Journal of Physics: Conference Series **99** (2008) 012006 doi:10.1088/1742-6596/99/1/012006

Theory of the dynamical Casimir effect in nonideal cavities with time-dependent parameters

V V Dodonov[1] **and A V Dodonov**[2]

[1] Instituto de Física, Universidade de Brasília, Caixa Postal 04455, 70910-900 Brasília, DF, Brazil

[2] Departamento de Física, Universidade Federal de São Carlos, Via Washington Luiz, Km 235, São Carlos, 13565-905, SP, Brazil

E-mail: vdodonov@fis.unb.br and adodonov@df.ufscar.br

Abstract.

This is a brief review of the recent progress in the theory of dynamical (non-stationary) Casimir effect in non-ideal cavities and of existing proposals to observe this effect in a laboratory, with an emphasis on the experiment which is under preparation in the University of Padua. The main idea of this experiment is to simulate periodical displacements of the cavity wall by periodical strong changes of conductivity of a thin semiconductor slab illuminated by picosecond laser pulses. In this connection the theory of quantum damped oscillator with arbitrary time dependence of the frequency and damping coefficient has been developed in order to take into account intrinsic losses in the semiconductor slab due to a finite conductivity during the intermediate part of the excitation-recombination cycle. The influence of different parameters, such as the diffusion and mobility coefficients of carriers, surface recombination velocity, absorption coefficient of laser radiation, thickness of the slab and geometry of the cavity, is analysed. Analytical and numerical evaluations show that under realistic experimental conditions, several thousand of quanta of EM field ('Casimir photons') could be produced from the initial vacuum state in a high-quality cavity at the lowest resonance frequency 2.5 GHz.

1. Introduction

Various quantum effects caused by a time dependence of dielectric properties of different media were subjects of numerous studies for the past decades. For example, the photon generation from vacuum due to temporal variations of the dielectric function was discussed in [1–6] while the generation of photons due to a motion of dielectric boundaries was studied in [7–9].

We consider the problem of photon generation in a selected mode of electromagnetic field inside a closed high-Q cavity due to periodical variations of the conductivity of a thin semiconductor layer deposited on the plane surface of a cavity wall. The main motivation for studying this problem is its relation to the experiment on observation of the nonstationary (or dynamical) Casimir effect (NSCE or DCE) which is under preparation in the university of Padua [10]. The idea is to use an effective electron-hole 'plasma mirror' created periodically on the surface of a semiconductor slab by illuminating it with a sequence of short laser pulses. If the interval between pulses exceeds the recombination time of carriers in the semiconductor, a highly conducting layer will periodically appear and disappear on the surface of the semiconductor film, thus simulating periodical displacements of the boundary.

Another scheme of simulating the dynamical Casimir effect, where periodical changes of the cavity eigenfrequency can be achieved by changing the surface impedance of a *superconducting film* illuminated by laser pulses, was proposed recently in [11]. Besides, a possibility of excitation of true surface vibrations of a cavity in the GHz band was claimed in [12] (this idea was formulated for the first time in [13, 14], but its realizability was questioned for a long time). However, the scheme of [10] seems to be the most promising from the point of view of reaching the final result in the nearest future, so we concentrate on it.

It is worth remembering that the whole story began almost 40 years ago, when Moore showed that the motion of ideal boundaries can result in creation of quanta of the electromagnetic field from the initial vacuum state [15]. But he concluded that the effect should be extremely small, if the velocities of boundaries are much less than the velocity of light. However, even earlier, a possibility of a significant amplification of *classical* electromagnetic fields inside cavities with oscillating boundaries under the conditions of parametric resonance was pointed out for the first time by Askar'yan in 1962 [16]. Later, a possibility of enhancement of vacuum (zero point) fluctuations under the conditions of resonance between field modes and oscillations of boundaries was discussed in [17, 18]. However, the first evaluations of the effect gave unrealistic numbers for two reasons: (i) approximate perturbation approaches, used in that papers, are invalid, as a matter of fact, under resonance conditions; (ii) the chosen amplitudes of oscillations of the cavity length were many orders of magnitude bigger than those which could be actually achieved in practice.

More precise and realistic calculations were performed only in 1990s in the frameworks of different approaches [13, 14, 19–22]. It was shown that a significant amount of photons could be created from vacuum, if boundaries of a high-Q cavity perform small oscillations at a frequency which is multiple of some cavity eigenfrequency. In particular, if a plane boundary of a *three-dimensional* cavity performs *harmonical* oscillations with an amplitude a at the frequency $\omega_w = 2\omega_0$, where ω_0 is the eigenfrequency of the lowest electromagnetic mode in the cavity with fixed geometry, then the mean number of photons created from vacuum in this mode is given by the formula [13, 14]

$$\langle n \rangle(t) = \sinh^2\left(\varepsilon\omega_0 t\eta^3\right), \tag{1}$$

where $\varepsilon = a/\lambda$ is the maximal relative displacement of the boundary (with respect to the wavelength $\lambda = 2\pi c/\omega_0$) and $\eta = \lambda/(2L_0) < 1$ is a numerical coefficient, which depends on the cavity geometry (L_0 is the average distance between vibrating walls).

