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Dispersive limit of the dissipative Jaynes–Cummings model with a squeezed reservoir

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Abstract

We consider the dynamics of an atom–field system in a cavity described by means of the Jaynes–Cummings model in the dispersive limit, assuming that the electromagnetic field is interacting with a squeezed, or phase-sensitive, reservoir. After obtaining the solution of the master equation in terms of Wigner functions, we do an analysis of the system evolution and its asymptotical behaviour in terms of the so-called Mandel Q parameter and the linear entropies of the total system and the reduced system, associated with field and atom.

Keywords: Jaynes–Cummings model, dispersive limit, linear entropy, Wigner function, master equation, squeezing, photon statistics

1. Introduction

Quantum information is encoded in quantum states of its material carriers—modes of the electromagnetic field, atoms and ions. Therefore, knowledge of the details of the interaction between these carriers, including the processes of relaxation caused by different environments, is extremely important for quantum computing. One of the simplest, but very effective, models of such interactions was proposed by Jaynes and Cummings (Jaynes–Cummings model, JCM) 40 years ago [1]. Over the past decades, it has been a subject of numerous publications, reviewed in [2–4]. However, the JCM turned out to be so rich that until now some of its features have not been investigated completely.

One of the many different directions of studies is a search for simple approximate analytical solutions [4–9]. An important special case of JCM admitting explicit closed solutions corresponds to the so-called 'dispersive limit' [10–15]. In particular, different kinds of 'Schrödinger cat states' can be constructed in this regime [10–13]. Recently, the dispersive limit of JCM was studied in [16–18] in order to find relations between the entanglement of the atomic and field degrees of freedom and the decoherence caused by coupling the field mode to a *zero-temperature reservoir*.

The aim of our paper is to analyse the dynamics of the dispersive limit of the dissipative JCM in a more general case, where a cavity field mode is coupled to a *squeezed* reservoir [19–25] (also called a 'rigged' [26] or *phase-sensitive* [27, 28] reservoir). In contrast to thermal reservoirs, that destroy quantum superpositions very quickly [29, 30], phase-sensitive reservoirs allow one to increase the decoherence time of different superposition states [21, 28, 31].

The case of reservoirs with nonzero temperature has not only quan

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however small [32]. This is in accordance with the third law of thermodynamics, which tells us that zero temperature cannot be attainable, so the point $T = 0$ may be singular in some respects.

The plan of the paper is as follows. In section 2 we discuss the JCM Hamiltonian in the dispersive approximation and the master equation describing the interaction of the system 'atom + field mode' with a squeezed reservoir. In section 3 we introduce different useful relations for the 'linear entropies' of atomic and field subsystems, which can serve as measures of quantum purity and entanglement, putting the emphasis on the usefulness of the Wigner function to calculate these entropies (a similar approach was used recently in [18]). The evolution of the field mode Wigner functions connected with diagonal elements of the atomic density matrix is considered in section 4, where the field linear entropy, squeezing and photon distribution function in the asymptotical state are also analysed. The 'off-diagonal' Wigner functions and the evolution of partial atomic and total linear entropies are studied in section 5. Section 6 contains comments and conclusions. Some details of the calculations (a proof of the Araki–Lieb inequality for linear entropy and the method of solving the master equation with the aid of linear time-dependent operator integrals of motion) are given in appendices.

2. Dispersive limit of JCM in the presence of a squeezed reservoir

The Hamiltonian of the Jaynes–Cummings model is [1, 3, 4, 33, 34] (we assume $\hbar \equiv 1$)

$$\hat{H} = \omega a^\dagger a + \tfrac{1}{2}\omega_{\rm ef}\sigma_z + \mathcal{G}(\sigma_- a^\dagger + \sigma_+ a), \tag{1}$$

where ω is the frequency of a monomodal field trapped in a cavity and $\omega_{\rm ef}$ is the frequency of the atomic transition between the excited (e) and fundamental (f) levels. The Rabi frequency $\mathcal{G}$ describes the dipole coupling between atom and field. The bosonic operators a and $a^\dagger$ with $[a, a^\dagger] = 1$ describe the monomodal field, while σ operators describe the atomic transitions:

$$\sigma_+ = |{\rm e}\rangle\langle{\rm f}|, \qquad \sigma_+|{\rm f}\rangle = |{\rm e}\rangle, \qquad \sigma_+|{\rm e}\rangle = 0,$$

$$\sigma_- = |{\rm f}\rangle\langle{\rm e}|, \qquad \sigma_-|{\rm e}\rangle = |{\rm f}\rangle, \qquad \sigma_+|{\rm f}\rangle = 0,$$

$$\sigma_z = [\sigma_+, \sigma_-] = |{\rm e}\rangle\langle{\rm e}| - |{\rm f}\rangle\langle{\rm f}|, \qquad [\sigma_z, \sigma_\pm] = \pm\sigma_\pm.$$

It is well known that the Hamiltonian (1) can be written as a sum of two commuting operators, $H = H_1 + H_2$:

$$H_1 = \omega(a^\dagger a + \tfrac{1}{2}\sigma_z), \tag{2}$$

$$H_2 = \frac{\delta}{2}\sigma_z + \mathcal{G}(\sigma_- a^\dagger + \sigma_+ a), \tag{3}$$

where $\delta = \omega_{\rm ef} - \omega$ is the detuning between atomic and field frequencies. Therefore the evolution operator corresponding to the Hamiltonian (1) can be factorized as $U(t) = U_1(t)U_2(t)$, with $U_k(t) = \exp(-{\rm i}H_k t)$. The first operator is trivial:

$$U_1(t) = \exp(-{\rm i}\omega a^\dagger a t)\exp(-{\rm i}\omega\sigma_z t/2), \tag{4}$$

whereas the second operator is given by the matrix (in the atomic basis $|{\rm e}\rangle$, $|{\rm f}\rangle$) [2]

$$U_2(t) = \begin{bmatrix} \cos\hat{}_{n+1}t - \dfrac{{\rm i}\delta\sin\hat{}_{n+1}t}{2\hat{}_{n+1}} & -{\rm i}\mathcal{G}\dfrac{\sin\hat{}_{n+1}t}{\hat{}_{n+1}}a \\ -{\rm i}\mathcal{G}\dfrac{\sin\hat{}_n t}{\hat{}_n}a^\dagger & \cos\hat{}_n t + \dfrac{{\rm i}\delta\sin\hat{}_n t}{2\hat{}_n} \end{bmatrix} \tag{5}$$

with

$$\hat{}_n = \sqrt{\mathcal{G}^2\hat{n} + \delta^2/4}, \qquad \hat{n} = a^\dagger a. \tag{6}$$

One can essentially simplify the operator (5) in the so-called *dispersive limit* [4, 10–16]

$$|\delta| \gg \mathcal{G}\sqrt{\overline{n} + 1}, \tag{7}$$

where $\overline{n}$ is the mean number of photons (more precisely, some quantity characterizing the maximal Fock state number, giving a noticeable contribution to the dynamics of the system). Under condition (7) one can, first, neglect off-diagonal elements in the matrix (5), for being a small perturbation, and second, get rid of square roots, taking into account the linear term of the Taylor expansion of the frequency operator (6)

$$\hat{}_n \approx \delta/2 + \quad \hat{n}, \qquad \equiv \mathcal{G}^2/\delta. \tag{8}$$

In this approximation, the operator (5) becomes

$$U_2(t) = \begin{bmatrix} \exp\{-{\rm i}[\frac{\delta}{2} + \quad (\hat{n}+1)]t\} & 0 \\ 0 & \exp\{{\rm i}[\frac{\delta}{2} + \quad \hat{n}]t\} \end{bmatrix}$$

and the total evolution operator assumes the form

$$U(t) = \exp(-{\rm i}\omega a^\dagger a t)\exp\left(-\frac{\rm i}{2}\omega_{\rm ef}\sigma_z t\right)\exp(-{\rm i}H_d t) \tag{9}$$

with the *effective dispersive interaction Hamiltonian* [4, 10, 11, 16]

$$H_d = \quad [(a^\dagger a + 1)|{\rm e}\rangle\langle{\rm e}| - a^\dagger a|{\rm f}\rangle\langle{\rm f}|]. \tag{10}$$

If the electromagnetic field mode trapped in the cavity is coupled to some reservoir, then the dynamics of the whole system in the interaction picture can be described by means of some kind of master equation:

$$\frac{{\rm d}\rho}{{\rm d}t} = -\frac{\rm i}{\hbar}[H_d, \rho] + \mathcal{L}\rho. \tag{11}$$

So it depends not only on the effective interaction Hamiltonian H_d, but also on the relaxation operator $\mathcal{L}\rho$. We assume that the field mode is coupled to a *squeezed vacuum reservoir*, which can be described by means of the relaxation operator of the form [20, 23, 27, 28, 34, 35]

$$\begin{aligned}\mathcal{L}\rho = {} & \gamma(\nu+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \\ & + \gamma\nu(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger) \\ & + \gamma\mathcal{M}{\rm e}^{{\rm i}\varphi}(2a^\dagger\rho a^\dagger - a^\dagger a^\dagger\rho - \rho a^\dagger a^\dagger) \\ & + \gamma\mathcal{M}{\rm e}^{-{\rm i}\varphi}(2a\rho a - aa\rho - \rho aa),\end{aligned} \tag{12}$$

where the coefficient $\gamma > 0$ characterizes the strength of coupling with the reservoir and ν is the average number of squeezed photons in this reservoir (for more general master equations see, e.g., [24]). The real nonnegative coefficient $\mathcal{M}$, which determines the correlation between photons of the reservoir, must satisfy the inequality $\mathcal{M} \leqslant \sqrt{\nu(\nu+1)}$ in order

that positive definiteness of the statistical operator could not be violated during the evolution. The phase φ is frequently [20, 23, 27, 28, 35] assumed to be constant. Such a choice corresponds to the case when the central frequency ω_r of the objects (e.g. oscillators) constituting the reservoir coincides with the frequency ω of the field mode under consideration. However, studying the dispersive limit, when the transition frequency of the atom ω_{ef} is shifted from ω, it seems reasonable to consider a more general case of some detuning between the field mode and the reservoir. Then

$$\varphi = \varphi(t) = \varphi_0 + 2\zeta t, \qquad \zeta = \omega - \omega_r. \tag{13}$$

The physical meaning of parameters becomes more transparent, if one takes into account the relations [20, 34]

$$\langle b_\omega b_{\omega'}\rangle = \mathcal{M}\mathrm{e}^{\mathrm{i}\varphi(t)}\delta(\omega+\omega'-2\omega_r),$$

$$\langle b_\omega^\dagger b_{\omega'}\rangle = \nu\delta(\omega-\omega'),$$

where operators b_ω describe the reservoir oscillators. For the maximally squeezed reservoir we have $\nu = \sinh^2 r$ and

$$\mathcal{M} = \sqrt{\nu(\nu+1)} = \sinh r\cosh r = \tfrac{1}{2}\sinh(2r),$$

where r is the usual squeezing coefficient.

