

# Proposal to produce long-lived mesoscopic superpositions through an atom–driven field interaction

C J Villas-Bôas, F R de Paula, R M Serra and M H Y Moussa

Departamento de Física, Universidade Federal de São Carlos, PO Box 676, São Carlos, 13565-905, SP, Brazil

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## Abstract

We present a proposal for the production of longer-lived mesoscopic superpositions which relies on two requirements: parametric amplification and a squeezed vacuum reservoir for cavity-field states. Our proposal involves the interaction of a two-level atom with a cavity field which is simultaneously subjected to amplification processes.

**Keywords:** Decoherence, squeezed states, squeezed reservoir, mesoscopic states, cavity-field state engineering

The mastery of techniques for preparing cavity-field states through an atom–field interaction in cavity quantum electrodynamics (QED) is crucial to many useful applications in quantum optics. As high- $Q$  cavities have permitted the preparation of cavity-field superpositions of the form  $|\Psi\rangle = (|\alpha e^{i\phi}\rangle + |\alpha e^{-i\phi}\rangle)/\sqrt{2}$ , with a mean number of oscillator quanta  $|\alpha|^2 \approx 10$ , mesoscopic quantum coherence in cavity QED has been investigated: the progressive decoherence of the superposition  $|\Psi\rangle$ , involving radiation fields with classically distinct phases, was observed through the atom–field interaction [1] and the reversible decoherence of such a cavity-field state has been conjectured [2]. In this paper we present a proposal for the achievement of long-lived mesoscopic superposition states in cavity QED which relies on two basic requirements: parametric amplification and an engineered squeezed-vacuum reservoir for cavity-field states. Our proposal considers the dispersive interaction of a two-level atom with a cavity field which is simultaneously undergoing amplification processes. The parametric amplification is employed to achieve the required high degrees of squeezing and excitation of what we actually want to be a mesoscopic superposition state. Such long-lived squeezed-mesoscopic states, under the action of a similarly squeezed reservoir, exhibit a decoherence time orders of magnitude longer than those for non-squeezed (NS) cavity-field states subjected to the influence of (i) a squeezed reservoir and (ii) a NS reservoir.

## Atom–driven field interaction

The proposed configuration for engineering driven cavity-field states, based on the scheme of Brune *et al* [3], consists of a two-

level Rydberg atom which crosses a Ramsey-type arrangement, i.e. a high- $Q$  micromaser cavity  $C$  located between two Ramsey zones. After interacting with this arrangement, the atom is counted by ionization chambers, projecting the cavity-field state in  $C$ . The transition  $|2\rangle \rightarrow |1\rangle$  of the two-level atom (excited state  $|2\rangle$  and ground state  $|1\rangle$ ) is far from resonant with the cavity mode frequency, allowing for a dispersive atom–field interaction. In addition to the interaction with the two-level atom, the cavity mode is simultaneously submitted to linear and parametric amplifications so that the Hamiltonian of our model is given by ( $\hbar = 1$ )

$$H = \omega a^\dagger a + \frac{\omega_0}{2} \sigma_z + \chi a^\dagger a \sigma_z + \zeta(t) a^{\dagger 2} + \zeta^*(t) a^2 + \xi(t) a^\dagger + \xi^*(t) a, \quad (1)$$

where  $\sigma_z = |2\rangle\langle 2| - |1\rangle\langle 1|$ , and  $a$  and  $a^\dagger$  are, respectively, the creation and annihilation operators for the cavity mode of frequency  $\omega$  which lies between the two atomic energy levels, which are separated by  $\omega_0$ , such that the detuning  $\delta = |\omega - \omega_0|$  is large enough (compared to the dipole atom–field coupling  $\Omega$ , i.e.  $\delta \gg \Omega$ ) to enable only virtual transitions to occur between the states  $|1\rangle$  and  $|2\rangle$ . In this regime, the effective atom–field coupling parameter inside the cavity is  $\chi = \Omega^2/\delta$  [4]. We suppose, for simplicity, that the atom–field coupling is turned on (off) suddenly at the instant the atom enters (leaves) the cavity region, such that  $\chi = 0$  when the atom is outside the cavity. The time-dependent (TD) functions  $\zeta(t)$  and  $\xi(t)$  allow for the parametric and linear amplifications, respectively. We consider the atom, prepared at time  $t_0$  by the first Ramsey zone in a  $|1\rangle$ ,  $|2\rangle$  superposition, to reach  $C$  at time  $t_1$  and leave it

at  $t_2$ . The linear and parametric pumping are supposed to be turned on also at  $t_0$  and turned off at a convenient time  $t \geq t_2$ .