Formula (1) can be derived from a general solution for a quantum harmonic oscillator with an arbitrary time-dependent frequency obtained for the first time in the seminal paper by Husimi [23] in 1953, if one remembers that field modes behave as a set of harmonic oscillators. Husimi showed that all dynamical properties of the *quantum* oscillator are determined by the fundamental system of solutions of the *classical* equation of motion

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0. \tag{2}$$

In particular, if $\omega(t) = \omega_i$ for $t \to -\infty$ and initially (at $t \to -\infty$) the oscillator was in the vacuum state, then the mean energy at the moment t equals

$$\mathcal{E}(t) = \frac{1}{4}\left[|\dot{\varepsilon}(t)|^2 + \omega^2(t)|\varepsilon(t)|^2\right], \tag{3}$$

where the function $\varepsilon(t)$ satisfies Eq. (2) and the initial condition

$$\varepsilon_{t\to-\infty} = \omega_i^{-1/2} e^{-i\omega_i t}. \tag{4}$$

Formula (3) holds for an arbitrary function $\omega(t)$, provided this function is *real*, i.e., the quantum evolution is *unitary*.

According to (1), one of the most important parameters which determines a possible number of created photons is an achievable value of the wall displacement amplitude. For the cavity dimensions of the order of $1 \div 100$ cm, the field resonance frequencies $(\omega_0/2\pi)$ belong to the band from 30 GHz to 300 MHz. An idea of [13, 14] was not to force the wall to oscillate as a whole at such a high frequency, but to excite oscillations of the *surface* of the cavity wall. In such a case, the amplitude a of a standing acoustic wave at frequency $\omega_w = 2\omega_0$ (coinciding with the amplitude of oscillations of the free surface) is related to the relative deformation amplitude δ inside the wall as $\delta = \omega_w a/v_s$, where v_s is the sound velocity. Since usual materials cannot bear the deformations exceeding the value $\delta_{max} \sim 10^{-2}$, the maximal possible velocity of the boundary is $v_{max} \sim \delta_{max} v_s \sim 50$ m/s (independent of the frequency). The maximal relative displacement $\varepsilon = a/L_0$ is $\varepsilon_{max} \sim (v_s/2\pi c)\delta_{max} \sim 3 \cdot 10^{-8}$ for the lowest mode with the frequency $\omega_0 \sim c\pi/L_0$. Then, taking $\varepsilon = 10^{-9}$, $\omega_0/(2\pi) = 10\,\mathrm{GHz}$ and $\eta^3 = 1/3$, in $t = 1$ s one could get a big number $\sinh^2(10) \sim 10^8$ photons in an empty cavity. However, in such a case one needs a cavity with the Q-factor of the order of 10^{10}. Moreover, it is necessary to maintain the resonance condition, which means that the frequency of the wall oscillations must not deviate from $2\omega_0/(2\pi)$ by more than $\delta/(2\pi) < \varepsilon\,\omega_0\eta^3/(2\pi) \sim 3\,\mathrm{Hz}$ during the time 1 s.

On the other hand, what we really need to create photons from vacuum, it is a possibility to change the resonance frequency in a periodical way. But this can be achieved not only by changing the geometry, but by changing the electric properties of the walls or some medium inside the cavity. Hence the idea of simulating DCE and other quantum effects arose about two decades ago in the article by Yablonovitch [1], who proposed to use a medium with a rapidly decreasing in time refractive index ('plasma window') to simulate the so-called Unruh effect. Also, he pointed out that fast changes of electric properties can be achieved in semiconductors illuminated by laser pulses. This idea was propagandized by Man'ko [24], who proposed to use semiconductors with time-dependent properties to produce an analogue of the *nonstationary Casimir effect* (see also [5, 25]). A more developed scheme, based on the creation of an electron-hole 'plasma mirror' inside a semiconductor slab, illuminated by a femtosecond laser pulse, was proposed in [26] (in the single-pulse case). But only recently a possibility of creating the effective 'plasma mirror' in a semiconductor slab was confirmed experimentally [27].

Quantum effects caused by a time dependence of properties of thin slabs were studied by several authors [28–31]. However, only very simple models of the media were considered in that papers: lossless homogeneous dielectrics with time-dependent permeability [31], ideal dielectrics or ideal conductors suddenly removed from the cavity [28, 29] or infinitely thin conducting slabs modeled by δ-potentials with time-dependent strength [30]. Moreover, all that models, as well as the estimations of the photon generations rate based on the simple formula (1), did not take into account inevitable losses inside the semiconductor slab during the excitation-recombination process. This is the immediate consequence of the fact that the dielectric permeability $\epsilon(x)$ of the semiconductor medium is a *complex function*: $\epsilon = \epsilon_1 + i\epsilon_2$, where $\epsilon_2 = 2\sigma/f_0$, σ and f_0 being the conductivity (in the CGS units) and frequency in Hz, respectively. Good conductors have $\epsilon_2 \sim 10^8$ at microwave frequencies, which is essentially bigger than $\epsilon_1 \sim 1 \div 10$. Although ϵ_2 is negligibly small in the non-excited semiconductor at low temperatures, it rapidly and continuously grows up to the values of the order of $10^5 \div 10^6$ during the laser pulse, returning to zero after the recombination time. In order to predict possible results of experiments and to suggest optimal choices of different parameters, one has to take into account the internal dissipation in the slab, which means that one has to generalize the Husimi solution to the case of *damped* quantum nonstationary oscillator.