The density operator, $\rho(t)$, of the total system 'atom + field' belongs to the Hilbert space that is a direct product of the atomic and field subspaces. Thus:

$$\begin{aligned}\rho(t) &= \rho_{\mathrm{ee}}(t)\otimes|\mathrm{e}\rangle\langle\mathrm{e}| + \rho_{\mathrm{ff}}(t)\otimes|\mathrm{f}\rangle\langle\mathrm{f}| \\ &\quad + \rho_{\mathrm{ef}}(t)\otimes|\mathrm{e}\rangle\langle\mathrm{f}| + \rho_{\mathrm{fe}}(t)\otimes|\mathrm{f}\rangle\langle\mathrm{e}|,\end{aligned} \tag{14}$$

where

$$\begin{aligned}\rho_{\mathrm{ee}}(t) &= \langle\mathrm{e}|\rho(t)|\mathrm{e}\rangle, \qquad & \rho_{\mathrm{ff}}(t) &= \langle\mathrm{f}|\rho(t)|\mathrm{f}\rangle, \\ \rho_{\mathrm{ef}}(t) &= \langle\mathrm{e}|\rho(t)|\mathrm{f}\rangle, \qquad & \rho_{\mathrm{fe}}(t) &= \langle\mathrm{f}|\rho(t)|\mathrm{e}\rangle.\end{aligned} \tag{15}$$

Once the effective Hamiltonian (10) is diagonal in the atomic basis, the evolution of each partial field statistical operator does not depend on the evolution of other operators. In particular, the partial field statistical operators ρ_{ee} and ρ_{ff}, related to the diagonal elements of the atom density matrix, obey similar equations:

$$\frac{\mathrm{d}\rho_{\mathrm{ee}}}{\mathrm{d}t} = -\mathrm{i}\quad[a^\dagger a, \rho_{\mathrm{ee}}] + \mathcal{L}\rho_{\mathrm{ee}}, \tag{16}$$

$$\frac{\mathrm{d}\rho_{\mathrm{ff}}}{\mathrm{d}t} = \mathrm{i}\quad[a^\dagger a, \rho_{\mathrm{ff}}] + \mathcal{L}\rho_{\mathrm{ff}}, \tag{17}$$

from which it follows that the f-level field statistical operator $\rho_{\mathrm{ff}}(t)$ can be obtained from its e-level partner $\rho_{\mathrm{ee}}(t)$ by means of a simple replacement $\quad\to -\quad$. The equations for the operators ρ_{ef} and ρ_{fe} are

$$\frac{\mathrm{d}\rho_{\mathrm{ef}}}{\mathrm{d}t} = -\mathrm{i}\quad(aa^\dagger\rho_{\mathrm{ef}} + \rho_{\mathrm{ef}}a^\dagger a) + \mathcal{L}\rho_{\mathrm{ef}}, \tag{18}$$

$$\frac{\mathrm{d}\rho_{\mathrm{fe}}}{\mathrm{d}t} = \mathrm{i}\quad(a^\dagger a\rho_{\mathrm{fe}} + \rho_{\mathrm{fe}}aa^\dagger) + \mathcal{L}\rho_{\mathrm{fe}}, \tag{19}$$

and they do not preserve the normalization and Hermiticity of the statistical operators. Note that all four equations (16)–(19) have the same relaxation term $\mathcal{L}\rho$.

3. Statistical operators and linear entropies of atom and field

One of our goals is to study the evolution of *linear entropies*

$$s_{\mathrm{A}} = 1 - \mathrm{Tr}\,\rho_{\mathrm{A}}^2, \qquad s_{\mathrm{C}} = 1 - \mathrm{Tr}\,\rho_{\mathrm{C}}^2, \tag{20}$$

where the reduced density operator of each subsystem is obtained by taking a partial trace of the total statistical operator ρ over the complementary subsystem:

$$\rho_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{C}}\,\rho, \qquad \rho_{\mathrm{C}} = \mathrm{Tr}_{\mathrm{A}}\,\rho.$$

We use the suffixes A and C to denote the atom and the field, respectively. If the total system is in a *pure* quantum state (that happens in the absence of dissipation, i.e. for $\gamma = 0$), then each of the partial linear entropies can be used as a measure of entanglement between the atomic and field subsystems [36, 37]. An advantage of using the linear entropy instead of the von Neumann entropy $S = -\mathrm{Tr}(\rho\ln\rho)$ is in the great simplification of calculations, without changing the qualitative physical conclusions. For example, one can easily check by direct inspection that, for any *pure* entangled state of two subsystems of the form $|\quad\rangle = \sum_i |\phi_i\rangle_{\mathrm{C}}\otimes|f_i\rangle_{\mathrm{A}}$, both partial traces coincide:

$$\mathrm{Tr}\,\rho_{\mathrm{A}}^2 = \mathrm{Tr}\,\rho_{\mathrm{C}}^2 = \sum_{ijkl}\langle\phi_l|\phi_i\rangle\langle\phi_j|\phi_k\rangle\langle f_l|f_k\rangle\langle f_j|f_i\rangle. \tag{21}$$

If one chooses an orthonormalized basis for the atomic subsystem with finite-dimensional Hilbert space:

$$|\quad\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|\phi_i\rangle_{\mathrm{C}}\otimes|f_i\rangle_{\mathrm{A}}, \qquad \langle f_i|f_j\rangle = \delta_{ij},$$

then

$$\mathrm{Tr}\,\rho_{\mathrm{A}}^2 = \mathrm{Tr}\,\rho_{\mathrm{C}}^2 = \frac{1}{N^2}\sum_{i,j=1}^{N}|\langle\phi_i|\phi_j\rangle|^2, \tag{22}$$

and

$$s_{\mathrm{A}} = s_{\mathrm{C}} = 1 - \frac{1}{N} - \frac{2}{N^2}\sum_{i>j=1}^{N}|\langle\phi_i|\phi_j\rangle|^2. \tag{23}$$

In particular, in the case of a two-level atom, with

$$|\quad\rangle = \frac{1}{\sqrt{2}}(|\phi_{\mathrm{e}}\rangle_{\mathrm{C}}\otimes|\mathrm{e}\rangle_{\mathrm{A}} + |\phi_{\mathrm{f}}\rangle_{\mathrm{C}}\otimes|\mathrm{f}\rangle_{\mathrm{A}})$$

we have a simple formula:

$$s_{\mathrm{A}} = s_{\mathrm{C}} = \tfrac{1}{2}(1 - |\langle\phi_{\mathrm{e}}|\phi_{\mathrm{f}}|^2). \tag{24}$$

On the other hand, the proof of the equality $S_{\mathrm{A}} = S_{\mathrm{C}}$ for the von Neumann entropies of pure quantum states is based on the Araki–Lieb inequality [38]:

$$|S_{\mathrm{A}} - S_{\mathrm{C}}| \leqslant S \leqslant S_{\mathrm{A}} + S_{\mathrm{C}}, \tag{25}$$

which is also valid for linear entropy, as shown in appendix A.

In the generic case of the statistical operator as displayed in equation (14) we have

$$\rho_{\mathrm{C}} = \rho_{\mathrm{ee}} + \rho_{\mathrm{ff}} \tag{26}$$

and

$$\rho_{\rm A} = {\rm Tr}(\rho_{\rm ee})|{\rm e}\rangle\langle{\rm e}| + {\rm Tr}(\rho_{\rm ff})|{\rm f}\rangle\langle{\rm f}| + {\rm Tr}(\rho_{\rm ef})|{\rm e}\rangle\langle{\rm f}| + {\rm Tr}(\rho_{\rm fe})|{\rm f}\rangle\langle{\rm e}|. \quad (27)$$

Calculating all necessary traces we obtain the following expressions for the linear entropy of the total system $s_{\rm AC} = 1 - {\rm Tr}(\rho^2)$ and entropies of the field and atom:

$$s_{\rm AC} = 1 - {\rm Tr}(\rho_{\rm ee}^2) - {\rm Tr}(\rho_{\rm ff}^2) - 2\,{\rm Tr}(\rho_{\rm ef}\rho_{\rm fe}), \quad (28)$$

$$s_{\rm C} = 1 - {\rm Tr}(\rho_{\rm ee}^2) - {\rm Tr}(\rho_{\rm ff}^2) - 2\,{\rm Tr}(\rho_{\rm ee}\rho_{\rm ff}), \quad (29)$$

$$s_{\rm A} = 2\,{\rm Tr}(\rho_{\rm ee})\,{\rm Tr}(\rho_{\rm ff}) - 2|\,{\rm Tr}\,\rho_{\rm fe}|^2. \quad (30)$$

