

A consistent quantum model for continuous photodetection processes

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Abstract

We modify some aspects of the continuous photodetection theory proposed by Srinivas and Davies (SD) (1981 *Opt. Acta* **28** 981), which describes the non-unitary evolution of a quantum field state subjected to a continuous photocount measurement. In order to remedy inconsistencies that appear in their approach, we redefine the ‘annihilation’ and ‘creation’ operators that enter in the photocount super-operators. We show that this new approach not only still satisfies all the requirements for a consistent photocount theory according to SD precepts, but also avoids some weird result appearing when previous definitions are used.

Keywords: Continuous photodetection, exponential phase operators, master equation, thermal, coherent, binomial and negative binomial states

1. Introduction

The subject of quantum measurements is as old as the very foundation of quantum mechanics. For a long time the scheme proposed by von Neumann [1] has been prevalent. According to this scheme, there are two kinds of evolution of quantum states: a unitary evolution obeying the rules of quantum dynamics in the absence of measurements, and an ‘extra-quantum’ dynamics resulting from a measurement, when the state vector suffers a sudden, instantaneous and irreversible transformation (or reduction, or collapse) to one of possible eigenstates compatible with the observable measured.

When one considers, for instance, a photocount process, the radiation impinges a photomultiplier tube and each read burst of electrons (current) is viewed as a manifestation of a single photon. A sequence of bursts is associated with photocounts. A classical theory describing this process was proposed by Mandel [2]. However, it resulted, under certain particular circumstances, in negative probabilities of counts. Thence, refined quantum photocount theories were developed by Mandel *et al* [3, 4], Glauber [5], Kelley and Kleiner [6], Mollow [7], Scully and Lamb [8], and others (see the review [9] for more references).

However, these theories relied on the assumption of instantaneous measurement, although in reality photons are counted sequentially, one by one. In the 1980s Srinivas and Davies [10] (SD) developed a theory describing actual photocounting events, which was adopted for estimating the outcome in several problems, such as quantum non-demolition measurements [11, 12], determination of field states under continuous photodetection process [13], quantum theory of field-quadrature measurements [14], conditional generation of special states [15], and for the control of the amount of entanglement between two fields [16].

SD theory considers photodetection as a continuous measurement, with no reference to a ‘meter state’. The main quantities to be calculated in this theory are probability distributions for counts (or no counts). This is based on the assumption that in an infinitesimal time interval τ only two processes may occur: either one-count, characterized by a super-operator J acting on the field density operator ρ , or no-count, characterized by another super-operator, S_τ . Super-operator J in the SD theory has the form

$$J\rho = \gamma a\rho a^\dagger \quad (1)$$

where γ is the detector efficiency and a (a^\dagger) is the field ‘annihilation’ (‘creation’) operator ($[a, a^\dagger] = 1$). Such a choice is based on the assumption that operator a subtracts

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one photon from the field. It is assumed [13] that just after one count (conceived as *subtraction* of a single photon from the field) the system state is given by

$$\rho(t^+) = \frac{J\rho(t)}{\text{Tr}[J\rho(t)]}, \quad (2)$$

where t^+ stands for t plus an infinitesimal time after. (From another point of view, the states described by means of the statistical operators of the form (2) were considered in [17–19] under the name ‘photon-subtracted states’.)

One can easily check that the mean number of photons in the state (2) (i.e. after counting one photon) equals

$$\bar{n}(t^+) = \bar{n}(t) + \left[\frac{\overline{\Delta n^2}(t) - \bar{n}(t)}{\bar{n}(t)} \right] = \bar{n}(t) + q, \quad (3)$$

where $\overline{\Delta n^2} = \overline{n^2} - \bar{n}^2$ and q is Mandel’s q parameter [20] characterizing the type of photon statistics in the initial state of field: for $q < 0$ ($q > 0$) the field statistics is said to be sub-Poissonian (super-Poissonian) and for $q = 0$ the statistics is Poissonian. However, equation (3) clearly shows that the mean number of photons *may increase* after one count, if the field statistics is super-Poissonian. Otherwise, the mean number will remain the same if the statistics is Poissonian, and will decrease only if the statistics is sub-Poissonian. For example, for a field represented by a Fock state $|m\rangle$, one gets $\bar{n}(t^+) = m - 1$, exactly one photon less, whereas for any other field state this is not true. This point received special attention in [21]. Thus, we see that in general the common choice for J does not really mean counting one less photon in the field. Similar observations were made in [13, 18, 22, 23], however without attempts to modify the theory of photocounting processes.

In fact, SD themselves [10] perceived that the super-operator J does not satisfy assumption (V) of their theory, namely the boundedness property $\text{Tr}(J\rho) < \infty$. In fact, J is an unbounded linear transformation and consequently the counting rate is unbounded, thus not defined for all possible states. This fact leads to an ill-defined coincidence probability density. We return to this point in the next section.

In this paper we propose some modifications in the SD photocount theory in order to satisfy *all* the assumptions proposed by its authors. Our motivation finds ground in the recent discussion about the role of the ‘annihilation’ operator a in quantum optics [21], since state $a|\psi\rangle$ is not always a state whose mean number of photons is less than in $|\psi\rangle$. Depending on the field statistics, state $a|\psi\rangle$ may show much higher mean number of photons than state $|\psi\rangle$. In [21] the authors suggested that instead of a and a^\dagger the *exponential phase operators* E_- and E_+ should be considered as real ‘annihilation’ and ‘creation’ operators in the photocounting theory. The introduction of these operators in the continuous photocounting theory, besides eliminating inconsistencies in the SD proposal, leads to new interesting results related to the counting statistics.

This paper is organized as follows. In section 2 we briefly revise the main aspects of quantum photodetection theories. In section 3 we present our model based on exponential phase operators. In section 4 we consider the field state evolution under continuous monitoring, but when no information about

the number of counted photons is read out, or the ‘pre-selection’ state evolution. Also, in this section we solve the master equation generated by the exponential phase operators instead of the annihilation/creation ones (at zero temperature) and consider several important special cases. In section 5 we discuss a physical meaning of the results obtained, pointing at the principal differences in the predictions of the SD theory and our model, which could be verified experimentally.