Journal of Physics: Conference Series **99** (2008) 012006 doi:10.1088/1742-6596/99/1/012006

2. Quantum damped oscillator: a generalization of the Husimi solution via the Heisenberg–Langevin approach

An immediate consequence of the time variation of electromagnetic properties of the cavity walls is the time dependence of the eigenmode frequencies. Hence it follows a simple idea that one could understand the main features of the behavior of the quantum field in the cavity by considering a single selected mode and describing it as a quantum oscillator with 'instantaneous' time-dependent frequency [32, 33]. Later on, it was justified (see, e.g., [13, 14, 34, 35]) for three-dimensional cavities without accidental degeneracy of the spectrum of eigenmode frequencies and for harmonic variations of the effective frequency. We *assume* that even in the presence of dissipation and non-monochromatic periodical variations, the field problem still can be reduced approximately to the dynamics of a *single selected mode* described in the classical limit as a harmonic oscillator with time-dependent *complex* frequency $\Omega(t) = \omega(t) - i\gamma(t)$, which can be found from the solution of the classical electrodynamical problem by taking the *instantaneous* geometry and material properties (as was done in the non-dissipative case in [31, 36]).

The scheme presented below was developed in [37–39]. It can be considered as a generalization of the *quantum noise operator* approach, first proposed in [40–43], to the case of arbitrary time dependence of the frequency. In this approach, dissipative quantum systems are described within the framework of the Heisenberg–Langevin operator equations. In the case concerned these equations can be written as

$$d\hat{x}/dt = \hat{p} - \gamma_x(t)\hat{x} + \hat{F}_x(t), \qquad d\hat{p}/dt = -\gamma_p(t)\hat{p} - \omega^2(t)\hat{x} + \hat{F}_p(t). \tag{5}$$

Here $\hat{x}$ and $\hat{p}$ are the dimensionless quadrature operators of the selected mode, normalized in such a way that the mean number of photons equals

$$\mathcal{N} = \frac{1}{2}\langle \hat{p}^2 + \hat{x}^2 - 1\rangle. \tag{6}$$

In other words, in the subsequent formulas ω and γ are the frequency and damping coefficient normalized by the initial frequency ω_i. The noise operators $\hat{F}_x(t)$ and $\hat{F}_p(t)$ are supposed to commute with $\hat{x}$ and $\hat{p}$. The system of linear equations (5) can be solved explicitly for arbitrary time-dependent functions $\gamma_{x,p}(t)$, $\omega(t)$ and $\hat{F}_{x,p}(t)$:

$$\hat{x}(t) = e^{-\Gamma(t)}\left\{\hat{x}_0 \mathrm{Re}\,[\xi(t)] - \hat{p}_0 Im\,[\xi(t)]\right\} + \hat{X}(t), \tag{7}$$

$$\hat{p}(t) = e^{-\Gamma(t)}\left\{\hat{x}_0 \mathrm{Re}\,[\eta(t)] - \hat{p}_0 Im\,[\eta(t)]\right\} + \hat{P}(t), \tag{8}$$

where $\hat{x}_0$ and $\hat{p}_0$ are the initial values of operators at $t = -\infty$ (taken as the initial instant),

$$\Gamma(t) = \int_{-\infty}^{t} \gamma(\tau)d\tau, \qquad \gamma(t) = \frac{1}{2}\left[\gamma_x(t) + \gamma_p(t)\right], \tag{9}$$

$$\hat{X}(t) = e^{-\Gamma(t)} \int_{-\infty}^{t} d\tau e^{\Gamma(\tau)} \mathrm{I}m \left\{\xi^*(t)\left[\hat{F}_p(\tau)\xi(\tau) - \hat{F}_x(\tau)\eta(\tau)\right]\right\}, \tag{10}$$

$$\hat{P}(t) = e^{-\Gamma(t)} \int_{-\infty}^{t} d\tau e^{\Gamma(\tau)} \mathrm{I}m \left\{\eta^*(t)\left[\hat{F}_p(\tau)\xi(\tau) - \hat{F}_x(\tau)\eta(\tau)\right]\right\}. \tag{11}$$

$\xi(t)$ is the special solution to Eq. (2) with $\omega^2(t)$ replaced by the effective frequency

$$\omega_{ef}^2(t) = \omega^2(t) + \dot{\delta}(t) - \delta^2(t), \qquad \delta(t) = \frac{1}{2}\left[\gamma_x(t) - \gamma_p(t)\right]. \tag{12}$$

