* **82** 5229
Harris S E and Hau L V 1999 *Phys. Rev. Lett.* **82** 4611
[24] van Enk S J and Hirota O 2001 *Phys. Rev.* A **64** 022313
[25] Ikram M, Zhu S Y and Zubairy M S 2000 *Phys. Rev.* A **62** 022307
[26] Lombardi E, Sciarrino F, Popescu S and De Martini F 2002 *Phys. Rev. Lett.* **88** 070402
[27] Moussa M H Y 1996 *Phys. Rev.* A **54** 4661
Zubairy M S 1998 *Phys. Rev.* A **58** 4368
Stenholm S and Bardroff P J 1998 *Phys. Rev.* A **58** 4373
[28] Bouwmeester D, Pan J-W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575
[29] Boschi D, Branca S, De Martini F, Hardy L and Popescu S 1998 *Phys. Rev. Lett.* **80** 1121
[30] Popescu S 2002 at press
[31] Kim Y H, Kulik S P and Shih Y 2001 *Phys. Rev. Lett.* **86** 1370
[32] Furasawa A, Sorensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[33] Barnett S, Jeffers J and Gatti A 1998 *Phys. Rev.* A **57** 2134
[34] Özdemir S K, Miranowicz A, Koashi M and Imoto N 2001 *Phys. Rev.* A **64** 063818
Barnett S M, Phillips L S and Pegg D T 1998 *Opt. Commun.* **158** 45
[35] Vitali D, Fortunato M and Tombesi P 2000 *Phys. Rev. Lett.* **85** 445
[36] Banaszek K, Radzewicz C, Wodkiewicz K and Krasinski J S 1999 *Phys. Rev.* A **60** 674
[37] de Almeida N G, Napolitano R and Moussa M H Y 2000 *Phys. Rev.* A **62** 033815
de Almeida N G, Ramos P B, Serra R M and Moussa M H Y 2000 *J. Opt. B: Quantum Semiclass. Opt.* **2** 792
Serra R M, Ramos P B, de Almeida N G, José W D and Moussa M H Y 2001 *Phys. Rev.* A **63** 053813