2. Fundamentals of conventional quantum photodetection theories

The first quantum photodetection theory, developed independently by Mandel *et al* [3, 4], Glauber [5], and Kelley and Kleiner [6] as a simple extension of the classical theory [2], gave the following probability of k counts:

$$\begin{aligned} P(k, t) &= \text{Tr} \left\{ \rho : \frac{1}{k!} [\gamma t I(t)]^k e^{-\gamma t I(t)} : \right\} \\ &= \sum_{n=k}^{\infty} \binom{n}{k} (1 - \gamma t)^{n-k} (\gamma t)^k p_n. \end{aligned} \quad (4)$$

Here ρ is the statistical operator of the field (we consider a simplified model of a one-mode field), γ is the detector efficiency, $I(t)$ is the average field intensity, $::$ stands for operator normal ordering and $p_n = \langle n|\rho|n\rangle$ is the probability to have n photons in the given field mode. However, formula (4) becomes obviously meaningless if $\gamma t > 1$, when it can result in negative probabilities or unlimited mean number of counted photons as $t \rightarrow \infty$. These troubles were removed in studies [7, 8, 10], whose authors, using different approaches, arrived at the same result, which consists, from the formal point of view, in the substitution $\gamma t \rightarrow 1 - \exp(-\gamma t)$ in equation (4):

$$P^{SD}(k, t) = \sum_{n=k}^{\infty} \binom{n}{k} (1 - e^{-\gamma t})^k (e^{-\gamma t})^{n-k} \langle n|\rho|n\rangle. \quad (5)$$

The SD photocount theory does not refer to a specific detector state. It was built by considering two kinds of events represented by super-operators acting continuously on the field state. The first one, represented by J , is a single instantaneous count event, while the other, represented by S_t , is a no-count event for a time interval t . Thus, the operation (a super-operator)

$$N_t(k) = \int_0^t dt_k \int_0^{t_k} dt_{k-1} \cdots \int_0^{t_2} dt_1 S_{t-t_k} J S_{t_k-t_{k-1}} \cdots J S_{t_1} \quad (6)$$

stands for the count of k photons from the field. The operator (6) acts continuously the initial field state ρ , and $P(k, t) = \text{Tr}[N_t(k)\rho]$ is the probability of counting exactly k -photons in a time interval t .

The assumptions or properties of the SD theory are as follows.

- (I) $\rho \rightarrow N_t(k)\rho$ is a linear positive map on the space $\mathcal{T}(\mathcal{H})$ of trace class operators on the Hilbert space \mathcal{H} , such that for any positively definite ρ satisfying $\text{Tr} \rho = 1$, one has

$$0 \leq \text{Tr}[N_t(k)\rho] \leq 1. \quad (7)$$

(II) The sum over all possible counted photons satisfies the normalization condition $\text{Tr}[T_t \rho] = 1$ where

$$T_t = \sum_{k=0}^{\infty} N_t(k) = S_t + \int_0^t T_{t-t'} J S_{t'} dt'. \quad (8)$$

(III) Semigroup associative property

$$N_{t_1+t_2}(k) = \sum_{k_1+k_2=k} N_{t_2}(k_2) N_{t_1}(k_1). \quad (9)$$

(IV) Identity

$$\lim_{t \rightarrow 0} N_t(0) \rho = \rho. \quad (10)$$

(V) Assumption of bounded interaction rate: there exists a number $K < \infty$ such that

$$\sum_{k=1}^{\infty} \text{Tr}[N_t(k) \rho] < Kt, \quad \text{for } t > 0. \quad (11)$$

(VI) Assumption of ideality: the operation

$$S_t = N_t(0) \quad (12)$$

transforms pure states into pure states, noting that $S_t \rho$ is a pure state if ρ is a pure state.

As a matter of fact the process of photocounting by a macroscopic detector is the resultant of many microscopic fundamental interactions between the electromagnetic field and atoms composing the detector. But how can these fundamental interactions be specified by a reasonable model for the count (or no-count) jump super-operator in a phenomenological macroscopic photodetection theory? It is worth quoting from the SD paper [10]: ‘As a simple model for the measurement performed by the photodetector, where an n -photon state will be converted to an $(n-1)$ -photon state whenever a photon is detected, it is reasonable to set’ the single photon counting super-operator J in the form (1). We see that this operator was not derived from fundamental processes, but set *ad hoc* due to its apparent simplicity and reasonability.

As soon as J is chosen, the super-operator for the state evolution between consecutive counts can be derived as

$$S_t \rho = e^{Yt} \rho e^{Y^\dagger t}, \quad Y = -iH_0 - \gamma a^\dagger a / 2, \quad (13)$$

once it conserves the probability in a regular point process:

$$\text{Tr}[J\rho + Y\rho + \rho Y^\dagger] = 0. \quad (14)$$

Note, however, that the super-operators J and S_t defined above *do not satisfy* condition (V), since

$$\text{Tr}(J\rho) = \gamma \text{Tr}(\rho a^\dagger a), \quad (15)$$

or more generally,

$$\text{Tr}[J^k \rho] = \gamma^k \text{Tr}[\rho : n^k :] \quad (16)$$

is unbounded, so not defined for all states. The violation of assumption (V) prevents a consistent definition for all states of some important functions. For example, although one can define the elementary probability density of counting k photons

in a time interval t , attempts to calculate the coincidence probability density

$$h(t_1, \dots, t_k) = \text{Tr}(T_{t-k} J \dots J T_{t_1} \rho) \quad (17)$$

of counts observed at each of the times t_1, \dots, t_k together with other possible counts in between, when the detector is making measurements for a time interval t , result in serious problems in the SD theory.

One of the goals of our paper is to show how one can avoid the violation of assumption (V) using other operators instead of a and a^\dagger .

3. Quantum counting processes with exponential phase operators

Our idea is to use, instead of operators a and a^\dagger , the so-called *exponential phase operators*

$$E_- \equiv (a^\dagger a + 1)^{-1/2} a, \quad (18)$$

$$E_+ \equiv a^\dagger (a^\dagger a + 1)^{-1/2}, \quad (19)$$

introduced, as a matter of fact, by London [24] at the dawn of quantum mechanics, although their systematic use began only after the paper by Susskind and Glogower [25] (for the history see [26]). The commutator of E_- and E_+ is the vacuum state projector,

$$[E_-, E_+] = |0\rangle\langle 0| \equiv \Lambda_0. \quad (20)$$

The normal ordered product of these operators is a complementary projector

$$E_+ E_- = 1 - \Lambda_0 = \Lambda = \Lambda^\dagger, \quad \Lambda^2 = \Lambda. \quad (21)$$

Applying E_- and E_+ on the number states one gets

$$E_- |n\rangle = |n-1\rangle, \quad E_+ |n\rangle = |n+1\rangle \quad (22)$$

with $E_- |0\rangle = \Lambda |0\rangle = 0$, and $[\Lambda, n] = 0$ (where $n = a^\dagger a$ is the number operator). A useful property is

$$e^{\alpha \Lambda} = \Lambda_0 + e^\alpha \Lambda. \quad (23)$$

For other properties and generalizations see, e.g., [27–36]. Finally, it is worth recalling that E_- has as eigenstate the ‘coherent phase state’

$$|\psi\rangle = \sqrt{1 - |z|^2} \sum_{n=0}^{\infty} z^n |n\rangle, \quad |z| < 1 \quad (24)$$

as introduced in [37] and studied in [38–47] (see also [48]).