The Schrödinger state vector associated with Hamiltonian (1) can be written using

$$|\Psi(t)\rangle = e^{i\omega_0 t/2}|1\rangle|\Phi_1(t)\rangle + e^{-i\omega_0 t/2}|2\rangle|\Phi_2(t)\rangle, \quad (2)$$

where  $|\Phi_\ell(t)\rangle = \int (d^2\alpha/\pi) \mathcal{A}_\ell(\alpha, t)|\alpha\rangle$ ,  $\ell = 1, 2$ , the complex quantity  $\alpha$  standing for the eigenvalues of  $a$ , and  $\mathcal{A}_\ell(\alpha, t) = \langle\alpha, \ell|\Psi(t)\rangle$  are the expansion coefficients for  $|\Phi_\ell(t)\rangle$  in the basis of coherent state,  $\{|\alpha\rangle\}$ . Using the orthogonality of the atomic states and equations (1) and (2) we obtain the uncoupled TD Schrödinger equations

$$i\frac{d}{dt}|\Phi_\ell(t)\rangle = \mathbf{H}_\ell|\Phi_\ell(t)\rangle, \quad (3)$$

$$\mathbf{H}_\ell = \omega_\ell(t)a^\dagger a + \zeta(t)a^{\dagger 2} + \zeta^*(t)a^2 + \xi(t)a^\dagger + \xi^*(t)a, \quad (4)$$

with  $\omega_\ell(t) = [\omega + (-1)^\ell \chi]$ . Note that the problem has been reduced to that of a cavity field under parametric and linear amplifications, whose frequency  $\omega$  is shifted by  $-\chi$  ( $+\chi$ ) when interacting with the atomic state  $|1\rangle$  ( $|2\rangle$ ), during the time interval  $\tau = t_2 - t_1$ .

#### Time-dependent invariants

To solve the Schrödinger equation (3) we employ the TD invariants of Lewis and Riesenfeld [5]. However, instead of proposing an invariant associated with the Hamiltonian (4), we first perform a unitary transformation on equation (3) in order to reduce it to a form which already has an associated known invariant. Thus, under a unitary transformation represented by the operator  $S(\varepsilon_\ell)$  ( $\varepsilon_\ell$  standing for a set of TD group parameters which may also depend on the atomic state  $\ell$ ), we obtain from equation (3)

$$i\frac{d}{dt}|\Phi_\ell^S(t)\rangle = \mathcal{H}_\ell^S|\Phi_\ell^S(t)\rangle, \quad (5)$$

where the transformed Hamiltonian and wavevector are given by

$$\mathcal{H}_\ell^S = S^\dagger(\varepsilon_\ell)\mathcal{H}_\ell S(\varepsilon_\ell) + i\frac{dS^\dagger(\varepsilon_\ell)}{dt}S(\varepsilon_\ell), \quad (6a)$$

$$|\Phi_\ell^S(t)\rangle = S^\dagger(\varepsilon_\ell)|\Phi_\ell(t)\rangle. \quad (6b)$$

In what follows we employ two theorems to obtain the solution of the TD Schrödinger equation (3): (a) on the one hand, a theorem presented in [6] asserts that if  $I_\ell(t)$  is an invariant associated to  $\mathcal{H}_\ell$  (i.e.  $dI_\ell(t)/dt = \partial I_\ell/\partial t + i[\mathcal{H}_\ell, I_\ell(t)] = 0$ ), then the transformed operator  $I_\ell^S(t) = S^\dagger(\varepsilon_\ell)I_\ell(t)S(\varepsilon_\ell)$  becomes an invariant associated to  $\mathcal{H}_\ell^S$ ; (b) on the other hand, from Lewis and Riesenfeld's well-known theorem [5], it follows that a solution of the Schrödinger equation is an eigenstate of the Hermitian invariant  $I_\ell(t)$  multiplied by a TD phase factor. It follows from (a) and (b) that the solutions of equation (3) are given by  $|\Phi_\ell(t)\rangle = S(\varepsilon_\ell)|\Phi_\ell^S(t)\rangle = S(\varepsilon_\ell)\exp[i\phi_{\ell,m}^S(t)]|m, t\rangle_S$  ( $m = 0, 1, 2, \dots$ ), where  $|m, t\rangle_S$  is the eigenstate of the invariant [7] and the Lewis and Riesenfeld phase [5] obeys