This special solution is selected by the initial condition $\xi(t) = \exp(-it)$ for $t \to -\infty$, which is equivalent to fixing the value of the Wronskian:

$$\xi\dot{\xi}^* - \dot{\xi}\xi^* = 2i. \tag{13}$$

The function $\eta(t)$ is defined as

$$\eta(t) = \dot{\xi}(t) + \delta(t)\xi(t). \tag{14}$$

It seems natural to identify the functions $\omega(t)$ and $\gamma(t)$ in equations (5), (9) and (12) with the real and imaginary parts of the instantaneous cavity eigenfrequency. Now we shall try to find the best set of other coefficients. The first step is to calculate the commutator $[\hat{x}(t), \hat{p}(t)]$. An immediate consequence of solution (8) is the formula

$$[\hat{x}(t), \hat{p}(t)] = i\hbar e^{-2\Gamma(t)} + \left[\hat{X}(t), \hat{P}(t)\right]. \tag{15}$$

In proving (15) we used the Wronskian formula (13) and its consequence

$$\mathrm{Im}\,[\xi(t)\eta^*(t)] \equiv 1. \tag{16}$$

We suppose that the noise operators are *delta-correlated* (the Markov approximation):

$$\langle \hat{F}_j(t)\hat{F}_k(t')\rangle = \delta(t-t')\chi_{jk}(t), \qquad j,k = x,p, \tag{17}$$

Using equations (10), (11), (16) and (17), we find

$$\left\langle \left[\hat{X}(t), \hat{P}(t)\right]\right\rangle = e^{-2\Gamma(t)} \int_{-\infty}^{t} \left[\chi_{xp}(\tau) - \chi_{px}(\tau)\right] e^{2\Gamma(\tau)} d\tau. \tag{18}$$

If we assume now that

$$\chi_{xp}(t) - \chi_{px}(t) = 2i\hbar\dot{\Gamma}(t) \equiv 2i\hbar\gamma(t), \tag{19}$$

then

$$\left\langle \left[\hat{X}(t), \hat{P}(t)\right]\right\rangle = i\hbar\left[1 - e^{-2\Gamma(t)}\right], \tag{20}$$

and the commutator $[\hat{x}(t), \hat{p}(t)] = i\hbar$ is preserved exactly (after averaging over noise operators) for any function $\gamma(t)$. In contrast to the classical Langevin equations, which contain a single stochastic force, in the quantum case one must use *two* noise operators, otherwise the canonical commutation relations cannot be saved.

For the mean values of the operators $\hat{x}^2(t)$ and $\hat{p}^2(t)$ we find the following expressions (from now on we shall use dimensionless variables, putting $\hbar = 1$; besides, we replace the symbol of function $f(t)$ by a short form f_t):

$$\begin{aligned}\langle \hat{x}^2(t)\rangle \;=\; & e^{-2\Gamma(t)}\left\{\langle\hat{x}^2\rangle_0\left[\mathrm{Re}\,(\xi_t)\right]^2 + \langle\hat{p}^2\rangle_0\left[\mathrm{Im}\,(\xi_t)\right]^2 - \langle\hat{x}\hat{p}+\hat{p}\hat{x}\rangle_0 \mathrm{Re}\,(\xi_t)\,\mathrm{Im}\,(\xi_t)\right\} \\ & + e^{-2\Gamma(t)}\int_{-\infty}^{t} d\tau e^{2\Gamma(\tau)}\Big\{\chi_{xx}(\tau)\left[\mathrm{Im}\,(\xi_t\eta_\tau^*)\right]^2 + \chi_{pp}(\tau)\left[\mathrm{Im}\,(\xi_t\xi_\tau^*)\right]^2 \\ & - \left[\chi_{xp}(\tau)+\chi_{px}(\tau)\right]\mathrm{Im}\,(\xi_t\eta_\tau^*)\,\mathrm{Im}\,(\xi_t\xi_\tau^*)\Big\},\end{aligned} \tag{21}$$

$$\begin{aligned}\langle \hat{p}^2(t)\rangle \;=\; & e^{-2\Gamma(t)}\left\{\langle\hat{x}^2\rangle_0\left[\mathrm{Re}\,(\eta_t)\right]^2 + \langle\hat{p}^2\rangle_0\left[\mathrm{Im}\,(\eta_t)\right]^2 - \langle\hat{x}\hat{p}+\hat{p}\hat{x}\rangle_0 \mathrm{Re}\,(\eta_t)\,\mathrm{Im}\,(\eta_t)\right\} \\ & + e^{-2\Gamma(t)}\int_{-\infty}^{t} d\tau e^{2\Gamma(\tau)}\Big\{\chi_{xx}(\tau)\left[\mathrm{Im}\,(\eta_t\eta_\tau^*)\right]^2 + \chi_{pp}(\tau)\left[\mathrm{Im}\,(\eta_t\xi_\tau^*)\right]^2 \\ & - \left[\chi_{xp}(\tau)+\chi_{px}(\tau)\right]\mathrm{Im}\,(\eta_t\eta_\tau^*)\,\mathrm{Im}\,(\eta_t\xi_\tau^*)\Big\}.\end{aligned} \tag{22}$$