3.1. One-count event

We redefine the one-count operator equation (1) as

$$J\rho = E_- \rho E_+. \quad (25)$$

Now J is bounded operator and the system state immediately after the one-count process in the time interval $[0, t)$ is transformed into

$$\tilde{\rho}(t^+) = \frac{J\rho(t)}{\text{Tr}[J\rho(t)]} = \frac{J\rho(t)}{1 - p_0}, \quad (26)$$

where $p_0 \equiv \langle 0|\rho(t)|0\rangle$ is the probability for the vacuum state. (Note that pure states $\hat{E}_+^m|\psi\rangle$ were considered in another context in [49]. Mixed *shifted thermal states* $\hat{\rho}_{th}^{(shift)} = \hat{E}_+^m \hat{\rho}_{th} \hat{E}_-^m$ were studied in [50], whereas methods of generating such states in a micromaser were discussed in [51].)

The mean number of photons in the state $\tilde{\rho}(t)$ (26) is

$$\tilde{n}(t^+) = \frac{\bar{n}(t)}{1 - p_0} - 1, \quad (27)$$

so, whenever a state ρ has none, or very small, contribution from the vacuum state, the counting operation extracts exactly one photon from the system, independently of the field statistics. For example, for the *number state* $\rho = |m\rangle\langle m|$ ($m \neq 0$), $\tilde{n} = m - 1$, and for the *coherent state* $\rho = |\alpha\rangle\langle\alpha|$ ($\alpha \neq 0$), $\tilde{n} = \bar{n}/(1 - e^{-\bar{n}}) - 1$, with $\bar{n} = |\alpha|^2$.

On the other hand, for the *thermal state*

$$\rho = \frac{1}{1 + \bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1 + \bar{n}} \right)^n |n\rangle\langle n| \quad (28)$$

we obtain $\tilde{n} = \bar{n}$, i.e. the mean number of photons is not changed, as expected. This is a correct description of a thermal system since taking out a single photon from a reservoir should not change its average number. Note that using the SD definition, $J\rho = a\rho a^\dagger$, one obtains the weird result $\tilde{n} = 2\bar{n}$. So one perceives that using a and a^\dagger for constructing a continuous photocount measurement leads to some inconsistent results.

3.2. No-count event

The time evolution between sequential counts is represented by $S_t \equiv N_t(0)$, a super-operator defined in terms of ordinary Hilbert space operators

$$S_t \rho = e^{Yt} \rho e^{Y^\dagger t}, \quad (29)$$

where $B_t = e^{Yt}$ is a semigroup element given in terms of the generator Y . The deduction of S_t is conditioned to the relation

$$\text{Tr}[J\rho] = \text{Tr}[\rho R], \quad (30)$$

where R is the rate operator, related to Y by $\text{Tr}(\rho R) = \text{Tr}(Y\rho + \rho Y^\dagger)$, which substituted in (30) gives equation (14). The theory requires that in the absence of counts the system has a unitary evolution, whose dynamics is governed by the free-field Hamiltonian $H = \hbar\omega a^\dagger a$. Thus, the convenient choice satisfying (14) is

$$Y = -iH - R/2 = -iH - \frac{\gamma}{2} E_+ E_-. \quad (31)$$

Taking into account equations (21) and (23), as well as the commutativity of operators H and Λ , one can easily calculate the result of action of nonunitary operator $e^{Y\tau}$ on a pure state $|\psi\rangle$ (here τ is an interval of time between counts)

$$|\psi_S(\tau)\rangle = e^{Y\tau} |\psi\rangle = \langle 0|\psi\rangle |0\rangle + e^{-\gamma\tau/2} \Lambda |\psi_H(\tau)\rangle, \quad (32)$$

where $|\psi_H(\tau)\rangle = \exp(-iH\tau)$ is the freely evolved state vector (in the absence of measurements). Thus the probability of *no-count* event equals

$$\mathcal{P}_0(\tau) = \|\psi_S(\tau)\|^2 = |\langle 0|\psi\rangle|^2 + e^{-\gamma\tau} \langle \psi|\Lambda|\psi\rangle. \quad (33)$$

For a mixed state, the same probability is given by $\text{Tr}[S_\tau \rho]$, and simple calculations result in the formula

$$\mathcal{P}_0(\tau) = e^{-\gamma\tau} + p_0(1 - e^{-\gamma\tau}), \quad (34)$$

which, of course, coincides with (33) in the case of pure state. Note that $\lim_{\tau \rightarrow \infty} \mathcal{P}_0(\tau) = p_0$, which means that the probability of no counts registered during an infinite time interval is equal to the probability of finding the vacuum state in the measured state ρ . Formula (34) should be compared with analogous formula of the SD theory based on the operator Y of the form (13)

$$\mathcal{P}_0(\tau) = \sum_{n=0}^{\infty} p_n e^{-n\gamma\tau}. \quad (35)$$

Although equations (34) and (35) give the same limits for $\tau \rightarrow \infty$, the intermediate time dependencies are different.

3.3. Continuous counting

The continuous counting of k -photons from a field in a time interval t is represented by a linear operator $N_t(k)$ acting on the system state during a time interval $[0, t)$,

$$\tilde{\rho}^{(k)}(t) = \frac{N_t(k)\rho(0)}{\text{Tr}[N_t(k)\rho(0)]}, \quad (36)$$

where $\rho(0)$, or simply ρ , is the state of field prior to the counting process and $P(k, t) = \text{Tr}(N_t(k)\rho)$ is the probability of counting k photons in t . The linear operator $N_t(k)$ can be written in terms of the operators S_t and J as

$$N_t(k) = \int_0^t dt_k \int_0^{t_k} dt_{k-1} \cdots \int_0^{t_2} dt_1 S_{t-t_k} J S_{t_k-t_{k-1}} \cdots J S_{t_1}. \quad (37)$$

Noticing, however, that

$$J S_t \rho = e^{-\gamma t} \mathcal{U}_t(J\rho), \quad (38)$$

one gets

$$S_{t-t_k} J S_{t_k-t_{k-1}} \cdots J S_{t_1} = e^{-\gamma t_k} \mathcal{U}_{t_k} S_{t-t_k} J^k, \quad (39)$$

where

$$\mathcal{U}_t \rho = e^{-iHt} \rho e^{iHt}. \quad (40)$$

It is convenient to introduce short notation for two partial sums of probabilities:

$$\mathcal{A}_k = \sum_{n=0}^k p_n, \quad \mathcal{Z}_{k+1} = \sum_{n=k+1}^{\infty} p_n \equiv 1 - \mathcal{A}_k. \quad (41)$$