$$\phi_{\ell,m}^S(t) = \int_{t_i}^t dt'_S \langle m, t'_S | (i\partial/\partial t' - \mathcal{H}_\ell^S) | m, t'_S \rangle. \quad (7)$$

It is straightforward to verify that under the unitary transformation carried out by the operator  $S(\varepsilon_\ell)$  the TD phase is invariant:  $\phi_{\ell,m}^S(t) = \phi_{\ell,m}(t)$ .

#### The transformed Hamiltonian

Next, we associate the unitary transformation with the squeeze operator  $S(\varepsilon_\ell) = \exp[\frac{1}{2}(\varepsilon_\ell a^{\dagger 2} - \varepsilon_\ell^* a^2)]$ , where the complex TD function  $\varepsilon_\ell(t) = r_\ell(t)e^{i\varphi_\ell(t)}$  includes the squeeze parameters  $r_\ell(t)$  and  $\varphi_\ell(t)$ . ( $r_\ell(t)$  is associated with a squeeze factor while  $\varphi_\ell(t)$  defines the squeezing direction in phase space.) Moreover, the TD parameters for the parametric and linear amplifications are written as  $\zeta(t) = \kappa(t)e^{i\eta(t)}$  and  $\xi(t) = \varkappa(t)e^{i\varpi(t)}$ , respectively. The squeeze parameters ( $r_\ell(t)$ ,  $\varphi_\ell(t)$ ), the amplification amplitudes ( $\kappa(t)$ ,  $\varkappa(t)$ ) and frequencies ( $\eta(t)$ ,  $\varpi(t)$ ) are real TD functions. From the above assumptions and after a lengthy calculation, the transformed Hamiltonian becomes

$$\mathcal{H}_\ell^S = \Omega_\ell(t)a^\dagger a + \Lambda_\ell(t)a^\dagger + \Lambda_\ell^*(t)a + F_\ell(t), \quad (8)$$

provided that its TD coefficients satisfy

$$\Omega_\ell(t) = \omega_\ell(t) + 2\kappa(t) \tanh r_\ell(t) \cos(\eta(t) - \varphi_\ell(t)), \quad (9a)$$

$$\Lambda_\ell(t) = \xi(t) \cosh r_\ell(t) + \xi^*(t)e^{i\varphi_\ell(t)} \sinh r_\ell(t), \quad (9b)$$

$$F_\ell(t) = \varkappa(t) \tanh r_\ell(t) \cos(\eta(t) - \varphi_\ell(t)), \quad (9c)$$

while the squeeze parameters  $r_\ell(t)$  and  $\varphi_\ell(t)$  are determined by solving the coupled differential equations

$$\dot{r}_\ell(t) = 2\kappa(t) \sin(\eta(t) - \varphi_\ell(t)), \quad (10a)$$

$$\dot{\varphi}_\ell(t) = -2\omega_\ell(t) - 4\kappa(t) \coth(2r_\ell(t)) \cos(\eta(t) - \varphi_\ell(t)). \quad (10b)$$

It is evident from these relations that the TD group parameters  $\varepsilon_\ell(t)$ , defining the unitary operator  $S(\varepsilon_\ell)$ , depend on the atomic state  $\ell$ , as assumed from the beginning.