Note that partial traces ${\rm Tr}(\rho_{\rm ee})$ and ${\rm Tr}(\rho_{\rm ff})$ are conserved in time in view of equations (16) and (17). Therefore, in the simplest variant of the dispersive limit of JCM there is no change in the atomic populations or the atomic inversion: only off-diagonal elements of the atomic density matrix depend on time, which corresponds to *decoherence* of the atomic subsystem. This means that the dispersive limit is the most adequate in the case where initial populations of the upper and lower atomic levels are close to each other. For this reason, we confine ourselves in this paper to the case when initially ($t = 0$) an atom enters the cavity in a (pure) superposition state with equal amplitudes of the excited and ground levels, whereas the cavity field is set initially in a coherent state:

$$|\quad(0)\rangle = \frac{1}{\sqrt{2}}[|{\rm e}\rangle + |{\rm f}\rangle] \otimes |\alpha\rangle, \quad (31)$$

$$|\alpha\rangle = {\rm e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{(n!)^{1/2}}|n\rangle. \quad (32)$$

Such a situation was realized, for example, in the experiment of [39]. Then the initial conditions for the partial density matrices of the field $\rho_{ij}(t)$ are

$$\rho_{\rm ee}(0) = \rho_{\rm ff}(0) = \rho_{\rm ef}(0) = \rho_{\rm fe}(0) = \tfrac{1}{2}|\alpha\rangle\langle\alpha|. \quad (33)$$

In this special case the atomic entropy equals

$$s_{\rm A} = \tfrac{1}{2} - 2|{\rm Tr}\rho_{\rm fe}|^2. \quad (34)$$

In the absence of dissipation, by acting with the evolution operator (9) on the state (31) we obtain (in the interaction picture)

$$|\quad(t)\rangle = \frac{1}{\sqrt{2}}[{\rm e}^{-{\rm i}\ \ t}|{\rm e}\rangle|\alpha{\rm e}^{-{\rm i}\ \ t}\rangle + |{\rm f}\rangle|\alpha{\rm e}^{{\rm i}\ \ t}\rangle]. \quad (35)$$

Then, in accordance with (24), we have

$$s_{\rm A}(t) = s_{\rm C}(t) = \tfrac{1}{2}[1 - {\rm e}^{-D^2(t)}], \quad (36)$$

where

$$D(t) = 2|\alpha|\sin\quad t \quad (37)$$

is a measure of the distance between two different coherent components of the field generated during the evolution. The *excess entropy* is defined as $I = s_{\rm A} + s_{\rm C} - s$, so, as $s = 0$ for pure states, it follows that $I = [1 - {\rm e}^{-D^2(t)}]$, which varies with time periodically, going to zero at the disentanglement times $t_{\rm d} = k\pi/\quad$, with k being an integer.

Projecting the state (35) on the initial atomic superposition state $[|{\rm e}\rangle + |{\rm f}\rangle]/\sqrt{2}$ or its orthogonal partner $[|{\rm e}\rangle - |{\rm f}\rangle]/\sqrt{2}$, one obtains the superposition states of the *field mode* in the form [10–13]

$$|\psi\rangle_{\rm cat} = \tfrac{1}{2}[{\rm e}^{-{\rm i}\ \ t}|\alpha{\rm e}^{-{\rm i}\ \ t}\rangle \pm |\alpha{\rm e}^{{\rm i}\ \ t}\rangle].$$

In the generic case, we have the following *conditional* field statistical operator (arising after an appropriate measurement of the atomic state):

$$\rho_{\pm} = \tfrac{1}{2}(\rho_{\rm ee} + \rho_{\rm ff} \pm \rho_{\rm ef} \pm \rho_{\rm fe}). \quad (38)$$

The two last terms are responsible for the coherence effects. They go eventually to zero in the dissipative case. Therefore the quantity

$$\mathcal{R} = |{\rm Tr}(\rho_{\rm ef})|^2 = 1 - 2s_{\rm A} \quad (39)$$

(which equals 1 at $t = 0$) can also be used as a quantitative measure of *coherence* of the Schrödinger cat states $\rho_{\pm}$.

One of the simplest ways to calculate traces of partial statistical operators and their products is to use the Wigner function associated with the operator ρ [40]

$$W(\xi,\xi^*) = \frac{1}{\pi}\int {\rm Tr}\{\exp[\eta(a^\dagger - \xi^*) - \eta^*(a - \xi)]\rho\}\,{\rm d}^2\eta$$

(or, better, the Weyl transform in the case of non-Hermitian operators as $\rho_{\rm fe}$ and $\rho_{\rm ef}$), taking into account the relations [40]

$${\rm Tr}\,\rho = \frac{1}{\pi}\int {\rm d}^2\xi\; W(\xi,\xi^*), \quad (40)$$

$${\rm Tr}(\rho_a\rho_b) = \frac{1}{\pi}\int {\rm d}^2\xi\; W_a(\xi,\xi^*)W_b(\xi,\xi^*). \quad (41)$$

In the above equations, ξ and ξ^* are mutually conjugated complex arguments. In many cases it is more convenient to use real arguments $q = \sqrt{2}\,{\rm Re}(\xi)$ and $p = \sqrt{2}\,{\rm Im}(\xi)$. Then the integration measure is ${\rm d}p\,{\rm d}q/(2\pi)$.

The integrals in (40) and (41) can be calculated in the case concerned with the aid of the known formula for the Gaussian integrals of real or complex arguments (in a generic case, $\mathcal{Q}$ and $\mathcal{A}$ are $2N \times 2N$ symmetrical matrices, $\mathbf{q}$ and $\mathbf{b}$ are $2N$-dimensional vectors, whereas α and δ are N-dimensional complex vectors):

$$\int \exp\left(-\frac{1}{2}\mathbf{q}\mathcal{Q}\mathbf{q} + \mathbf{b}\mathbf{q}\right)\frac{{\rm d}\mathbf{q}}{(2\pi)^N} = (\det\mathcal{Q})^{-1/2}\exp(\tfrac{1}{2}\mathbf{b}\mathcal{Q}^{-1}\mathbf{b}), \quad (42)$$

$$\int \exp\left\{-\frac{1}{2}(\alpha^*,\alpha)\mathcal{A}\begin{pmatrix}\alpha^*\\ \alpha\end{pmatrix} + \alpha\delta^* + \alpha^*\delta\right\}\frac{{\rm d}\alpha\,{\rm d}\alpha^*}{\pi} = [-\det(\mathcal{A})]^{-1/2}\exp\left\{\tfrac{1}{2}(\delta,\delta^*)\mathcal{A}^{-1}\begin{pmatrix}\delta\\ \delta^*\end{pmatrix}\right\}. \quad (43)$$

Note that all four 'partial' Wigner functions $W_{ab}(\xi,\xi^*)$ (where $a, b = {\rm e}, {\rm f}$) have the same initial form, corresponding to the Wigner function of the coherent state [40] multiplied by the factor 1/2:

$$W_{ab}(\xi,\xi^*,0) = \exp(-2|\xi - \alpha_0|^2), \quad (44)$$

due to the specific initial conditions (33). However, their evolutions are quite different, as shown in the following sections.

4. Unconditional field evolution

4.1. Diagonal Wigner functions

An advantage of using the Wigner function, over other representations, becomes especially clear in the case of the partial field statistical operators, ρ_{ee} and ρ_{ff}, related to the diagonal elements of the atomic density matrix, because master equations with quadratic Liouvillians, such as (16), transform any initial Gaussian statistical operator (in particular, the operator corresponding to the initial Gaussian Wigner function (44)) into another Gaussian operator (for the general theory and references see, e.g., [41]). But any Gaussian state is completely determined by the first-order (complex) average value $\overline{a} \equiv \mathrm{Tr}(a\rho)$, the real covariance

$$\widetilde{a^\dagger a} \equiv \tfrac{1}{2}\,\mathrm{Tr}[(a^\dagger a + a a^\dagger)\rho] - |\overline{a}|^2,$$

and the complex variance $\overline{a^2} \equiv \mathrm{Tr}(a^2\rho) - \overline{a}^2$ [41–43]:

$$W(\xi,\xi^*) = [(\widetilde{a^\dagger a})^2 - |\overline{a^2}|^2]^{-1/2} \times \exp\left\{-\frac{|\xi-\overline{a}|^2\widetilde{a^\dagger a} - \mathrm{Re}[(\xi-\overline{a})^2(\overline{a^2})^*]}{(\widetilde{a^\dagger a})^2 - |\overline{a^2}|^2}\right\}. \tag{45}$$

Therefore, to find the correct Wigner function at $t > 0$ one has only to solve simple equations for the first- and second-order moments resulting from equation (16):

$$\mathrm{d}\overline{a}/\mathrm{d}t = -(\gamma + \mathrm{i}\ \ )\overline{a},$$

$$\mathrm{d}\overline{a^2}/\mathrm{d}t = -2(\gamma + \mathrm{i}\ \ )\overline{a^2} - 2\gamma\mathcal{M}\mathrm{e}^{\mathrm{i}\varphi},$$

$$\mathrm{d}\widetilde{a^\dagger a}/\mathrm{d}t = -2\gamma\widetilde{a^\dagger a} + \gamma(1+2\nu).$$

By substituting the solutions of these equations (with the time-dependent phase $\varphi(t)$ (13) of the squeezed reservoir) into equation (45), we obtain the following explicit expression (taking into account the normalization $\mathrm{Tr}(\rho_{ee}) = 1/2$):

$$W_{ee}(\xi,\xi^*,t) = \frac{1}{2\sqrt{F\ (t)}}\exp\left\{-\frac{1}{F\ (t)}(B(t)|\xi-\alpha\ \ (t)|^2 - \mathrm{Re}[\mathcal{M}\mathrm{e}^{-\mathrm{i}\varphi_0}A^*\ (t)(\xi-\alpha\ \ (t))^2])\right\}, \tag{46}$$

where

$$F\ \ (t) = B^2(t) - \mathcal{M}^2|A\ \ (t)|^2, \tag{47}$$

$$A\ \ (t) = -\quad (\mathrm{e}^{2\mathrm{i}\zeta t} - \mathrm{e}^{-2\gamma\ t}), \tag{48}$$

$$\overline{a^2}(t) = \mathcal{M}\mathrm{e}^{\mathrm{i}\varphi_0}A\ \ (t), \tag{49}$$

$$B(t) \equiv \widetilde{a^\dagger a}(t) = \tfrac{1}{2} + \nu(1-\mathrm{e}^{-2\gamma t}), \tag{50}$$

$$\overline{a}(t) = \alpha\ \ (t) = \alpha\mathrm{e}^{-\gamma\ t}, \qquad \gamma\ \ \equiv \gamma + \mathrm{i}\ \ , \tag{51}$$

$$= \frac{\gamma}{\gamma + \mathrm{i}(\quad + \zeta)}. \tag{52}$$

The replacement $\to -$ transforms the function $W_{ee}(\xi,\xi^*,t)$ into $W_{ff}(\xi,\xi^*,t)$.