Using (39) we can calculate the elementary probability distribution (EPD) of counts at the instants t_1, t_2, \dots, t_k , if the total measurement time is t ,

$$\begin{aligned} \mathcal{P}(t_1, t_2, \dots, t_k; t) &\equiv \text{Tr}[S_{t-t_k} J S_{t_k-t_{k-1}}, \dots, J S_{t_1} \rho] \\ &= \gamma^k (e^{-\gamma t_k} p_k + e^{-\gamma t} \mathcal{Z}_{k+1}), \end{aligned} \quad (42)$$

which should be compared to the EPD in SD theory,

$$\begin{aligned} \mathcal{P}_{SD}(t_1, t_2, \dots, t_m; t) &= \gamma^m m! e^{-\gamma(t_1+t_2+\dots+t_m-mt)} \\ &\times \sum_{n=m}^{\infty} \binom{n}{m} e^{-\gamma n t} p_n. \end{aligned} \quad (43)$$

In particular, for a sufficiently long counting time $t \rightarrow \infty$ we obtain

$$\mathcal{P}(t_1, t_2, \dots, t_k; \infty) = \gamma^k e^{-\gamma t_k} p_k, \quad (44)$$

whereas the SD theory yields an essentially different result for $k \geq 2$:

$$\mathcal{P}_{SD}(t_1, \dots, t_m; \infty) = \gamma^m m! e^{-\gamma(t_1+t_2+\dots+t_m)} p_m. \quad (45)$$

Only for the one-photon event, $k = 1$, do both models predict the same exponential probability distribution

$$\mathcal{P}(t_1) = \gamma e^{-\gamma t_1} p_1 \equiv \tilde{\mathcal{P}}(t_1) p_1.$$

We see that (42) corresponds to a Markovian process, in the sense that

$$\mathcal{P}(t_1, t_2, \dots, t_k; \infty) / p_k = \tilde{\mathcal{P}}(t_1) \tilde{\mathcal{P}}(t_2 - t_1) \cdots \tilde{\mathcal{P}}(t_k - t_{k-1}),$$

i.e. the EPD depends only on the last count, at time t_k . In contrast, in the SD theory the EPD depends on all times at which counts occur. Moreover, each new count enters with increasing weight:

$$\mathcal{P}_{SD}(t_1, t_2, \dots, t_m; \infty) / p_k = [\tilde{\mathcal{P}}(t_1)][2\tilde{\mathcal{P}}(t_2)] \cdots [m\tilde{\mathcal{P}}(t_m)].$$

Nonetheless, both distributions, (44) and (45), have the same normalization

$$\int_0^\infty dt_k \int_0^{t_k} dt_{k-1} \cdots \int_0^{t_2} dt_1 \mathcal{P}(t_1, t_2, \dots, t_k; \infty) = p_k.$$

We can write equation (37) as ($k = 1, 2, \dots$)

$$N_t(k) \rho = \mathcal{U}_t \int_0^t dt' e^{-\gamma t'} \frac{(t')^{k-1}}{(k-1)!} \exp\left[-\frac{\gamma}{2}(t-t')\Lambda\right] \times (J^k \rho) \exp\left[-\frac{\gamma}{2}(t-t')\Lambda\right], \quad (46)$$

where $t' \equiv t_k$ and the probability of occurrence of k counts in a time interval t is given by

$$P(k, t) = p_k \left[1 - e^{-\gamma t} \sum_{j=0}^k \frac{(\gamma t)^j}{j!} \right] + \frac{e^{-\gamma t} (\gamma t)^k}{k!} \sum_{n=k}^{\infty} p_n, \quad (47)$$

which verifies the normalization condition

$$\sum_{k=0}^{\infty} P(k, t) = 1. \quad (48)$$

The limiting value of equation (47)

$$\lim_{t \rightarrow \infty} P(k, t) = \langle k | \rho | k \rangle = p_k \quad (49)$$

means that, asymptotically, the counting statistics coincides with the photon statistics (as it should), since we do not consider here the possibility of photons lost to the surroundings or the failure to count any photon exiting the cavity. If a photon leaves the cavity it is detected and counted, that is certain.

The moments of distribution (47) are given by

$$\begin{aligned} \bar{k}_t^l &\equiv \sum_{k=0}^{\infty} P(k, t) k^l \\ &= \langle k^l \rangle - e^{-\gamma t} \left\{ \sum_{j=0}^{\infty} \frac{(\gamma t)^j}{j!} \sum_{k=j}^{\infty} k^l p_k - \sum_{k=0}^{\infty} \frac{k^l (\gamma t)^k}{k!} \sum_{j=0}^k p_j \right\}, \end{aligned} \quad (50)$$

where $\langle k^l \rangle \equiv \sum_{k=0}^{\infty} k^l p_k$. In particular, for the mean number of photons one verifies that $\lim_{t \rightarrow \infty} \bar{k}_t = \langle k \rangle$.

3.4. Examples of field states

Now let us consider specific field states, comparing the probabilities of k -counts resulting from the SD formula (5) and from our formula (47). For a field initially prepared in a number state $|m\rangle$, $m > k$, the probabilities are

$$P^{SD}(k, t) = \binom{m}{k} (1 - e^{-\gamma t})^k (e^{-\gamma t})^{m-k}, \quad (51)$$

$$P(k, t) = e^{-\gamma t} \frac{(\gamma t)^k}{k!}. \quad (52)$$

Different probabilities of a k -count reveal different physical schemes of counting. In SD's theory, $P^{SD}(k, t)$ is a binomial distribution reflecting an underlying one-dimensional 'random walk' process: the factor $(1 - e^{-\gamma t})^k$ is the probability of k photons leave the cavity, while $(e^{-\gamma t})^{m-k}$ is the probability that photons stay in the cavity, thus being not counted. On the other hand, the distribution (52) is Poissonian. That means that each photon leaving the cavity is counted, resulting in the same counting statistics as for falling raindrops in a small area, roughly the drop size. The last process is the only one consistent with the initial assumption that every photon leaving the cavity is counted. In figure 1 we compare both probability distributions, (51) and (52), for a field with $m = 5$, and selected k -counted photon numbers. $P(k, t)$ for (52) reveals a more widely spread shape than (51), a characteristic of Poissonian processes. In that figure we included the probability distributions for $k = m$. The SD theory gives the expression $P^{SD}(m, t) = (1 - e^{-\gamma t})^m$, while our model gives $P(m, t) = \Phi_{m-1}(\gamma t)$, where

$$\Phi_k(x) = 1 - e^{-x} \sum_{n=0}^k \frac{x^n}{n!} = e^{-x} \sum_{n=k+1}^{\infty} \frac{x^n}{n!}. \quad (53)$$

Notice that for $k > m$, both theories give a zero-valued probability distribution, which is a signature of the fixed number state for the field inside the cavity, meaning that, in any time interval t , it is not possible to count more photons than those present in the field at time $t = 0$.