#### The evolution operators

With the Hamiltonian (8) at hand we return to the solution of the Schrödinger equation (5). The invariant associated with this Hamiltonian is given by [7]

$$I_\ell^S(t) = a^\dagger a - \theta_\ell(t)a^\dagger - \theta_\ell^*(t)a + f_\ell(t), \quad (11)$$

$\theta_\ell(t)$  being a solution to the equation  $i\dot{\theta}_\ell(t) = \Omega_\ell(t)\theta_\ell(t) + \Lambda_\ell(t)$  while  $\dot{f}_\ell(t) = \theta_\ell^*(t)\Lambda_\ell(t) - \theta_\ell(t)\Lambda_\ell^*(t) = \text{id}|\theta_\ell(t)|^2/dt$ . The application of the invariant method leads to the wavevector [7]

$$|\Phi_\ell^S(t)\rangle = e^{i\phi_{\ell,m}(t)}|m, t\rangle_S = e^{i\phi_{\ell,m}(t)}D[\theta_\ell(t)]|m\rangle \quad (m = 0, 1, 2, \dots), \quad (12)$$

where  $|m\rangle$  is the number state and  $D[\theta_\ell(t)] = \exp[\theta_\ell(t)a^\dagger - \theta_\ell^*(t)a]$  is the displacement operator.

Therefore, the solutions of the Schrödinger equation (3), which form a complete set, are  $|\Phi_\ell(t)\rangle = S[\varepsilon_\ell(t)]|\Phi_\ell^S(t)\rangle = U_\ell(t)|m\rangle$ , where  $U_\ell(t) = \Upsilon_\ell(t)S[\varepsilon_\ell(t)]D[\theta_\ell(t)]R[\Omega_\ell(t)]$  is a unitary operator composed, in addition to the squeeze and the displacement operators, of a global phase factor  $\Upsilon_\ell(t) = \exp\{-\frac{i}{2}[\beta(t) - \omega t]\}$  and the rotation operator (coming from the TD Lewis and Riesenfeld phase factor)  $R[\Omega_\ell(t)] = \exp[-ia^\dagger a \beta_\ell(t)]$ , with  $\beta_\ell(t) = \int_{t_i}^t \Omega_\ell(t') dt'$ . Hence, for the solution of the Schrödinger equation (3), we find  $|\Phi_\ell(t)\rangle = U_\ell(t)U_\ell^\dagger(t_i)|\Phi_\ell(t_i)\rangle$ , which finally defines the evolution operators  $\mathbb{U}_\ell(t, t_i) = U_\ell(t)U_\ell^\dagger(t_i)$ .

### Evolution of the atom-field state

Let us assume that the micromaser cavity is prepared at time  $t_0$  in a single-mode coherent state  $|\alpha\rangle$  by a monochromatic source. As mentioned above, the linear and parametric pumping are supposed to be turned on also at  $t_0$ , at the time the atom is prepared by the first Ramsey zone in the superposition state  $c_1|1\rangle + c_2|2\rangle$ . Evidently, the evolution operators  $\mathbb{U}(t_1, t_0)$  and  $\mathbb{U}(t, t_2)$  governing the dynamics of the cavity-field state while the atom is outside the cavity do not depend on the state of the two-level atom. However, the operator  $\mathbb{U}_\ell(t_2, t_1)$ , given the evolution of the cavity-field state during its interaction with the atom, does depend on the atomic state and differs from the operators  $\mathbb{U}(t_1, t_0)$  and  $\mathbb{U}(t, t_2)$  only by the shifted frequency  $\omega_\ell(t)$ . With this in mind it is straightforward to verify that the measurement of the atomic state, after undergoing a  $\pi/2$  pulse in the second Ramsey zone, projects the cavity field in the ‘Schrödinger cat’-like state [8]

$$|\Psi(t)\rangle = \mathcal{N}_\pm [\pm e^{i\omega_0 t/2} c_1 \mathbf{U}_1(t, t_0) + e^{-i\omega_0 t/2} c_2 \mathbf{U}_2(t, t_0)] |\alpha\rangle, \quad (13)$$

where the sign + or – occurs if the atom is detected in state  $|2\rangle$  or  $|1\rangle$ , respectively,  $\mathcal{N}_\pm$  accounts for the normalization factors, and the evolution operator reads  $\mathbf{U}_\ell(t, t_0) = \mathbb{U}(t, t_2) \mathbb{U}_\ell(t_2, t_1) \mathbb{U}(t_1, t_0)$ . From equation (13) it follows that, after measuring the atomic level used to generate the superposition state of the radiation field, it is possible to control such superposition by adjusting the TD-driven parameters  $\kappa(t)$ ,  $\chi(t)$ ,  $\eta(t)$  and  $\varpi(t)$ .