4.2. Time evolution of the field linear entropy in the 'resonance' squeezed reservoir

According to equations (29), (41) and (46), the calculation of linear entropy of the field is reduced to several Gaussian integrals. However, the expressions in the case of arbitrary coefficients and ζ turn out to be rather cumbersome. Therefore we consider in detail the case of the conventional choice $\zeta = 0$, whereas for $\zeta \neq 0$ we bring here only formulae describing the asymptotical state as $t \to \infty$.

For $\zeta = 0$, all expressions are significantly simplified due to the relations

$$A\ \ = A_-^*\ , \qquad\qquad = \ {}_-^*\ , \qquad F\ \ = F_-\ \ \equiv F(t). \tag{53}$$

Taking them into account we obtain the formula

$$s_C(t) = 1 - \frac{1}{4\sqrt{F(t)}} - \frac{1}{4\sqrt{F_*(t)}}\exp\left\{-\frac{\mathrm{e}^{-2\gamma t}D^2(t)}{2F_*(t)} \times [B(t) + \mathcal{M}\,\mathrm{Re}[A\ \ (t)]\cos(2\chi - \varphi_0)]\right\}, \tag{54}$$

where $D(t)$ is given by (37), $\alpha = |\alpha|\mathrm{e}^{\mathrm{i}\chi}$ and

$$F_*(t) = B^2(t) - \mathcal{M}^2\{\mathrm{Re}[A\ \ (t)]\}^2. \tag{55}$$

When $\gamma t \gg 1$, the field entropy tends to a nonzero asymptotical value (if $\nu > 0$), which does not depend on the phases of the initial coherent state χ and the 'resonance' reservoir φ_0:

$$s_C^\infty = 1 - \tfrac{1}{2}[1 + 4\nu(\nu+1)(1-\mu|\ \ |^2)]^{-1/2} - \tfrac{1}{2}[1 + 4\nu(\nu+1)(1-\mu|\ \ |^4)]^{-1/2}, \tag{56}$$

where

$$\mu = \frac{\mathcal{M}^2}{\nu(\nu+1)}, \qquad |\ \ |^2 \equiv \frac{\gamma^2}{\gamma^2 + \ \ ^2}. \tag{57}$$

It is worth noticing that (56) is a monotonic increasing function for ν, > 0 and $\mathcal{M} \leqslant \sqrt{\nu(\nu+1)}$. Interestingly, for a non-physical choice $\mathcal{M} > \sqrt{\nu(\nu+1)}$ the entropy s_c may be turned to zero for certain relations between ν, and $\mathcal{M}$.

Actually, the dependence of $s_C(t)$ on the phase difference between the initial state and the state of the reservoir, $2\chi - \varphi_0$, is always rather weak for highly excited initial states of the field, $|\alpha| \gg 1$ (provided $|\ \ \alpha| \ll |\delta|$ in order to fulfil the condition of validity of the dispersive limit (7)), because the third term of the sum in (54) is small for almost all moments of time if $\gamma t \sim 1$ or less. This term is important only at the moments $t_k = k\pi/\ \ $ (we assume $\ \ > 0$), when sharp local minima of the field entropy are observed. However, these minima do not depend on $2\chi - \varphi_0$, as soon as $D(t_k) = 0$:

$$s_C^{(\min)}(k) = 1 - \tfrac{1}{2}[1 + 4\nu u_k + 4(\nu^2 - \mathcal{M}^2|\ \ |^2)u_k^2]^{-1/2} - \tfrac{1}{2}[1 + 4\nu u_k + 4(\nu^2 - \mathcal{M}^2|\ \ |^4)u_k^2]^{-1/2},$$

where $u_k = 1 - \exp(-2\pi k\gamma/\ \ )$.

In the case of $\nu = \mathcal{M} = 0$, equation (54) becomes

$$s_C(t) = \tfrac{1}{2}(1 - \exp[-\mathrm{e}^{-2\gamma t}D^2(t)]),$$

and $s_C(t) \to 0$ for $t \to \infty$ (the field occurs in the pure vacuum state). But the asymptotical field entropy can be made very small even when $\nu \gg 1$, if coupling with the reservoir is strong enough and the reservoir is taken in the maximally

squeezed state with $\mu = 1$. Namely, if $\gamma \gg \sqrt{\nu(\nu+1)}$, then $s_C^\infty \approx 3\nu(\nu+1)\ ^2/\gamma^2$.

If $\gamma \ll |\ \ |$, then $|\ \ | \ll 1$, and the squeezing properties of the reservoir (described by the parameter $\mathcal{M}e^{i\varphi_0}$) do not play any significant role in the evolution of the field. In this case we have the following simplified expressions for the Wigner function and the entropy of the field:

$$W_{ee}(\xi, \xi^*, t) \approx \frac{1}{2B(t)} \exp\left[-\frac{|\xi - \alpha\ (t)|^2}{B(t)}\right], \tag{58}$$

$$s_C(t) \approx 1 - \frac{1}{4B(t)} - \frac{1}{4B(t)} \exp\left[-\frac{e^{-2\gamma t} D^2(t)}{2B(t)}\right], \tag{59}$$

$$s_C^{(\min)}(k) \approx \frac{2\nu u_k}{1 + 2\nu u_k}, \qquad s_C^\infty \approx \frac{2\nu}{1+2\nu}. \tag{60}$$

4.3. Asymptotical field state in the generic case

Asymptotically ($\gamma t \to \infty$), the total Wigner function of the field mode goes to the sum of two correlated (in the sense of [44]) squeezed Gaussian states, centred at the origin but having different (and *rotated*, in the generic case) orientations of the ellipses of equal quasi-probabilities (here we use the representation in terms of real phase space coordinates q and p as defined at the end of section 3:

$$W^\infty(p, q, t) = W_{ee}^\infty(p, q, t;\ \ ) + W_{ee}^\infty(p, q, t; -\ \ ), \tag{61}$$

where

$$W_{ee}^\infty(p, q, t;\ \ ) = \frac{1}{2\sqrt{F^\infty}} \exp[-a_+ p^2 - a_- q^2 - cpq],$$

$$a_\pm = \frac{1}{2F^\infty}\left[B_\infty \pm \frac{\mathcal{M}\gamma[\gamma\cos\varphi(t) + (\ \ + \zeta)\sin\varphi(t)]}{\gamma^2 + (\ \ + \zeta)^2}\right],$$

$$c = -\frac{1}{F^\infty}\frac{\mathcal{M}\gamma[(\ \ + \zeta)\cos\varphi(t) - \gamma\sin\varphi(t)]}{\gamma^2 + (\ \ + \zeta)^2},$$

$$F^\infty = B_\infty^2 - \mathcal{M}^2|\ \ \ |^2, \qquad B_\infty = \nu + \tfrac{1}{2}. \tag{62}$$

The expressions for $\varphi(t)$ and \ \ are given by equations (13) and (52), respectively. The integration of the Wigner function is finite only if $F^\infty > 0$ or $4a_+a_- > c^2$, which implies

$$\nu + 1/2 > \frac{\mathcal{M}\gamma}{[\gamma^2 + (\ \ + \zeta)^2]^{1/2}}.$$

The asymptotical value of the field entropy in the generic case $\zeta \neq 0$ equals

$$s_C^\infty = 1 - \frac{1}{8\sqrt{F^\infty}} - \frac{1}{8\sqrt{F_-^\infty}} - \frac{\sqrt{F^\infty F_-^\infty}}{2\sqrt{B_\infty^2(F^\infty + F_-^\infty)^2 - \mathcal{M}^2|\ \ F_-^\infty + \ _-\ F^\infty|^2}}. \tag{63}$$

The relative weights of two Gaussians in (61) are determined by the coefficients $F_\pm^\infty$ given by formula (62). These coefficients are different if $\zeta \neq 0$, and the asymmetry becomes maximal in the special case of $\zeta = -\ \ $, when $\equiv 1$ for any (however small but nonzero) coefficient γ, while $_-\ = \gamma/(\gamma - 2i\ \ )$. In contrast to the case of $\zeta = 0$ studied in the preceding subsection, the influence of the squeezed reservoir is very significant even if $\gamma \ll \ \ $, because now $|\ _-\ | \ll 1$. Assuming that $\mathcal{M}^2 = \nu(\nu+1)$ (maximally correlated squeezed reservoir) we have

$$F^\infty = 1/4, \qquad F_-^\infty \approx (\nu + 1/2)^2,$$

$$a_\pm^{(\ \ )} = 1 + 2\nu \pm 2\sqrt{\nu(\nu+1)}\cos\varphi(t), \qquad a_\pm^{(-\ \ )} \approx (1+2\nu)^{-1},$$

$$c^{(\ \ )} = 4\sqrt{\nu(\nu+1)}\sin\varphi(t), \qquad c^{(-\ \ )} \approx 0.$$

Under the conditions

$$\zeta = -\ \ , \qquad \gamma \ll \ \ , \qquad \mathcal{M}^2 = \nu(\nu+1), \tag{64}$$

the asymptotical value of the field entropy (63) goes to

$$s_C^\infty \approx \frac{1+3\nu}{2(1+2\nu)} - \frac{1}{[1 + 3(1+2\nu)^2]^{1/2}}. \tag{65}$$

If $\nu \ll 1$, then the function (65) grows as $5\nu/4$, compared with the dependence 2ν in the case of equation (60). Moreover, if $\nu \gg 1$, the function (65) tends not to the unit value, as happened for equation (60), but to the value 3/4. Consequently, an appropriately tuned squeezed reservoir can diminish the effect of decoherence.