For the coherent state $|\alpha\rangle$ SD theory gives

$$P^{SD}(k, t) = \frac{1}{k!} [|\alpha|^2 (1 - e^{-\gamma t})]^k \exp[-|\alpha|^2 (1 - e^{-\gamma t})],$$

while in our approach we get

$$P(k, t) = \frac{1}{k!} [(\gamma t)^k e^{-\gamma t} \Phi_{k-1}(|\alpha|^2) + |\alpha|^{2k} e^{-|\alpha|^2} \Phi_k(\gamma t)].$$

In figure 2 we display, for comparison, both distributions for a coherent state with average photon number $|\alpha|^2 = 5$, and selected counted photon numbers.

For the thermal state (28) we obtain the distributions

$$\begin{aligned} P^{SD}(k, t) &= \frac{[\bar{n}(1 - e^{-\gamma t})]^k}{[1 + \bar{n}(1 - e^{-\gamma t})]^{k+1}}, \\ P(k, t) &= \frac{e^{-\gamma t}}{k!} \left(\frac{\bar{n}\gamma t}{\bar{n} + 1} \right)^k + \frac{\bar{n}^k \Phi_k(\gamma t)}{(1 + \bar{n})^{k+1}}, \end{aligned}$$

which are displayed in figure 3, for $\bar{n} = 5$. For both the coherent and thermal states, the probability distribution for our

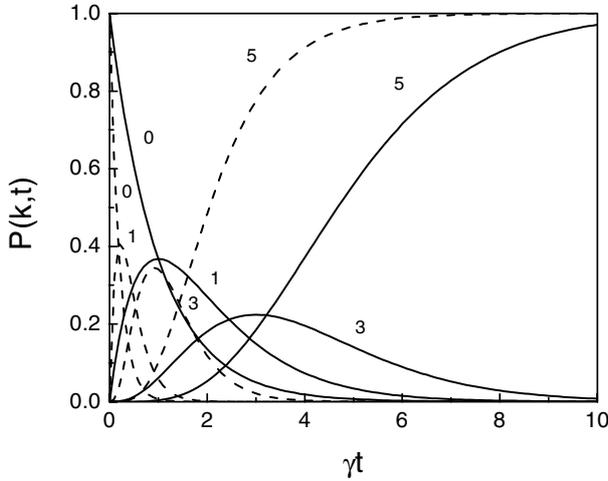


Figure 1. Conditional photocount probability distribution for the initial number state with $m = 5$ photons. Solid curves are for our model while dashed ones are for the original SD theory. Numbers above the curves correspond to the k -event.

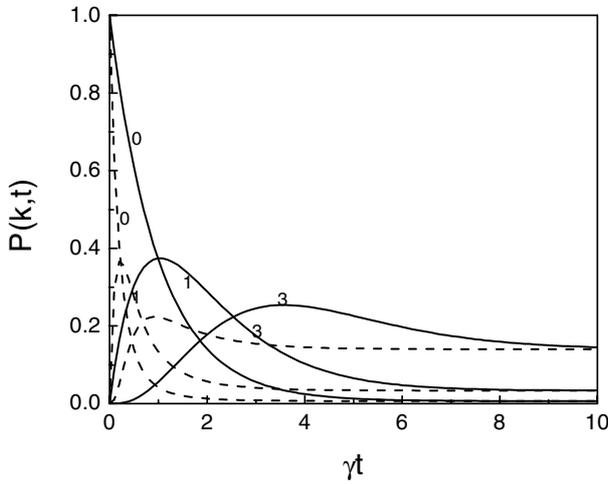


Figure 2. As figure 1, but for a coherent state with average $|\alpha|^2 = 5$ photons.

model shows a more widely spread shape than the distribution for the SD theory. This feature may be important in the distinction of both models for photocounting, by experimental evidence. By repeated experiments of photocounting the output of a leaking cavity, one may reconstruct these distributions, assuming that any other incoherent (dissipative) process is absent or negligible for the whole process.

4. ‘Pre-selection’ state evolution

Having no knowledge about the number of counted photons after a time interval t , the field state being continuously monitored but with no readout is given by

$$\tilde{\rho}(t) = T_t \rho = \sum_{k=0}^{\infty} N_t(k) \rho. \quad (54)$$

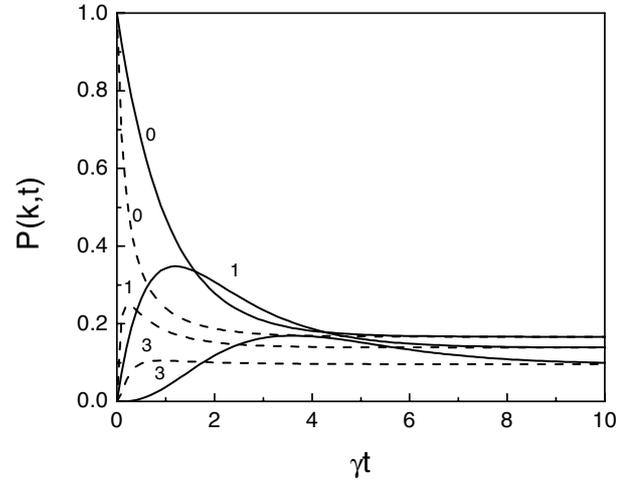


Figure 3. As figure 1, but for a thermal state with average $\bar{n} = 5$ photons.

This is referred to as the *pre-selection* state [52]. Summing over k in equation (46) we obtain

$$\tilde{\rho}(t) = S_t \rho + \sum_{k=1}^{\infty} N_t(k) \rho = \mathcal{U}_t \left\{ e^{-\gamma \Lambda t/2} \rho e^{-\gamma \Lambda t/2} + \int_0^t e^{-\gamma t'} e^{-\gamma \Lambda (t-t')/2} [e^{J t'} (J \rho)] e^{-\gamma \Lambda (t-t')/2} dt' \right\}. \quad (55)$$

The probability of having $n \neq 0$ photons in the cavity after continuous measurement for time t is equal to

$$\tilde{p}_n(t) = \langle n | \tilde{\rho}(t) | n \rangle = e^{-\gamma t} \sum_{l=0}^{\infty} \frac{(\gamma t)^l}{l!} p_{n+l}, \quad (56)$$

where p_{n+l} is the probability at $t = 0$. The probability to have no photons in the cavity at time t is obviously

$$\tilde{p}_0(t) = 1 - \sum_{n=1}^{\infty} \tilde{p}_n(t) = e^{-\gamma t} \sum_{l=0}^{\infty} \frac{(\gamma t)^l}{l!} \mathcal{A}_l, \quad (57)$$

where \mathcal{A}_l is defined in (41). So, if one waits a very long time the cavity will eventually end in the vacuum state, all photons having been absorbed by the detector and counted.