### Analytical solutions of the characteristic equations (10a), (10b)

Next, we investigate the situation where the cavity mode  $|\alpha\rangle$  is resonant with the driven fields during the time the atom is out of the cavity: from  $t_0$  to  $t_1$  and  $t_2$  to  $t$ . The parametric amplifier is assumed to operate in a degenerate mode in which the *signal* and the *idler* frequencies coincide, producing a single-mode driven field. In the resonant regime this single-mode field has the same frequency  $\omega$  as the cavity mode so that  $\eta(t) = -2\omega t$ . For the resonant linear amplifier it follows that  $\varpi(t) = \omega t$ . However, during the time interval the atom is inside the cavity, from  $t_1$  to  $t_2$ , it pulls the mode frequency out of resonance with the driven fields establishing a dispersive regime in the amplification process. In the resonant regime the solutions of the coupled differential equations (10a), (10b) are given by [6, 8]

$$\cosh(2r(t)) = \sqrt{1 + C_i^2} \cosh \left[ \operatorname{arccosh} \left( \frac{\cosh(2r(t_i))}{\sqrt{1 + C_i^2}} \right) + u(t) \right], \quad (14a)$$

$$\cos(\varphi(t) - \eta(t)) = - \frac{C_i}{\sqrt{(1 + C_i^2) \cosh^2(u(t)) - 1}}, \quad (14b)$$

where  $u(t) = 4 \int \kappa(t) dt$  and the constant of motion  $C_i = \cosh(\varphi(t) - \eta(t)) \sinh(2r(t))$  depends on the initial values  $r(t_i)$ ,  $\varphi(t_i)$  and  $\eta(t_i)$ , where  $i = 0, 2$ . It is possible to show [8] that in the dispersive regime equations (10a), (10b) can be solved by quadrature [6], leading to a constant of motion,  $C_1 = \cosh(2r_\ell(t)) + \mathfrak{P}_\ell \cos(\varphi_\ell(t) - \eta(t)) \sinh(2r_\ell(t))$ ,

which now depends on the initial values  $r(t_1)$ ,  $\varphi(t_1)$  and  $\eta(t_1)$ . Despite the assumption that the atom-field coupling is turned on (off) suddenly, these initial values must be computed from the solutions for the resonant amplification regime at time  $t_1$ . With this procedure we obtain the solutions for the resonant amplification ( $r(t_1)$ ,  $\varphi(t_1)$ ) as a limit of those for the dispersive amplification ( $r(t_1)$ ,  $\varphi(t_1)$ ) when  $\chi \rightarrow 0$ . The parameter  $\mathfrak{P}_\ell = (-1)^\ell 2\kappa/\chi$ , defined for a constant amplification amplitude  $\kappa$ , is an effective macroscopic coupling. Therefore, for the dispersive regime we find three different solutions depending on  $|\mathfrak{P}_\ell|$ : the strong ( $|\mathfrak{P}_\ell| > 1$ ), the weak ( $|\mathfrak{P}_\ell| < 1$ ) and the critical coupling ( $|\mathfrak{P}_\ell| = 1$ ). Considering the weak coupling regime, the TD squeeze parameters when  $C_1 > (1 - \mathfrak{P}_\ell^2)^{1/2}$  are given by

$$\cosh(2r_\ell(t)) = \frac{C_1}{1 - \mathfrak{P}_\ell^2} \left\{ 1 - |\mathfrak{P}_\ell| \frac{\sqrt{C_1^2 - (1 - \mathfrak{P}_\ell^2)}}{C_1} \times \sin \left[ \arcsin \left( \frac{C_1 |\mathfrak{P}_\ell|}{\sqrt{C_1^2 - (1 - \mathfrak{P}_\ell^2)}} \right) - \frac{\kappa \sqrt{1 - \mathfrak{P}_\ell^2}}{|\mathfrak{P}_\ell|} (t - t_1) \right] \right\}, \quad (15a)$$

$$\cos(\varphi_\ell(t) - \eta(t)) = \pm \frac{C_1 - \cosh(2r_\ell(t))}{\mathfrak{P}_\ell |\sinh(2r_\ell(t))|}. \quad (15b)$$