4.4. Squeezing

Introducing the field quadrature operator

$$\hat{X}(\phi) = \frac{1}{\sqrt{2}}(\hat{a}e^{i\phi} + \hat{a}^\dagger e^{-i\phi}), \tag{66}$$

one can easily verify that the minimal and maximal values of the variance of this operator as a function of the parameter ϕ are given by simple relations

$$\sigma_X^{(\min)} = \widetilde{a^\dagger a} - |\overline{a^2}|, \qquad \sigma_X^{(\max)} = \widetilde{a^\dagger a} + |\overline{a^2}| \tag{67}$$

(so that the denominator in the exponential of the Gaussian Wigner function (45) is nothing but the product $\sigma_X^{(\min)}\sigma_X^{(\max)}$). Formulae (67) were obtained under the name 'principal squeezing' in [45], and equivalent expressions were found also in [46–48].

In the case under study, i.e. for the initial states (31), the total variances $\widetilde{a^\dagger a}$ and $\overline{a^2}$ are equal to half of the sum of the corresponding 'partial' variances in the Gaussian states $W_{ee}(p, q, t;\ \ )$ and $W_{ee}(p, q, t; -\ \ )$. Putting the expressions (49) and (50) into formula (67) we obtain

$$\sigma_X^{(\min)}(t) = B(t) - \tfrac{1}{2}\mathcal{M}(|A\ \ (t)| + |A_-\ \ (t)|). \tag{68}$$

Obviously, the interaction with the atom cannot squeeze the field mode, if it was initially in the coherent state (in the framework of the dispersive approximation considered here). However, the squeezed reservoir can transfer a part of its degree of squeezing to the field.

In the asymptotical state we have

$$\sigma_X^{(\min)}(\infty) = \frac{1}{2} + \nu - \frac{\mathcal{M}}{2}\left(\frac{\gamma}{\sqrt{\gamma^2 + (\ \ + \zeta)^2}} + \frac{\gamma}{\sqrt{\gamma^2 + (\ \ - \zeta)^2}}\right). \tag{69}$$

Therefore, large squeezing ($\sigma_X^{(\min)} \ll 1$) can be obtained only in the limit of strong coupling with a highly squeezed reservoir, i.e. for $\gamma \gg |\ \pm\zeta|$, $\nu \gg 1$ and

$$\mathcal{M} = \mathcal{M}_{\max} = \sqrt{\nu(\nu+1)} \approx \nu + 1/2 - \tfrac{1}{8}(\nu+1/2)^{-1}.$$

Under these conditions

$$\sigma_X^{(\min)} \approx (\nu+1/2)\frac{^2+\zeta^2}{2\gamma^2} + \frac{1}{8}(\nu+1/2)^{-1}.$$

In this limit, the interaction between the field mode and the atom becomes insignificant, and the field evolution is governed by the interaction with the squeezed bath, which eventually imposes its own high degree of squeezing on the field mode.

4.5. Photon statistics

The photon statistics is frequently characterized by Mandel's Q factor:

$$Q \equiv \frac{\overline{n^2} - \overline{n}^2}{\overline{n}} - 1. \tag{70}$$

The mean values $\overline{n}$ and $\overline{n^2}$ of the photon number operator $\hat{n} = \hat{a}^\dagger\hat{a}$ for each of the two Gaussian components of the total field statistical operator (26) can be calculated with the aid of the relations [42, 43]

$$\overline{n}_{\rm Gauss} = \widetilde{a^\dagger a} - 1/2 + |\overline{a}|^2, \tag{71}$$

$$\overline{n^2}_{\rm Gauss} = 2(\widetilde{a^\dagger a})^2 - \widetilde{a^\dagger a} + |\overline{a^2}|^2 + |\overline{a}|^4 - |\overline{a}|^2 + 4\widetilde{a^\dagger a}|\overline{a}|^2 + 2\,{\rm Re}[\overline{a^2}(\overline{a}^*)^2]. \tag{72}$$

In the special case of $\zeta = 0$ and for the specific initial field state (33), the total mean values can be obtained by putting into equations (71) and (72) the solutions (49)–(51) and taking even parts of the resultant expressions with respect to . After some algebra, we arrive at the following final result:

$$\begin{aligned} Q\overline{n}_{\rm tot} &= \nu^2(1-{\rm e}^{-2\gamma t})^2 + 2\nu|\alpha|^2{\rm e}^{-2\gamma t}(1-{\rm e}^{-2\gamma t}) \\ &+ \frac{\mathcal{M}^2\gamma^2}{\gamma^2+\ ^2}[(1-{\rm e}^{-2\gamma t})^2 + 4{\rm e}^{-2\gamma t}\sin^2(\ t)] \\ &- \frac{2\mathcal{M}\gamma|\alpha|^2{\rm e}^{-2\gamma t}\cos(\varphi-2\chi)}{\gamma^2+\ ^2}\{\gamma(1-{\rm e}^{-2\gamma t}) \\ &+ 2\sin(\ t)[\ \cos(\ t) - \gamma\sin(\ t)]\}, \end{aligned} \tag{73}$$

where the total mean photon number does not depend on or $\mathcal{M}$:

$$\overline{n}_{\rm tot} = \nu(1-{\rm e}^{-2\gamma t}) + |\alpha|^2{\rm e}^{-2\gamma t}. \tag{74}$$

The first term of the Taylor expansion of (73) with respect to t is

$$Q = 4\gamma t[\nu - \mathcal{M}\cos(\varphi-2\chi)] + \mathcal{O}(t^2). \tag{75}$$

Consequently, one can observe sub-Poissonian statistics ($Q < 0$) at the initial stages of evolution: see figure 1, which shows damped oscillations of the Mandel parameter. However, as time goes on, this parameter stabilizes at an asymptotical *positive* constant value

$$Q^{(\infty)} = \nu + \tfrac{1}{2}\mu(\nu+1)(|\ \ ^2 + |\ \ _-\ |^2), \tag{76}$$

where the coefficient μ was defined by equation (57). Consequently, the photon statistics of the asymptotical state ($\gamma t \gg 1$) is always super-Poissonian (even in the case of strong squeezing). Another interesting fact is that the Q factor of the asymptotical state remains finite even in the limit $\nu \to 0$ if the reservoir is squeezed:

$$Q^{(\infty)} \to \tfrac{1}{2}\mu(|\ \ ^2 + |\ \ _-\ |^2) \qquad \text{for } \nu \to 0. \tag{77}$$

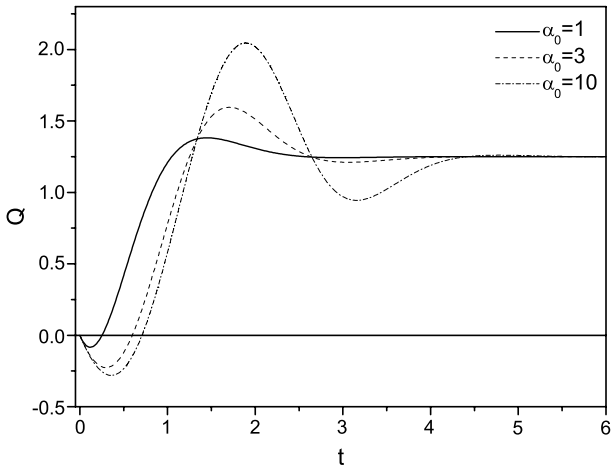


Figure 1. Time dependence of Mandel's parameter in the case of maximally squeezed reservoir ($\mathcal{M}^2 = \nu(\nu+1)$) for $\nu = 0.5$, $\gamma/\ \ = 1$, $\varphi = \chi = 0$ and different values of α_0.

4.6. Asymptotical photon distribution

The photon distribution function (PDF) is

$$p_n = \langle n|\hat{\rho}_{\rm ee} + \hat{\rho}_{\rm ff}|n\rangle, \tag{78}$$

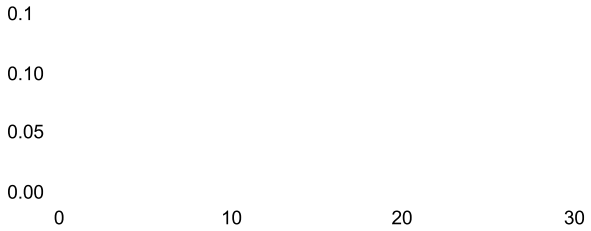
i.e. it is a sum of two distributions corresponding to the Gaussian state (46) with opposite signs of the frequency . Explicit formulae for the PDF of the most general Gaussian states in terms of the Hermite polynomials of two variables and their special cases (Laguerre, Legendre and the usual Hermite polynomials) were found in [43] (for some special cases see also [42, 49]). This PDF is determined completely by the first- and second-order moments of operators $\hat{a}$ and $\hat{a}^\dagger$ (or their quadrature components). Since the mean value of operator $\hat{a}$ in the asymptotical state (61) equals zero, the asymptotical partial PDFs depend only on two quantities, $\widetilde{a^\dagger a}$ and $|\widetilde{a^2}|$. Let us consider first the simplest case of $\zeta = 0$, when $\widetilde{a^\dagger a}$ and $|\widetilde{a^2}|$ are both invariant with respect to the change of sign of . Then, adopting the general expressions for the PDF of Gaussian states with zero mean values of quadratures, given in [43, 49], to the special case concerned, we obtain

$$p_n^{(\infty)} = \frac{[\nu^2-\beta^2]^{n/2}}{[(1+\nu)^2-\beta^2]^{(n+1)/2}} \times P_n\left(\frac{\nu(\nu+1)-\beta^2}{\sqrt{[\nu^2-\beta^2][(1+\nu)^2-\beta^2]}}\right), \tag{79}$$

where $P_n(z)$ stands for the Legendre polynomial and

$$\beta^2 = \frac{\mathcal{M}^2\gamma^2}{\gamma^2+\ ^2} \equiv \nu(\nu+1)\mu|\ \ ^2. \tag{80}$$