4.1. The master equation

Another way to obtain formula (56) is to use the phenomenological master equation of the Lindblad form (hereafter we suppress the tilde over the operator $\tilde{\rho}$)

$$\frac{\partial \rho(t)}{\partial t} = \frac{\gamma}{2} [2E_- \rho(t) E_+ - E_+ E_- \rho(t) - \rho(t) E_+ E_-]. \quad (58)$$

This should be compared with the ‘standard master equation’ [53, 54] for the amplitude damping model used in the SD theory, derivable from the interaction of a single electromagnetic (EM) mode (or 1D harmonic oscillator) with an environment made of many harmonic oscillators at $T = 0$ K (see, e.g., [55] for the most recent applications)

$$\frac{\partial \rho(t)}{\partial t} = \frac{\gamma}{2} [2a \rho(t) a^\dagger - a^\dagger a \rho(t) - \rho(t) a^\dagger a]. \quad (59)$$

If the EM mode is within a dissipative cavity, equation (59) can be viewed as describing the field state of an uncontrollable ‘photon-leaking’ process to the environment, although it could be not necessarily one-by-one, since $J = \gamma a \rho a^\dagger$ does not produce a minus-one-photon state. In contrast, if there is no vacuum, then it is certain that, inside the cavity, equation (58) describes the process of *subtracting photons from the cavity, sequentially and one-by-one*.

Equation (59) results in the equation for the mean photon number

$$-d\langle a^\dagger a \rangle^{(a)}/dt = \gamma \langle a^\dagger a \rangle = \gamma \sum_{n=1}^{\infty} n p_n, \quad (60)$$

whose solution

$$\langle a^\dagger a \rangle_t^{(a)} = \langle a^\dagger a \rangle_0^{(a)} e^{-\gamma t} \quad (61)$$

shows that the rate of decrease of the mean number of photons is proportional to the mean number of photons present inside the cavity (or in the beam, if beams are considered) resulting in an exponential decay as time goes on.

Equation (58) leads to a quite different differential equation for the mean photon number

$$-\partial \langle a^\dagger a \rangle^{(E)}/\partial t|_{t=0} = \gamma(1 - p_0) = \gamma \sum_{n=1}^{\infty} p_n, \quad (62)$$

according to which the rate of change in the mean number of photons is proportional to the probability that there are photons in the cavity, independent of their mean number. If initially $p_0 = 0$, this means that certainly there are photons in the cavity and equation (62) becomes

$$-\partial \langle a^\dagger a \rangle^{(E)}/\partial t|_{t=0} = \Gamma \langle a^\dagger a \rangle_{t=0}^{(E)} \langle a^\dagger a \rangle_{t=0}^{(E)} \quad (63)$$

with $\Gamma \langle a^\dagger a \rangle^{(E)} = \gamma / \langle a^\dagger a \rangle^{(E)}$. Or, to look at it another way, at initial times the rate does not depend on the mean photon number due to the nonconstant coupling parameter Γ , following the choice of operators E_- for picking out a single photon from the field.

Note that equations (60) and (62) practically coincide in the case of low field intensities, when $p_n \ll 1$ for $n \geq 2$ (this is especially clear from the expressions in the form of series over p_n). On the other hand, the ‘saturation’ of the decay rate in the case of high field intensities can be also understood if one takes into account a possibility of large dead time of the detector: in such a case, for short intervals of time, the detector can count (with some efficiency) only one photon, independent of the number of photons in the cavity or beam. This example shows that the decay rate (62) can by no means be considered as ‘unphysical’ *a priori*.

It is immediate to see that the solution to equation (62) is

$$\begin{aligned} \langle a^\dagger a \rangle^{(E)} &= \sum_{n=1}^{\infty} n \bar{p}_n(t) = e^{-\gamma t} \sum_{n=1}^{\infty} n \sum_{l=0}^{\infty} \frac{(\gamma t)^l}{l!} p_{n+l} \\ &= e^{-\gamma t} \left[\sum_{n=1}^{\infty} n p_n + (\gamma t) \sum_{n=1}^{\infty} n p_{n+1} + \dots \right] \end{aligned} \quad (64)$$

$$= e^{-\gamma t} \left[\bar{n} + (\gamma t) \sum_{n=1}^{\infty} n p_{n+1} + \dots \right]. \quad (65)$$

However, both (60) and (62) have the vacuum state as the asymptotic stationary state. Subtraction of (61) from (65) leads to

$$\langle a^\dagger a \rangle_t^{(E)} - \langle a^\dagger a \rangle_t^{(a)} = e^{-\gamma t} \sum_{l=1}^{\infty} \frac{(\gamma t)^l}{l!} \sum_{n=1}^{\infty} n p_{n+l} > 0, \quad (66)$$

which shows that if one uses the nonlinear operators E_- , E_+ , the calculated mean photon number of the remaining photons in the cavity is always higher than when a , a^\dagger operators are used. This is because, with E_- , E_+ -operators, the photons are always subtracted sequentially, one-by-one from any field state that does not contain the vacuum as one of its components, while when using a , a^\dagger operators, and depending on the field state, there exists the possibility that photon leak occurs through the escape of more than one photon at a time. Thus, the mean photon number reduction rate for E -operators does not follow an exponential law; it takes longer for a cavity to reach a given mean photon number than in the amplitude damping model. We will touch on this point again in the next section, in relation to some specific examples of field states.