### A protocol for engineering mesoscopic cavity-field states

To prepare a particular superposition state from (13) we follow a three-step protocol:

- (1) First, we adjust the amplitude  $\kappa$  of the parametric amplification and the atom-field interaction time  $\tau = t_2 - t_1$  in order to obtain a particular angle  $\Theta/2 = (\varphi_1(t_2) - \varphi_2(t_2))/2$  defined by the squeezing directions of the states composing the ‘Schrödinger cat’-like superposition.
- (2) Next, the desired excitation of the prepared state can be achieved by manipulating the excitation of the initial coherent state injected into the cavity and/or the amplitude of the linear amplification (the strength of the parametric amplification has been fixed in the first step) and/or the time interval of the amplification process.
- (3) Finally, the amplitude of both states composing the ‘Schrödinger cat’-like superposition can be adjusted through the probability amplitudes of the atomic superposition state prepared in the first Ramsey zone.

Clearly, the squeezed superposition in equation (13) was ideally prepared. In a real engineering process the dissipative mechanisms of both the cavity and the two-level atom, despite the fluctuations intrinsic in their interaction, must be taken into account. The complex calculations involved in the engineering process of quantum states under realistic quantum dissipation and fluctuation conditions can be surpassed by means of the phenomenological-operator approach as presented in [9, 10]. However, we will not consider in the present work the action of the reservoir in the preparation of the squeezed superposition (13), since the time interval required for this preparation, of order  $10^{-4}$ – $10^{-5}$  s, is considerably smaller than the relaxation times of both the cavity field and the two-level atom, around  $\tau_R \approx 10^{-2}$  s [3, 11]. Therefore, as usual

for estimating the decoherence time, we next consider that an ideally prepared state is submitted to the action of a quantum reservoir. In addition, we will be interested in the action of a vacuum-squeezed reservoir at absolute zero whose density operator is  $\prod_k S_k|0_k\rangle\langle 0_k|S_k^\dagger$ ,  $S_k$  being the squeezed operator for the  $k$ th bath oscillator mode. Here, we are considering that, somehow, it is possible to describe completely all the mechanisms of dissipation of the cavity by the action of a vacuum-squeezed reservoir. Describing the reservoir by a collection of harmonic oscillators  $\sum_k \hbar\omega_k b_k^\dagger b_k$  and its interaction with the cavity mode by  $\sum_k \hbar(\lambda_k a^\dagger b_k + \lambda_k^* a b_k^\dagger)$ , the decoherence time deduced from the idempotency defect of the reduced density operator of the cavity field, as suggested in [12], is given by

$$\tau = \{\tau_R\} \{2[(2N+1)(\langle a^\dagger \rangle \langle a \rangle - \langle a^\dagger a \rangle) + 2\text{Re}\{M(\langle a^\dagger \rangle^2 - \langle (a^\dagger)^2)\}) - N]\}^{-1}, \quad (16)$$

where  $\tau_R$  is the relaxation time,  $N = \sinh^2(\tilde{r})$  and  $M = -e^{i\tilde{\varphi}} \sinh(2\tilde{r})/2$ ,  $\tilde{r}$  and  $\tilde{\varphi}$  being the squeeze parameters of the vacuum reservoir [8]. The mean values are computed from the prepared squeezed superposition (13). Since the excitation of the initial coherent state  $\alpha$  and the squeeze parameters ( $r(t_2)$ ,  $\varphi_\ell(t_2)$ ) have been fixed by the engineering protocol, we note that equation (16) depends only on the reservoir squeeze parameters ( $\tilde{r}$ ,  $\tilde{\varphi}$ ). Considering the situation where  $\alpha$  is real and  $\exp(-2\alpha^2) \approx 0$  (implying  $\alpha \gtrsim \sqrt{2}$ ), the maximization of the decoherence time  $\tau$  with respect to these parameters leads to the results

$$\tilde{r}_A = r + \ln(1 + 4\alpha^2)/4, \quad \tilde{\varphi}_A = 0, \quad (17a)$$