Obviously, $\beta^2 < \nu(\nu+1)$. However, in the case of strong coupling with the maximally squeezed reservoir the inequality


0.1
0.10
0.05
0.00
0
10
20
30

Consequently, the problem is reduced to finding the solution of the Schrödinger equation for two independent oscillators (corresponding to the variables $\tilde{q}$ and $\tilde{p}$) described by means of generalized non-Hermitian quadratic Hamiltonians. This can be done in many different ways. The simplest approach is to use the general method of time-dependent linear operator integrals of motion, proposed in [50, 51] and exposed in detail in [41]. The scheme of calculations is given in appendix B. Here we write down the final result, i.e. the explicit form of the function $W_{\mathrm{ef}}(t)$:

$$W_{\mathrm{ef}}(q,p,t) = (R_+R_-)^{-1/2}\exp\left\{\gamma t - \frac{Y_+}{R_+}\tilde{p}^2 - \frac{Y_-}{R_-}\tilde{q}^2 + 2\sqrt{2}|\alpha|\left[\frac{\tilde{p}}{R_+}\sin\left(\chi-\frac{\varphi}{2}\right)+\frac{\tilde{q}}{R_-}\cos\left(\chi-\frac{\varphi}{2}\right)\right] - 2|\alpha|^2\left[\frac{\tilde{F}_+}{R_+}\sin^2\left(\chi-\frac{\varphi}{2}\right)+\frac{\tilde{F}_-}{R_-}\cos^2\left(\chi-\frac{\varphi}{2}\right)\right]\right\}. \tag{92}$$

The notations are as follows:

$$R_\pm = C_\pm + (\gamma \quad + 4\gamma\nu_\pm)S_\pm, \tag{93}$$

$$Y_\pm = C_\pm + \gamma \ \ S_\pm, \qquad \tilde{F}_\pm = C_\pm - \gamma S_\pm, \tag{94}$$

$$C_\pm = \cosh(\lambda_\pm t), \qquad S_\pm = \sinh(\lambda_\pm t)/\lambda_\pm, \tag{95}$$

$$\lambda_\pm = \sqrt{\gamma^2 + 4\mathrm{i}\gamma \quad \nu_\pm}. \tag{96}$$

The function W_{fe} can be obtained from (92) by changing the sign of $\quad$; consequently $W_{\mathrm{fe}}(q,p,t) = W^*_{\mathrm{ef}}(q,p,t)$.

5.2. Linear entropies of the atom and total system

We can better understand the atomic state evolution by looking at the entropies. For the partial atomic entropy and total atom–field entropy we obtain, using equations (28), (34), (41), (42) and (92), the following expressions:

$$s_{\mathrm{A}}(t) = \tfrac{1}{2} - \tfrac{1}{2}|Y_+Y_-|^{-1}\exp\left\{2\gamma t + 4 \quad |\alpha|^2 \times \mathrm{Im}\left[\frac{S_+}{Y_+}\sin^2\left(\chi-\frac{\varphi}{2}\right)+\frac{S_-}{Y_-}\cos^2\left(\chi-\frac{\varphi}{2}\right)\right]\right\}, \tag{97}$$

$$s(t) = 1 - \frac{1}{4\sqrt{F(t)}} - \frac{\exp(2\gamma t)}{2[\mathrm{Re}(Y_+R_+^*)\,\mathrm{Re}(Y_-R_-^*)]^{1/2}} \times \exp\left\{-4|\alpha|^2\sin^2\left(\chi-\frac{\varphi}{2}\right) \times\left[\mathrm{Re}\left(\frac{\tilde{F}_+}{R_+}\right)-\frac{[\mathrm{Re}(R_+^{-1})]^2}{\mathrm{Re}(Y_+/R_+)}\right] - 4|\alpha|^2\cos^2\left(\chi-\frac{\varphi}{2}\right)\left[\mathrm{Re}\left(\frac{\tilde{F}_-}{R_-}\right)-\frac{[\mathrm{Re}(R_-^{-1})]^2}{\mathrm{Re}(Y_-/R_-)}\right]\right\}. \tag{98}$$

The function $F(t)$ in (98) is the same as in equations (53) and (54).

In the case of $\nu = \mathcal{M} = 0$ we have $\lambda_+ = \lambda_- = \gamma$, and the formulae for the atomic and field entropies become significantly simplified:

$$s_{\mathrm{A}}(t) = \frac{1}{2} - \frac{1}{2}\exp\left\{-\frac{2 \ \ ^2|\alpha|^2}{\gamma^2 + \ \ ^2}\left(1 - \mathrm{e}^{-2\gamma t} + 2\sin(\ \ t)\mathrm{e}^{-2\gamma t} \times\left[\sin(\ \ t) - \frac{\gamma}{\ \ }\cos(\ \ t)\right]\right)\right\}, \tag{99}$$

$$s(t) = \frac{1}{2} - \frac{1}{2}\exp\left\{-\frac{2 \ \ ^2|\alpha|^2}{\gamma^2 + \ \ ^2}\left(1 - \mathrm{e}^{-2\gamma t} + 2\frac{\gamma}{\ \ }\sin(\ \ t)\mathrm{e}^{-2\gamma t} \times\left[\cos(\ \ t) - \frac{\gamma}{\ \ }\sin(\ \ t)\right]\right)\right\}. \tag{100}$$

As $\gamma t \to \infty$, the atom goes into a mixed state (although the field goes to the vacuum state, $s_{\mathrm{C}}^\infty = 0$, in this zero-temperature case). According to the Araki–Lieb inequality, the asymptotical total and atomic entropy coincide:

$$s^\infty = s_{\mathrm{A}}^\infty = \frac{1}{2} - \frac{1}{2}\exp\left(-\frac{2 \ \ ^2|\alpha|^2}{\gamma^2 + \ \ ^2}\right). \tag{101}$$

However, for a nonzero mean number of photons in the reservoir, the behaviours of the atomic and total entropies are qualitatively different from the zero-temperature idealization, even if $\nu \ll 1$. For $\nu \neq 0$ one should take into account not only the time dependence of the terms in the arguments of the exponential functions in (97) and (98) (which tend to some constant values when $\mathrm{Re}(\lambda_\pm)t \gg 1$), but also the time dependence of the *pre-exponential factors*, which both decay for $\mathrm{Re}(\lambda_\pm)t \gg 1$ as $\exp(-\kappa t)$, with

$$\kappa = \mathrm{Re}(\lambda_+ + \lambda_-) - 2\gamma. \tag{102}$$

It is important that $\kappa > 0$ as soon as $\nu > 0$. Therefore the linear entropy of the atomic subsystem tends to the maximal possible (for two-level systems) value $s_{\mathrm{A}}^{\max} = 1/2$ for *any nonzero value* of ν.[4] In other words, we observe complete decoherence of the atomic subsystem, because $|W_{\mathrm{ee}}| = |W_{\mathrm{ff}}|$ for the chosen initial conditions, whereas $|W_{\mathrm{ef}}| \to 0$ as $\kappa t \to \infty$.

For $\nu \ll 1$ and $\gamma t \gg 1$ we have, instead of (101),

$$s_{\mathrm{A}} \approx \frac{1}{2} - \frac{1}{2}\exp\left[-\frac{2 \ \ ^2}{\gamma^2 + \ \ ^2}(|\alpha|^2 + 2\gamma\nu t)\right] = \frac{1}{2} + \left(s_{\mathrm{A}}^\infty - \frac{1}{2}\right)\exp\left[-\frac{2 \ \ ^2}{\gamma^2 + \ \ ^2}(2\gamma\nu t)\right].$$

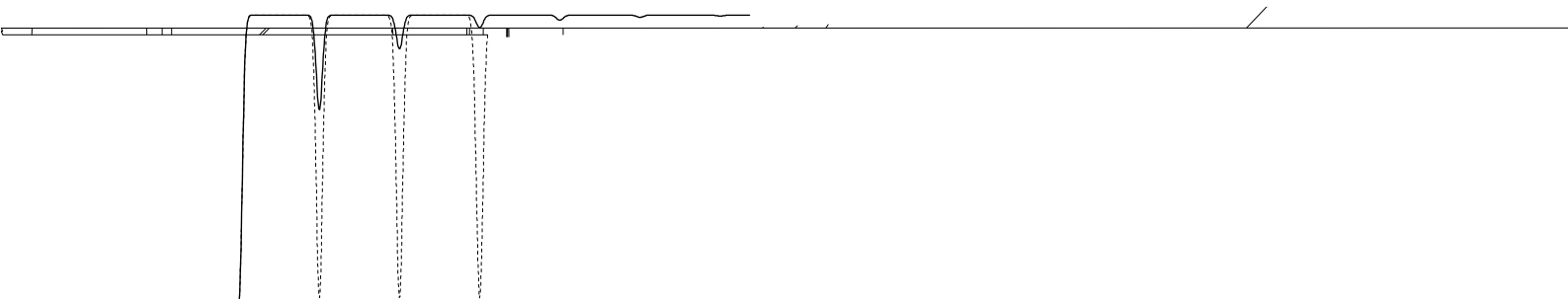
So, asymptotically for $\nu = 0$, s_{A} goes to equation (101), whereas for $\nu \neq 0$ it goes to 1/2. This means that, whenever the mean number of quanta in the reservoir is different from zero (even if very small), equation (101) gives in fact the entropy of some *metastable* (and only partially decohered) state, which finally goes to the completely decohered state in the limit $\kappa t \gg 1$.