Formulas (21) and (22) are exact, and they hold formally for arbitrary functions $\gamma_{x,p}(t)$, $\omega(t)$ and $\hat{F}_{x,p}(t)$. However, admissible *physical* sets of damping and force correlation coefficients must obey certain restrictions, which follow from the condition of non-negative definiteness of the statistical operator, or its consequence – the requirement of non-violation of the uncertainty relations. Detailed discussions of this subject can be found in [44–46]. It appears that the non-negative definiteness of the statistical operator can be preserved during the evolution for *arbitrary* physical initial states provided the 'noise matrix'

$$\mathcal{K} = \left\| \begin{array}{cc} \chi_{xx} & \chi_{xp} \\ \chi_{px} & \chi_{pp} \end{array} \right\| \tag{23}$$

is hermitian and non-negatively definite. The hermiticity of (23) together with equation (19) gives rise to the relations

$$\chi_{xp} = \chi_s + i\gamma, \qquad \chi_{px} = \chi_s - i\gamma \tag{24}$$

where χ_s is real parameter. Then the non-negativity of matrix (23) results in the restriction

$$\det \mathcal{K} = \chi_{xx}\chi_{pp} - \chi_s^2 \geq \gamma^2 \tag{25}$$

which tells us once again that one cannot preserve the uncertainty relations using only one noise operator (taking $\chi_{xx} = 0$ or $\chi_{pp} = 0$).

Let us consider the case of time-independent frequency $\omega = \omega_i = 1$ and time-independent damping and noise coefficients. Moreover, suppose that $\gamma \ll 1$ (small damping). Then one can neglect the correction $\delta^2 \sim \gamma^2$ in function $\omega_{ef}(t)$ (12) and use the solution $\xi(t) = \exp(-it)$. The integrals in equations (21) and (22) can be calculated exactly in this special case. We assume that $\gamma_{x,p} \sim \gamma$ and $\chi_{jk} \sim \gamma$ (in accordance with the fluctuation–dissipation theorem). Then neglecting terms of the order of γ^2, we obtain for $t \to \infty$

$$\langle \hat{x}^2 \rangle_\infty = \frac{1}{4\gamma}\left[\chi_{xx} + \chi_{pp} + 2\gamma_p\chi_s\right], \qquad \langle \hat{p}^2 \rangle_\infty = \frac{1}{4\gamma}\left[\chi_{xx} + \chi_{pp} - 2\gamma_x\chi_s\right], \tag{26}$$

$$\langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle_\infty = \frac{1}{2\gamma}\left[\gamma_x\chi_{pp} - \gamma_p\chi_{xx}\right]. \tag{27}$$

We see that the steady-state moments of the second order coincide with the thermodynamical equilibrium values

$$\langle \hat{x}^2 \rangle_{eq} = \langle \hat{p}^2 \rangle_{eq} = 1/2 + \langle n \rangle_{th}, \qquad \langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle_{eq} = 0$$

(where $\langle n \rangle_{th}$ is the mean number of quanta in the thermal state) with the accuracy of the order of γ^2 (i.e., without *linear* corrections with respect to the damping coefficients), provided the noise coefficients are chosen as follows

$$\chi_s = 0, \qquad \chi_{xx} = \gamma_x G, \qquad \chi_{pp} = \gamma_p G \tag{28}$$

where

$$G = 1 + 2\langle n \rangle_{th} = \coth\left(\frac{\hbar\omega_i}{2k_B T}\right) \tag{29}$$

and T is the temperature of the reservoir. In such a case, condition (25) takes the form

$$G^2\gamma_x\gamma_p \geq \left(\gamma_x + \gamma_p\right)^2/4. \tag{30}$$

At zero temperature of the reservoir ($G = 1$) inequality (30) can be satisfied for the only choice

$$\gamma_x = \gamma_p = \gamma. \tag{31}$$

For $G > 1$ the choice (31) is not the only possible one and we did not succeed in finding an additional mighty principle which could fix it. For example, one could think that more information could be extracted from the analysis of the evolution of such important quantities as the entropy $S = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$ or its equivalent – the so called linear entropy $S_1 = 1 - \mathrm{Tr}(\hat{\rho}^2)$. In this case it is sufficient to study the evolution of the *invariant uncertainty product* $\Delta \equiv \sigma_{xx}\sigma_{pp} - \sigma_{xp}^2$ (where $\sigma_{ab} \equiv \langle \hat{a}\hat{b} + \hat{b}\hat{a}\rangle/2 - \langle\hat{a}\rangle\langle\hat{b}\rangle$), because both S and S_1 are monotonous functions of Δ in the equilibrium state of a harmonic oscillator [44, 46, 47]. Suppose that the harmonic oscillator was in an equilibrium (thermal) state characterized by parameter G_0 (29) for $t < 0$ and a time-independent coupling with an environment characterized by parameter G is turned on at the 'initial' moment $t = 0$. Then an immediate consequence of equations (7), (8), (21) and (22) is the following formula for the rate of change of Δ at $t = 0$:

$$\left.\frac{d\Delta}{dt}\right|_{t=0} = \gamma G_0 \left(G - G_0\right). \tag{32}$$