$$\tilde{r}_B = r - \ln(1 + 4\alpha^2)/4, \quad \tilde{\varphi}_B = \pi, \quad (17b)$$

when fixing  $\Theta = 2n\pi$  ( $n$  integer), i.e. the states composing the superposition (13) are squeezed in the same direction. When  $\Theta \neq 2n\pi$ , the maximum of  $\tau$  turns out to be smaller than that for  $\Theta = 2n\pi$ , given either by the pair  $(\tilde{r}_A, \tilde{\varphi}_A)$  or  $(\tilde{r}_B, \tilde{\varphi}_B)$  when considering  $\varphi_\ell(t_2) = (2m_\ell + 1)\pi$  or  $\varphi_\ell(t_2) = 2m_\ell\pi$  ( $m_\ell$  integer), respectively (note that  $n = |m_1 - m_2|$ ). Observe that the direction of squeezing of both states composing the superposition (13), defined by the angle  $\varphi_1(t_2)$  or  $\varphi_2(t_2)$ , has to be perpendicular to the direction of squeezing of the vacuum reservoir.

Next, we compute the ‘distance’ in phase space between the centres of the quasi-probability distribution of the individual states composing the prepared superposition (13). This distance is defined by the quadratures of the cavity field  $X = (a^\dagger + a)/2$  and  $Y = (a - a^\dagger)/2i$ , as  $D = [(\langle X \rangle_2 - \langle X \rangle_1)^2 + (\langle Y \rangle_2 - \langle Y \rangle_1)^2]^{1/2}$ , the subscripts 1, 2 referring to the two states composing the superposition. When considering  $\varphi_1(t_2) = \varphi_2(t_2) = (2m+1)\pi$  or  $2m\pi$ , the distance becomes  $D = \langle X \rangle_2 - \langle X \rangle_1 = 2\alpha \exp(r)$  or  $2\alpha \exp(-r)$ , respectively. We will focus on the case  $\varphi_1(t_2) = (2m_1 + 1)\pi$ , since it results in a large distance  $D$  between the two states composing what we actually want to be a mesoscopic superposition. Assuming the squeezing factor  $r$  so that  $\exp(-2r) \approx 0$  in addition to  $\exp(-2\alpha^2) \approx 0$ , the decoherence time and the mean photon number of the prepared state, following from the values  $(\tilde{r}_A, \tilde{\varphi}_A)$ , is

$$\tau \approx \tau_R/\alpha, \quad \langle n \rangle = \langle a^\dagger a \rangle \approx \alpha^2 \exp(2r). \quad (18)$$

Remarkably, with the approximations  $\exp(-2r)$ ,  $\exp(-2\alpha^2) \approx 0$ , the decoherence time for the prepared cavity-field state when  $\varphi_1(t_2) = \varphi_2(t_2) = (2m_2 + 1)\pi$ —under the action of a vacuum reservoir squeezed in the direction  $\tilde{\varphi}_A = 0$ —turns out to be practically independent of the parameter  $r$  and thus of its own intensity  $\langle n \rangle$  and distance  $D$ . From the result in equation (18) we conclude that it is convenient to start from a coherent state  $\alpha$  as small as possible (within the limit  $\exp(-2\alpha^2) \approx 0$ ) and to adjust the macroscopic coupling parameter  $|\Re\{\ell\}|$  in order to obtain a large squeeze factor and so a large intensity of the prepared state, since we are actually interested in mesoscopic superpositions. We stress that even considering the weak coupling regime ( $|\Re\{\ell\}| < 1$ ), we obtain, from equations (14a) and (15a), large squeeze parameters. Considering  $|\Re\{\ell\}| = 0.1$ ,  $\alpha = \sqrt{2}$ , and an experimental running time of about  $2 \times 10^{-4}$  s, we get a superposition state where  $r \approx 2$  and  $\langle n \rangle \approx 10^2$  photons.