If the mean number of photons in the reservoir is so big such that $\gamma \quad \nu \gg \gamma^2 + \ \ ^2$, then $\lambda_+ \approx 2(1+\mathrm{i})\sqrt{\gamma \ \ \nu_+}$ and there is no metastable state, because $\kappa \gg \gamma$. The value of λ_- can be small (for example, $\lambda_- = \gamma \quad$ if $\mathcal{M} = \nu$), but it does not have much influence on the evolution of entropies in this case.

5.3. Total and atomic entropies in the case of a detuned reservoir

What happens in the case of a generic detuned reservoir with time-dependent phase (13)? Equation (84) remains the same, but the coefficients at the second derivatives become

[4] Remember that we consider the case of the initial state (31) with equal occupation probabilities for the upper and lower levels, which do not vary with time in the dispersive approximation.



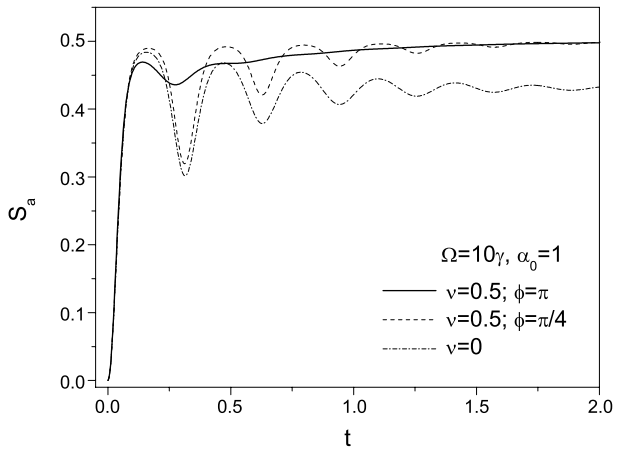
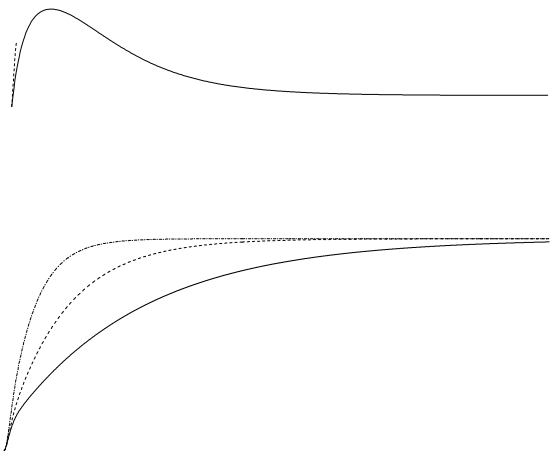


Figure 4. An example of the evolution of the atom linear entropy.



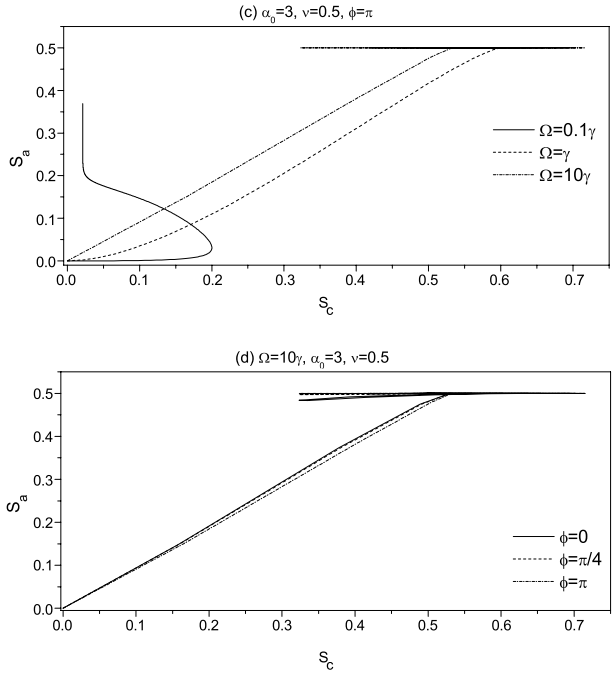


Figure 8. The 'trajectories in the entropic phase space' (S_a versus S_c) for different ratios $/\gamma$ (c) and different phases of the squeezed reservoir (d).

entropy is plotted for several reservoir phases: an increase of the phase from 0 to π causes a significant reduction of oscillations. Another distinguishing feature is the existence of different timescales of the evolution of the field and atom reduced entropies (see figures 5(a) and (b)), which strongly depend on the temperature (mean number of quanta in the reservoir ν). For example, for $\nu = 0.5$ the field attains its asymptotical value at $t \approx 3.0$, while the atom goes quite slowly, attaining the asymptotical value at $t \approx 60.0$ (the ratio of relaxation times is about 3:60). At higher temperature $\nu = 2.0$ the ratio is reduced to 2.5:20.

In figures 6(a) and (b) we show the entropies for two different values of $/\gamma = 1.0, 0.1$; for the latter the atom entropy increases slowly to a constant asymptotical value, whereas for the former one perceives a sharp increase, and the asymptotical value is attained at $t \approx 0.5$. The field does not seem to be sensitive to . However, the total entropy shows a dip before attaining the asymptotical value for small , an effect that vanishes for $= \gamma$. So, strong coupling constants or small detunings between the atom and field mode accelerate the stabilization of the atom system with respect to the entropies (compare the timescales in figures 3 and 6).

The existence of different timescales of the evolution of atomic and field subsystems is demonstrated in another way in figures 7 and 8, where we have plotted the 'trajectory' of the system in 'phase space' of the reduced entropies, S_a versus S_c. The presence of vertical parts of the trajectories in the case of strong damping, $= 0.1\gamma$ (figures 7(b) and 8(c)), indicates that the field entropy attains its steady value (after an initial rapid increase and the following decay) much faster than the atomic entropy. In contrast, for a small coupling with the reservoir ($\geqslant \gamma$) the atomic subsystem goes to the equilibrium state much faster than the field, and horizontal lines in figures 7(a) and 8 are visual confirmations of this fact: when S_a attains the steady level 0.5, the trajectory goes first to the right and then returns to the left, passing sometimes several times along the horizontal segment before reaching the extreme left final position. In figure 7(a) the trajectories are not quite sensible to the phase φ for $= \gamma$, although they are sensible to φ for small (figure 7(b)). In figure 8(c) we compare the trajectories in the 'entropic phase space' for several values of and we verify that they do not differ significantly for high values of . Figure 8(d) shows that, for high values of , the trajectories are not sensitive to the phase φ.

In summary, we did learn that the reduced entropies constitute a valuable tool for the analysis of the atom–field interaction. We verified that when the field is embedded in a phase-sensitive reservoir, they permit us to see clearly that, for high values of , the two timescales at which the atom and field evolve merge into a single timescale.

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Appendix A. Araki–Lieb inequality for the linear entropy

The Araki–Lieb inequality:

$$|S_A - S_C| \leqslant S_{AC} \leqslant S_A + S_C \tag{A.1}$$

was demonstrated to hold [38] for the von Neumann entropy, $S = -\mathrm{Tr}(\rho \ln \rho)$, for $\rho = \rho_{AC}$, $\rho_A = \mathrm{Tr}_C \rho_{AC}$ and $\rho_C = \mathrm{Tr}_A \rho_{AC}$. Here we show that it is also valid for the so-called linear entropy, $s = \mathrm{Tr}[\rho(1-\rho)] = 1 - \mathrm{Tr}(\rho^2)$, involving the two systems, field and atom. Let us consider the density operator

$$\rho = \sum_{i,j} p_{ij} |\varphi_i\rangle_C \langle\varphi_j| \otimes |f_i\rangle_A \langle f_j| \tag{A.2}$$

where the vector $|\varphi_i\rangle_C$ stands for the field states and $|f_i\rangle_A$ for the atomic states, while the coefficients $p_{ij} = p_{ji}$ are the joint probabilities. We assume $|{}_C\langle\varphi_j|\varphi_i\rangle_C| \leqslant 1$, ${}_C\langle\varphi_i|\varphi_i\rangle_C = 1$ and ${}_A\langle f_j|f_i\rangle_A = \delta_{i,j}$. The squared ρ and its trace are given by

$$\rho^2 = \sum_{ijl} p_{ij} p_{jl} |\varphi_i\rangle\langle\varphi_l| \otimes |f_i\rangle\langle f_l|, \qquad \mathrm{Tr}_{AC}\, \rho^2 = \sum_{ij} p_{ij}^2.$$

So the linear entropy is written as

$$s_{AC} = 1 - \sum_{ij} p_{ij}^2. \tag{A.3}$$

The reduced density operators and their respective linear entropies are as follows:

$$\rho_C = \sum_i p_{ii} |\varphi_i\rangle\langle\varphi_i|, \qquad \rho_A = \sum_{ij} p_{ij} \langle\varphi_i|\varphi_j\rangle |f_i\rangle\langle f_j|,$$

$$s_C = 1 - \sum_{ij} p_{ii} p_{jj} |\langle\varphi_i|\varphi_j\rangle|^2, \qquad s_A = 1 - \sum_{ij} p_{ij}^2 |\langle\varphi_i|\varphi_j\rangle|^2.$$

Since

$$|s_{\rm A}-s_{\rm C}| = \left|\sum_{ij}(p_{ii}p_{jj}-p_{ij}^2)|\langle\varphi_i|\varphi_j\rangle|^2\right| \leqslant \left|\sum_{ij}(p_{ii}p_{jj}-p_{ij}^2)\right| = \left|\left(\sum_i p_{ii}\right)^2-\sum_{ij}p_{ij}^2\right| = 1-\sum_{ij}p_{ij}^2 = s_{\rm AC}$$

and

$$s_{\rm A}+s_B = 2-\sum_{ij}(p_{ii}p_{jj}+p_{ij}^2)|\langle\varphi_i|\varphi_j\rangle|^2 \geqslant 2-\sum_{ij}(p_{ii}p_{jj}+p_{ij}^2) = 1-\sum_{ij}p_{ij}^2 = s_{\rm AC},$$

one verifies that the Araki–Lieb inequality holds for the linear entropy, too.