4.2. Solutions of the master equation

The operator master equation (58) results in the following infinite set of coupled equations for the diagonal elements $p_n \equiv \langle n | \rho | n \rangle$ of the statistical operator in the Fock basis:

$$\dot{p}_n = p_{n+1} - (1 - \delta_{n0}) p_n, \quad (67)$$

where dot means the derivative with respect to the ‘slow time’ $\tau = \gamma t$. Making the Laplace transformation

$$\bar{p}_n(s) = \hat{\mathcal{L}}\{p_n(t)\} = \int_0^{\infty} e^{-s\tau} p_n(t) d\tau$$

and recalling that

$$\hat{\mathcal{L}}\{\dot{p}_n(\tau)\} = s \bar{p}_n(s) - p_n(0),$$

we obtain from (67) the following set of algebraic equations:

$$\bar{p}_{n+1}(s) = (s+1) \bar{p}_n(s) - p_n(0), \quad n \geq 1, \quad (68)$$

$$\bar{p}_1(s) = s \bar{p}_0(s) - p_0(0). \quad (69)$$

Their consequence is the relation

$$\begin{aligned} \bar{p}_k(s) &= s(s+1)^{k-1} \bar{p}_0(s) - (s+1)^{k-1} p_0(0) \\ &\quad - (s+1)^{k-2} p_1(0) - \dots - p_{k-1}(0). \end{aligned} \quad (70)$$

Suppose that only the states with $n \leq k$ were excited initially. Then $p_{k+1}(\tau) \equiv 0$ and consequently $\bar{p}_{k+1}(s) \equiv 0$. In this case the consequence of (70) is

$$\bar{p}_0(s) = \frac{p_0(0)}{s} + \frac{p_1(0)}{s(s+1)} + \frac{p_2(0)}{s(s+1)^2} + \dots + \frac{p_k(0)}{s(s+1)^k}. \quad (71)$$

Taking into account the relations

$$\hat{\mathcal{L}}\{\exp(-at)\} = (a+s)^{-1},$$

$$\hat{\mathcal{L}}\{t^m e^{-t}\} = m!(s+1)^{-m-1},$$

and the expansion

$$\frac{1}{s(s+1)^k} = \frac{1}{s} - \frac{1}{(s+1)} - \frac{1}{(s+1)^2} - \dots - \frac{1}{(s+1)^k},$$

we can find the inverse Laplace transform of (71):

$$p_0(\tau) = p_0(0) + p_1(0)[1 - e^{-\tau}] + p_2(0)[1 - e^{-\tau}(1 + \tau)] + \dots + p_k(0)\Phi_{k-1}(\tau), \quad (72)$$

where the function $\Phi_k(x)$ was defined in (53). Having arrived at the expression (72), one can verify that it does not depend on the initial auxiliary assumptions that $p_n(0) = 0$ for $n > k$, but it holds for any initial distribution. Knowing $p_0(\tau)$ one can find all other functions $p_n(\tau)$ from equations (67). Finally, one arrives at the expression which coincides exactly with (56):

$$p_m(\tau) = e^{-\tau} \sum_{k=0}^{\infty} \frac{\tau^k}{k!} p_{m+k}(0), \quad m \geq 1. \quad (73)$$

The reduced generating function of the diagonal matrix elements depends on time as follows:

$$\tilde{G}(z; \tau) \equiv \sum_{n=1}^{\infty} z^n p_n = e^{-\tau} \sum_{k=1}^{\infty} p_k(0) \sum_{n=1}^k \frac{z^n \tau^{k-n}}{(k-n)!}. \quad (74)$$

Thus the mean number of photons evolves as

$$\bar{n}(\tau) = \left. \frac{\partial \tilde{G}}{\partial z} \right|_{z=1} = e^{-\tau} \sum_{k=1}^{\infty} p_k(0) \sum_{n=1}^k \frac{n \tau^{k-n}}{(k-n)!}. \quad (75)$$

For the initial k -photon Fock state $|k\rangle$ we obtain

$$p_m^{(Fock)}(\tau) = e^{-\tau} \frac{\tau^{k-m}}{(k-m)!}, \quad 1 \leq m \leq k, \quad (76)$$

$$\bar{n}^{(Fock)}(\tau) = e^{-\tau} \sum_{n=1}^k \frac{n \tau^{k-n}}{(k-n)!}. \quad (77)$$

4.3. Special cases

The series in the right-hand side of equation (73) can be calculated analytically for the initial *negative binomial distribution* (the corresponding pure *negative binomial states* were introduced independently in [56–58])

$$p_n^{(negbin)}(0) = \frac{\Gamma(\mu + n) \mu^\mu \bar{n}_0^\mu}{\Gamma(\mu) n! (\bar{n}_0 + \mu)^{\mu+n}}, \quad \mu > 0, \quad (78)$$

where \bar{n}_0 is the initial average number of photons. The result is expressed in terms of the confluent hypergeometric function:

$$p_n(\tau) = p_n(0) e^{-\tau} \Phi\left(\mu + n; n + 1; \frac{\bar{n}_0 \tau}{\bar{n}_0 + \mu}\right) \quad (79)$$

$$= p_n(0) \exp\left(-\frac{\mu \tau}{\bar{n}_0 + \mu}\right) \Phi\left(1 - \mu; n + 1; \frac{-\bar{n}_0 \tau}{\bar{n}_0 + \mu}\right). \quad (80)$$

For integral values $\mu = 1, 2, \dots$ formula (80) can be written in terms of the associated Laguerre polynomials:

$$p_n(\tau) = \frac{\mu^\mu \bar{n}_0^\mu}{(\bar{n}_0 + \mu)^{\mu+n}} \exp\left(-\frac{\mu \tau}{\bar{n}_0 + \mu}\right) L_{\mu-1}^n\left(\frac{-\bar{n}_0 \tau}{\bar{n}_0 + \mu}\right).$$

The special case of $\mu = 1$ corresponds to the initial *thermal distribution* (coherent phase states in the case of pure quantum states)

$$p_n(0) = \bar{n}_0^n / (1 + \bar{n}_0)^{n+1}. \quad (81)$$

In this case the photon number distribution preserves its form:

$$p_n^{(th)}(\tau) = p_n(0) \exp[-\tau/(1 + \bar{n}_0)], \quad (82)$$

and the mean number of photons decreases with time exponentially, although the rate of decrease diminishes with increase of the initial mean number:

$$\bar{n}^{(th)}(\tau) = \bar{n}_0 \exp[-\tau/(1 + \bar{n}_0)]. \quad (83)$$

A formal substitution $\mu \rightarrow -M$ with an integer M transforms the negative binomial distribution (78) to the *binomial distribution* (pure *binomial states* were considered in [59] and rediscovered in [60, 61])

$$p_n^{(bin)}(0) = \frac{M! \bar{n}_0^n (M - \bar{n}_0)^{M-n}}{(M - n)! n! M^M}, \quad \bar{n}_0 \leq M. \quad (84)$$

Then equation (79) is transformed into the formula

$$p_n(\tau) = \left(\frac{\bar{n}_0}{M}\right)^n \left(1 - \frac{\bar{n}_0}{M}\right)^{M-n} e^{-\tau} L_{M-n}^n\left(\frac{\bar{n}_0 \tau}{\bar{n}_0 - M}\right).$$