It contains only the sum $\gamma = \gamma_x + \gamma_p$. An account of the second order terms with respect to γ_x and γ_p performed in [44, 45, 48] also did not lead to a convincing unique choice of the damping and diffusion coefficients for nonzero reservoir temperature.

An exact formula for the time dependence of the mean number of quanta (6) can be split in two parts: $\mathcal{N}(t) = \mathcal{N}_s(t) + \mathcal{N}_r(t)$, where the first term depends on the initial state ('signal') while the second term is determined completely by the interaction with the reservoir (it is represented by the integrals containing the coefficients χ_{jk}). For example, in the special case of the initial *coherent* state $|\alpha\rangle$ (which corresponds to the initial 'classical' signal in the cavity) we obtain

$$\mathcal{N}_s^{(coh)}(t) = e^{-2\Gamma(t)}\left\{ \left(\mathrm{Re}[\alpha\xi(t)]\right)^2 + \left(\mathrm{Re}[\alpha\eta(t)]\right)^2 + \frac{1}{2}E_\eta(t)\right\} - \frac{1}{2} \tag{33}$$

where

$$E_\eta(t) = \frac{1}{2}\left[|\xi(t)|^2 + |\eta(t)|^2\right]. \tag{34}$$

For the initial thermal state we have

$$\mathcal{N}_s^{(th)}(t) = \frac{1}{2}\left\{G_0 e^{-2\Gamma(t)} E_\eta(t) - 1\right\}. \tag{35}$$

Let us introduce the 'asymmetry parameter' y according to the relations

$$y = \frac{\gamma_p - \gamma_x}{\gamma_p + \gamma_x}, \qquad \gamma_p = \gamma(1+y), \quad \gamma_x = \gamma(1-y). \tag{36}$$

If this parameter does not depend on time (but it can depend on the reservoir temperature), then the contribution of the noise terms to the mean number of created quanta is equal to

$$\begin{aligned} \mathcal{N}_r(t) \;=\; & G e^{-2\Gamma(t)} \int_{-\infty}^{t} d\tau e^{2\Gamma(\tau)}\gamma(\tau)\Big(E_\eta(t)\left[E_\eta(\tau) + y D_\eta(\tau)\right] \\ & -\mathrm{Re}\left(\tilde{E}_\eta^*(t)\left[\tilde{E}_\eta(\tau) + y\tilde{D}_\eta(\tau)\right]\right)\Big) \end{aligned} \tag{37}$$

where

$$\tilde{E}_\eta(\tau) = \frac{1}{2}\left[\xi_\tau^2 + \eta_\tau^2\right], \quad D_\eta(\tau) = \frac{1}{2}\left[|\xi_\tau|^2 - |\eta_\tau|^2\right], \quad \tilde{D}_\eta(\tau) = \frac{1}{2}\left[\xi_\tau^2 - \eta_\tau^2\right] \tag{38}$$

and we assume that the noise coefficients are given by equation (28). In the special case $y = 0$ equation (37) assumes the following simple form:

$$\mathcal{N}_r(t) = G e^{-2\Gamma(t)} \int_{-\infty}^{t} d\tau e^{2\Gamma(\tau)}\gamma(\tau)\left\{E(t)E(\tau) - \mathrm{Re}\left[\tilde{E}^*(t)\tilde{E}(\tau)\right]\right\} \tag{39}$$

where

$$E(\tau) = \frac{1}{2}\left[|\varepsilon(\tau)|^2 + |\dot{\varepsilon}(\tau)|^2\right], \qquad \tilde{E}(\tau) = \frac{1}{2}\left[\varepsilon^2(\tau) + \dot{\varepsilon}^2(\tau)\right] \tag{40}$$

and function $\varepsilon(t)$ satisfies equation (2) with unmodified frequency $\omega(t)$. One should remember that formulas (33), (35), (37) and (39) hold for sufficiently big values of time t, when the time dependent normalized frequency $\omega(t)$ returns to its initial value $\omega(-\infty) = 1$.

All the formulas given above are exact (of course, in the frameworks of the model considered). However, in the case of small damping one can always use a simple formula (39) instead of (37), because the replacement of function $\eta(t)$ by $\dot{\xi}(t)$ results in an error of the order of γ. Besides, the terms containing functions $D_\eta(\tau)$ and $\tilde{D}_\eta(\tau)$ in (37) become very small after the integration, because these functions exhibit fast oscillations. Nonetheless, it is necessary to emphasize that for $y \neq 0$ function $\varepsilon(t)$ satisfying equation (2) with frequency $\omega^2(t)$ must be replaced by the function $\xi(t)$, which satisfies the same equation, but with the effective frequency

$$\omega_{ef}^2(t) = \omega^2(t) - y\dot{\gamma}(t) \tag{41}$$

(the term $y^2\gamma^2(t)$ can be neglected).