Appendix B. Solving the equation for 'off-diagonal' Wigner function

The evolution equation (90) can be considered as a 'Schrödinger equation' for the 'wavefunction' $W(q,p,t)$ with some effective quadratic non-Hermitian Hamiltonian [41, 52]:

$$\frac{\partial W}{\partial t} = \hat{\mathcal{H}}W, \qquad \hat{\mathcal{H}} = \tfrac{1}{2}\hat{\mathbf{z}}\mathcal{B}\hat{\mathbf{z}}+\gamma, \tag{B.1}$$

where the four-dimensional operator vector $\hat{\mathbf{z}}$ is defined as $\hat{\mathbf{z}} = (\partial/\partial q, \partial/\partial p, q, p)$, and the symmetrical 4×4 matrix $\mathcal{B}$ consists of the following 2×2 blocks:

$$\mathcal{B} = \begin{Vmatrix} b_1 & b_2 \\ b_3 & b_4 \end{Vmatrix}, \qquad b_1 = \begin{Vmatrix} \tilde{D}_q & 0 \\ 0 & \tilde{D}_p \end{Vmatrix}, \tag{B.2}$$

$$b_2 = b_3 = \gamma E_2, \qquad b_4 = -2{\rm i}\quad E_2. \tag{B.3}$$

E_n stands for the $n\times n$ unit matrix.

To find the propagator of equation (B.1), we introduce the operator (defined on the solutions of equation (B.1))

$$\hat{\mathbf{Z}}(t) = \hat{U}(t)\hat{\mathbf{z}}\hat{U}^{-1}(t), \tag{B.4}$$

where $\hat{U}(t)$ is the evolution operator of equation (B.1). $\hat{\mathbf{Z}}(t)$ is the *operator integral of motion*, in the sense that it transforms any solution to equation (B.1) into another solution of the same equation. A consequence of equations (B.1) and (B.4) is the equation $\partial\hat{\mathbf{Z}}/\partial t = [\hat{\mathcal{H}},\hat{\mathbf{Z}}]$ (which holds for the solutions of the evolution equation). Therefore, since the 'Hamiltonian' $\hat{\mathcal{H}}$ is *quadratic* with respect to the components of vector $\hat{\mathbf{z}}$, one can easily verify that operator $\hat{\mathbf{Z}}(t)$ can be expressed as a linear transformation of the 'initial' operator vector $\hat{\mathbf{z}}$:

$$\hat{\mathbf{Z}}(t) = \quad(t)\hat{\mathbf{z}}, \qquad = \begin{Vmatrix} \lambda_1 & \lambda_2 \\ \lambda_3 & \lambda_4 \end{Vmatrix}, \tag{B.5}$$

provided the matrix (t) satisfies the following equation and initial condition:

$$\dot{} = - \quad \mathcal{B}, \qquad (0) = E_4, \qquad = \begin{Vmatrix} 0 & E_2 \\ -E_2 & 0 \end{Vmatrix}. \tag{B.6}$$

The 2×2 blocks of matrix (t) satisfy the equations

$$\dot{\lambda}_1 = -\lambda_1 b_3+\lambda_2 b_1, \qquad \dot{\lambda}_2 = -\lambda_1 b_4+\lambda_2 b_2, \qquad \dot{\lambda}_3 = -\lambda_3 b_3+\lambda_4 b_1, \qquad \dot{\lambda}_4 = -\lambda_3 b_4+\lambda_4 b_2. \tag{B.7}$$

An immediate consequence of the definition (B.4) is the operator equation $\hat{\mathbf{Z}}(t)\hat{U} = \hat{U}\hat{\mathbf{z}}$, which is equivalent, in view of (B.5), to the following first-order partial differential equations:

$$\lambda_3(t)\partial G/\partial\mathbf{q}+\lambda_4(t)\mathbf{q}G = \mathbf{q}'G, \tag{B.8}$$

$$\lambda_1(t)\partial G/\partial\mathbf{q}+\lambda_2(t)\mathbf{q}G = -\partial G/\partial\mathbf{q}', \tag{B.9}$$

where $\mathbf{q} = (q,p)$. Solving equations (B.8) and (B.9) together with (B.1) (which determines the time dependence of the pre-exponential factor) we obtain (for details see [41, 52, 50])

$$G(\mathbf{q};\mathbf{q}';t) = [\det(\lambda_3)]^{-1/2}\exp[\gamma t-\tfrac{1}{2}\mathbf{q}\lambda_3^{-1}\lambda_4\mathbf{q} + \mathbf{q}\lambda_3^{-1}\mathbf{q}'-\tfrac{1}{2}\mathbf{q}'\lambda_1\lambda_3^{-1}\mathbf{q}'], \tag{B.10}$$

where the normalization is chosen according to the relation

$$W(\mathbf{q},t) = \int G(\mathbf{q};\mathbf{q}';t)W(\mathbf{q}',0)\frac{{\rm d}\mathbf{q}'}{(2\pi)^N}, \tag{B.11}$$

N being the number of components of the vector $\mathbf{q}$. Note that the matrix λ_2 does not enter the right-hand side of (B.10), although it is contained in equation (B.9). This is because the elements of matrix (t) are not independent. This matrix is *symplectic* (albeit complex):

$$(t)\quad\tilde{}\,(t) \equiv \quad , \tag{B.12}$$

as a consequence of equation (B.6), the symmetry of the matrix $\mathcal{B}$ and the antisymmetry of the matrix (here $\tilde{}$ means a transposed matrix). One of the consequences of the identity (B.12) is the symmetry of matrices $\lambda_3^{-1}\lambda_4$ and $\lambda_1\lambda_3^{-1}$. Moreover, in the case of the time-independent matrix $\mathcal{B}$ we have the solution $(t) = \exp(-\mathcal{B}t)$, which gives rise to additional identities:

$$(-t) = \quad^{-1}(t) = \quad\tilde{}\,(t)\quad^{-1}. \tag{B.13}$$

These identities enable us to further simplify the propagator, writing it in terms of only two matrices λ_3 and λ_4 (one of which, in turn, can be expressed in terms of another and its time derivative):

$$G(\mathbf{q};\mathbf{q}';t) = (\det[\lambda_3(t)])^{-1/2}\exp[\gamma t-\tfrac{1}{2}\mathbf{q}\lambda_3^{-1}(t)\lambda_4(t)\mathbf{q} + \mathbf{q}\lambda_3^{-1}(t)\mathbf{q}'+\tfrac{1}{2}\mathbf{q}'\lambda_3^{-1}(-t)\lambda_4(-t)\mathbf{q}']. \tag{B.14}$$

In the case of matrices b_j given by (B.2) and (B.3) we have

$$\lambda_3 = (\dot{\lambda}_4-\gamma\lambda_4)/(2{\rm i}\quad), \tag{B.15}$$

$$\ddot{\lambda}_4 = \lambda_4\begin{Vmatrix} \gamma^2+2{\rm i}\quad\tilde{D}_q & 0 \\ 0 & \gamma^2+2{\rm i}\quad\tilde{D}_p \end{Vmatrix}, \tag{B.16}$$

and the characteristic equation connected with (B.16) turns out to be biquadratic, with the roots $\pm\lambda_\pm$, where $\lambda_\pm$ are given by (96). Taking into account the initial conditions, we arrive at the following expressions for the matrices $\lambda_4(t)$ and $\lambda_3(t)$:

$$\lambda_4 = \begin{Vmatrix} F_- & 0 \\ 0 & F_+ \end{Vmatrix}, \qquad \lambda_3 = \begin{Vmatrix} K_- & 0 \\ 0 & K_+ \end{Vmatrix},$$

where

$$F_\pm = C_\pm + \gamma S_\pm, \qquad K_\pm = \tfrac{1}{2}[2\gamma(1 + 2\nu_\pm) + \mathrm{i}\ \ ]S_\pm,$$

and other symbols are defined by equations (95) and (96). Putting these matrices in (B.14) we obtain the following explicit expression for the propagator of equation (84):

$$G(q, p; q', p'; t) = (K_+K_-)^{-1/2}\exp\left\{\gamma t + \frac{\tilde{p}\tilde{p}'}{K_+} + \frac{\tilde{q}\tilde{q}'}{K_-}\right.$$
$$\left. - \frac{1}{2}\left[\frac{F_+}{K_+}\tilde{p}^2 + \frac{F_-}{K_-}\tilde{q}^2\right] + \frac{1}{2}\left[\frac{\tilde{F}_+}{K_+}\tilde{p}'^2 + \frac{\tilde{F}_-}{K_-}\tilde{q}'^2\right]\right\}, \quad \text{(B.17)}$$

where the rotated variables $\tilde{q}$ and $\tilde{p}$ are defined as in equations (88) and (89), and functions $\tilde{F}_\pm$ are defined in (94). Multiplying the propagator (B.17) by the initial function (44) written in terms of real variables as

$$W_{\mathrm{ef}}(q', p', 0) = \exp[-(q' - \sqrt{2}|\alpha|\cos\chi)^2$$
$$- (p' - \sqrt{2}|\alpha|\sin\chi)^2]$$

and integrating over $\mathrm{d}q'\,\mathrm{d}p'$ with the aid of formula (42), we arrive at the expression (92).

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