In the case of the initial *Poissonian distribution* (*coherent pure states*),

$$p_n(0) = \frac{\bar{n}_0^n}{n!} \exp(-\bar{n}_0), \quad (85)$$

the series (73) is reduced to the modified Bessel function:

$$p_n^{(Pois)}(\tau) = e^{-\bar{n}_0 - \tau} (\bar{n}_0/\tau)^{n/2} I_n(2\sqrt{\bar{n}_0 \tau}), \quad (86)$$

$$\bar{n}^{(Pois)}(\tau) = e^{-\bar{n}_0 - \tau} \sum_{n=1}^{\infty} n (\bar{n}_0/\tau)^{n/2} I_n(2\sqrt{\bar{n}_0 \tau}). \quad (87)$$

The expressions (86) and (87) can be simplified in the asymptotical case $\bar{n}_0 \tau \gg 1$, when the modified Bessel functions can be replaced by exponentials. Actually, one needs an additional condition $\tau \gg \bar{n}_0$ to ensure that the simplified probabilities result in a convergent series whose value does not exceed 1. Thus for $\tau \gg \bar{n}_0 + \bar{n}_0^{-1}$ we can write

$$p_n^{(Pois)}(\tau) \approx \frac{(\bar{n}_0/\tau)^{n/2}}{(4\pi \sqrt{\bar{n}_0 \tau})^{1/2}} \exp[-(\sqrt{\bar{n}_0} - \sqrt{\tau})^2], \quad (88)$$

$$\bar{n}^{(Pois)}(\tau) \approx \frac{\bar{n}_0^{-1/4}}{\sqrt{4\pi}} \tau^{-3/4} \exp[-(\sqrt{\bar{n}_0} - \sqrt{\tau})^2]. \quad (89)$$

In figure 4 we compare $\bar{n}(\tau)/\bar{n}(0)$ for Fock, thermal and coherent states from (77), (83) and (87), respectively, with the corresponding result from the amplitude damping model (61). While the amplitude damping model presents an exponential decay independent of the field amplitude the exponential phase damping does not, the field taking longer to relax in this situation. Only for $k = 1$ and Fock states do the two processes coincide. On comparing the curves for Fock, thermal and coherent states we observe that the decay rate shows a strong dependence on the field state statistics. The more super-Poissonian is the field, the longer it will take to decay in the model based on the exponential phase operators. It is

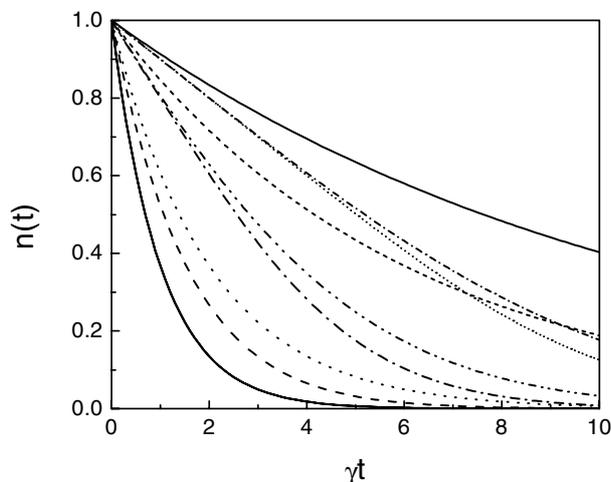


Figure 4. Normalized mean number of photons $n(t) \equiv \bar{n}(\gamma t)/\bar{n}(0)$ in the cavity under continuous measurement, for mean initial number of photons $\bar{n}(0) = 1, 5$ and 10 . Different line styles represent different states for each mean initial photon number. The three lower curves correspond to the Fock state (solid curve), coherent state (dashed), and thermal state (dotted), for $\bar{n}(0) = 1$. The three middle curves relate to the case of $\bar{n}(0) = 5$: Fock state (dash-dotted curve), coherent state (dash-dot-dotted), thermal state (short dashed). The three upper curves relate to the case of $\bar{n}(0) = 10$: Fock state (short dotted curve), coherent state (short dash-dotted), thermal state (solid). The lowest solid curve (the Fock state with $\bar{n}(0) = 1$) coincides with the exponential decay for the amplitude damping model.

interesting to note this dependence of the relaxation process on the field statistics. In discussions about the amplitude relaxation process (e.g. [62, 63]), this basic dependence is only attributed to coherence properties of quantum fields. In this respect the coherent state is said to be selected from all the states as the more robust to decoherence from the amplitude damping model [63]. We leave the discussion of coherence properties of the exponential phase model to a future publication, but we can anticipate from figure 4 that coherent states are not the most robust to dissipation in this model.

5. Conclusion

Summarizing, in this paper we have proposed modifications in the SD photocount theory in order satisfy *all* the precepts, as proposed by SD for a consistent theory. Our central assumption was the choice of the exponential phase operators E_- and E_+ as real ‘annihilation’ and ‘creation’ operators in the photocounting process, instead of a and a^\dagger . The introduction of those operators into the continuous photocount theory, besides eliminating inconsistencies, leads to new interesting results related to the counting statistics. A remarkable result, which is responsible for all the physical consistency of the model, is that in this new form an infinitesimal photocount operation J really takes out *one photon* from the field, if the vacuum state is not present. Consequently, the photocounting probability distribution for a Fock field state is Poissonian, evidencing again the direct correspondence of number of counted photons and number of photons taken from the field.

We have also investigated the evolution of the field state when photons are counted, but with no readout, leading to

the pre-selected state. The mean photon number change now shows (in contrast to the exponential law obtained for the amplitude damping model) a non-exponential law, which depends only on the condition that photons are present in the field, independent of their mean number.

An advantage of the proposed model is its mathematical consistency. Since many of its predictions, especially those related to multiphoton events, are significantly different from the predictions of the SD theory, it can be verified experimentally. One of the first questions which can be answered is whether the decrease of number of photons in the cavity due to their continuous counting always obeys the exponential law (61) (i.e. the rate of change is proportional to the instantaneous mean number of photons), or whether nonexponential dependences can be also observed (for example, in the case of detectors with large dead times).

We leave for a future work a detailed study of photocounting processes in the presence of other incoherent (dissipative) processes and discussion about coherence properties of the field under the exponential phase damping model. Also, an open question is how an extremely nonlinear (with respect to a and a^\dagger) one-count operator (25) can be derived from some microscopical model, starting from fundamental interactions which are linear with respect to operators a and a^\dagger (the model considered in our paper is purely phenomenological, as is the SD model).

We would like to emphasize that many preceding studies, such as [11–16], adopted the SD theory of photodetection. We believe that applications of our consistent model of photodetection to these problems may bring new and important results for both the quantum measurement theory and experiment.

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