2.1. Periodical variations of parameters

Having in mind applications to the dynamical Casimir effect, we are interested in the special case when the functions $\omega(t)$ and $\gamma(t)$ have the form of *periodical* pulses, separated by intervals of time with $\omega = 1$ and $\gamma = 0$ (we neglect the damping of the field between pulses, supposing that the quality factor of the cavity is big enough). Since the relative change of the frequency $\omega(t)$ is very small, one can consider the functions $E(\tau)$ and $\tilde{E}(\tau)$ defined in (40) as *approximate integrals of motion* within the duration of each pulse (they are exact integrals of motion if $\omega \equiv 1$). Thus we assume that $E(\tau)$ and $\tilde{E}(\tau)$ are constant during each pulse, but their values for different pulses are slightly different, so that performing the integration over the duration of the kth pulse we replace $E(\tau)$ and $\tilde{E}(\tau)$ in the integrand of (39) by their values E_k and $\tilde{E}_k$ taken, say, at the end of the pulse. Then the integral over the period of pulse is calculated exactly (since $\gamma(t) = d\Gamma/dt$), and after n pulses we have [37–39]

$$\mathcal{N}_r(n) = Ge^{-2\Lambda n}\left(1 - e^{-2\Lambda}\right)\sum_{k=1}^{n} e^{2\Lambda k}\left[E_n E_k - \mathrm{Re}\left(\tilde{E}_n^* \tilde{E}_k\right)\right], \tag{42}$$

where

$$\Lambda = \int_{t_i}^{t_f} \gamma(\tau)d\tau, \tag{43}$$

t_i and t_f being the initial and final time moments of each pulse.

If $\varepsilon(t) = e^{-it}$ before the pulse represented by the 'effective potential barrier' $V_{ef}(t) = \omega_{ef}^2(t) - 1$, then after the pulse the solution can be written as $\varepsilon(t) = \rho_- e^{-it} + \rho_+ e^{it}$ with $|\rho_-|^2 - |\rho_+|^2 = 1$, so that the ratio $r = \rho_+/\rho_-$ can be interpreted as an effective complex amplitude reflection coefficient from the barrier, whereas the complex number $\rho_- = f$ has the meaning of the inverse amplitude transmission coefficient. In the case of a sequence of pulses we can write the function $\varepsilon(t)$ in the interval between the kth and $(k+1)$th pulses as

$$\varepsilon_k(t) = a_k e^{-it} + b_k e^{it}, \qquad a_0 = 1, \quad b_0 = 0, \tag{44}$$

where a_k and b_k are constant coefficients. Then we obtain the following expressions for the quantities E_k and $\tilde{E}_k$:

$$E_k = |a_k|^2 + |b_k|^2, \qquad \tilde{E}_k = 2a_k b_k. \tag{45}$$

Evidently, every two sets of the nearest constant coefficients, (a_{k-1}, b_{k-1}) and (a_k, b_k), are related by means of a linear transformation. Using the well known method of the transfer matrix, one can arrive at the following expressions [38]:

$$a_n = f\,\frac{\sinh(n\nu)}{\sinh(\nu)}e^{-iT(n-1)} - \frac{\sinh[(n-1)\nu]}{\sinh(\nu)}e^{-iTn}, \qquad b_n = g\,\frac{\sinh(n\nu)}{\sinh(\nu)}e^{iT(n-1)}, \tag{46}$$

where $g \equiv \rho_+$ and T is the periodicity of pulses. The coefficient ν is determined by the relations

$$\cosh(\nu) = |\mathrm{Re}[f\exp(iT)]| \equiv |f\cos(\delta)|, \tag{47}$$

$$\delta = \omega_0\left(T - T_{res}\right), \qquad T_{res} = \frac{1}{2}T_0\left(m - \varphi/\pi\right) \tag{48}$$

where $f = |f|\exp(i\varphi)$, T_0 is the period of oscillations in the selected field mode and T_{res} is the resonance periodicity ($m = 1, 2, \ldots$). It is assumed that coefficient ν is *real*. Note that $|f|^2 - |g|^2 \equiv 1$.