The mechanism behind this result is the degree of entanglement between the prepared state and the modes of the reservoir, which depends on the relative direction of their squeezing, defined by the angles  $\varphi_1(t_2) = \varphi_2(t_2)$  and  $\tilde{\varphi}_A$ . A result supporting this argument is presented in [13] where it is shown that the injection of two modes, squeezed in perpendicular directions, in a 50/50 beam splitter does not generate an entangled state. A careful analysis of the dependence on the degree of entanglement and the relative direction of squeezing between a prepared state and its multimode reservoir—a collection of independent beam splitters—is presented in [8]. Despite the fact that the mechanism behind the long-lived mesoscopic superpositions is mainly the perpendicular directions between the squeezing of the prepared state and the reservoir modes, the magnitude of the parameter  $r$  plays a crucial role in the present scheme for producing the mesoscopic superposition by increasing both its intensity  $\langle n \rangle$  and distance  $D$  in phase space.

The values presented above for  $\tau$ ,  $\langle n \rangle$  and  $D$  are to be compared with those when considering an NS cavity-field state ( $\langle n \rangle_{\text{NS}} = \alpha^2$ ,  $D_{\text{NS}} = 2\alpha$ ) under the influence of (i) a squeezed reservoir, resulting in the decoherence time  $\tau_i \approx \tau_R/\alpha$ , and (ii) a NS reservoir, such that  $\tau_{ii} \approx \tau_R/2\alpha^2$ . Note that in both cases (i) and (ii) we obtain the rates  $\langle n \rangle / \langle n \rangle_{\text{NS}} \approx \exp(2r)$  and  $D/D_{\text{NS}} \approx \exp(r)$ . Therefore, despite the exponential increase in the rates for both excitation and distance we still get  $\tau \approx \tau_i$  when comparing our results with previous schemes in the literature, where a squeezed reservoir is assumed for the enhancement of the decoherence time [14]; for NS cavity-field states and reservoir, we obtain a still better result,  $\tau \approx \alpha\tau_{ii}$ .

It is interesting to note that we could have separated the process of Hamiltonian (1) into two successive simpler processes, first creating the superposition state (with the first three terms of Hamiltonian (1)) and then applying the parametric amplification (fourth and fifth terms of (1), without any linear amplification). When considering the whole process simultaneously, as in this work, the squeezing directions in the phase space of both states composing the superposition can be adjusted independently. Evidently, this is not necessary for the proposal presented in this paper, where the components of the superposition have to be squeezed in the same direction. However, the possibility of squeezing the components of a superposition state in different directions can be considered

for other applications, such as for state engineering in cavity QED [8]. In this connection, the linear amplification process (sixth and seventh terms of equation (1)) can be employed to achieve a higher excitation of the engineered state. Moreover, when considering the whole process simultaneously, we decrease the time interval of the experiment, minimizing the noise effects coming from field and atomic decays, and thus achieving a higher fidelity for the generated squeezed superposition state. Even though we have not computed the noise effects during the preparation of the squeezed superposition state, it is always desirable to minimize the time interval of the engineering process in order to maximize the fidelity of the prepared state.

The experimental implementations of the proposed scheme rely on the possibility of engineering a squeezed reservoir as well as of parametrically driving cavity-field radiation. We stress that a scheme to realize physically a squeezed bath for cavity modes, via quantum–non-demolition-mediated feedback, has already been presented in [18]. However, the feedback process in [18] does not eliminate the standard non-squeezed bath and, as we have stressed, our scheme requires a resulting optimal squeezed-vacuum reservoir. The subject of quantum reservoir engineering has attracted attention especially in the domain of trapped ions [15, 16] and, specifically, a scheme has been presented for engineering squeezed-bath-type interactions for protecting a two-level system against decoherence [17].

Regarding parametric amplification of cavity fields, a technique was recently suggested based on the pulsed excitation of semiconductor layers (on the cavity walls) by laser radiation [19]. Moreover, a proposal to implement the parametric amplification of an arbitrary radiation-field state previously prepared in a high- $Q$  cavity is presented in [20]. In this work, the non-linear process is accomplished through the dispersive interactions of a three-level atom simultaneously with a classical driven field and a previously prepared cavity mode whose state is supposed to be squeezed. It is worth mentioning that all the treatment developed above in the context of cavity QED, for delaying the decoherence process of a squeezed superposition by coupling it to a vacuum-squeezed reservoir, can also be implemented in ion traps. We finally mention that the proposal presented here might

provide a motivation for future theoretical and experimental investigations.

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