One can check the fulfillment of the identity $|a_n|^2 - |b_n|^2 \equiv 1$. Consequently, one can replace the function $E_\eta(t)$ in equations (33)-(35) by

$$E_n = 1 + 2|g|^2\frac{\sinh^2(n\nu)}{\sinh^2(\nu)}. \tag{49}$$

Then the sum in (42) can be calculated exactly, giving the following formula for the noise contribution to the mean number of photons created after n pulses:

$$\begin{aligned}\mathcal{N}_r(n) \;=\; & \frac{G}{2}\left(1 - e^{-2n\Lambda}\right) + G|g|^2\left\{\frac{\exp(-\Lambda)}{4\sinh(\nu)}\left[\frac{\exp[2n(\nu-\Lambda)]}{\sinh(\nu-\Lambda)} + \frac{\exp[-2n(\nu+\Lambda)]}{\sinh(\nu+\Lambda)}\right]\right.\\ & \left. - \frac{1+\exp(-2\Lambda)}{4\sinh(\nu-\Lambda)\sinh(\nu+\Lambda)} - \frac{\sinh^2(n\nu)}{\sinh^2(\nu)}e^{-2n\Lambda}\right\}.\end{aligned} \tag{50}$$

Under the realistic experimental conditions the parameters $|g|$ and Λ are very small. Moreover, the parameter ν can be real only if the detuning coefficient δ is also small. Then one finds from (47) that $\nu \approx \sqrt{|g|^2 - \delta^2}$, so that no photons can be produced if the detuning coefficient exceeds the critical value $\delta_{max} = \sqrt{|g|^2 - \Lambda^2}$.

We see that photons can be generated provided $\nu > \Lambda$. If $n(\nu - \Lambda) \gg 1$ and the difference $\nu - \Lambda$ is not too small, then

$$\mathcal{N}_r(n) \approx \frac{G|g|^2\Lambda}{4\nu^2(\nu-\Lambda)}\exp[2n(\nu-\Lambda)] + \frac{G}{2}\left(1 - e^{-2n\Lambda}\right). \tag{51}$$

The asymptotical expression for the 'signal' contribution in the case of initial thermal state reads

$$\mathcal{N}_s^{(th)}(n) \approx \frac{G_0|g|^2}{4\nu^2}\exp[2n(\nu-\Lambda)] - \frac{1}{2}, \tag{52}$$

so the total number of quanta equals

$$\mathcal{N}^{(th)}(n) \approx \frac{|g|^2\exp[2n(\nu-\Lambda)]}{4\nu^2(\nu-\Lambda)}\left[G\Lambda + G_0(\nu-\Lambda)\right] + \frac{G}{2}\left(1 - e^{-2n\Lambda}\right) - \frac{1}{2}. \tag{53}$$

For the initial coherent state with $|\alpha| \gg 1$ and $\alpha = |\alpha|\exp(i\theta)$ we obtain

$$\mathcal{N}_s^{(coh)}(n) \approx |\alpha|^2 e^{-2n\Lambda}\left|\cosh(n\nu) - \sinh(n\nu)e^{i(\varphi+2\theta)}\right|^2. \tag{54}$$

This number is very sensitive to the initial phase θ of the classical signal. The phase averaged value for $n\nu \gg 1$ equals

$$\overline{\mathcal{N}_s^{(coh)}(n)} = \frac{1}{2}|\alpha|^2 e^{2n(\nu-\Lambda)}. \tag{55}$$

Consequently, one can find the amplification coefficient $\nu - \Lambda$ experimentally, studying the amplification of the initial classical signal.

2.2. Approximate formulas for effective single-pulse reflection and transmission coefficients

For small variations of the effective frequency $\omega(t)$ we can write $\omega(t) = \omega_0[1+\chi(t)]$ with $|\chi| \ll 1$. For the effective frequency (41) we thus have $\chi_{ef} = \chi - y\dot{\gamma}/2$. Then using the quasiclassical approximation for the solution of the effective wave equation equivalent to (2) [49] we can express the absolute value of the single-pulse effective amplitude reflection coefficient $|g|$ and the phase φ of the single-pulse effective inverse transmission coefficient f as [51]

$$|g| \approx \left|\int_{t_i}^{t_f} \omega_0 \chi_{ef}(t) e^{-2i\omega_0 t} dt\right| = \left|\int_{t_i}^{t_f} \omega_0 \left[\chi(t) - iy\gamma(t)\right] e^{-2i\omega_0 t} dt\right| \tag{56}$$

(we have made the integration by parts remembering that $\gamma(t_i) = \gamma(t_f) = 0$). We see that an asymmetry of the damping coefficients γ_x and γ_p can make a significant influence on the value of $|g|$. On the contrary, the phase φ does not depend on the asymmetry coefficient, because

$$\varphi \approx \omega_0 \int_{t_i}^{t_f} \chi_{ef}(t)dt = \omega_0 \int_{t_i}^{t_f} \chi(t)dt. \tag{57}$$

Consequently, the resonance periodicity of pulses T_{res} (48) also does not depend on the asymmetry parameter y.

3. Frequency shift of the cavity mode

Approximate explicit expressions for the functions $\chi(t)$ and $\gamma(t)$ can be found for a cylindrical cavity with an arbitrary cross section, if the dielectric permeability depends only on the longitudinal space variable x. We suppose that the axis of a cylinder is parallel to the x-direction and the main part of the cavity is empty, except for a thin slab of a semiconductor material. Then $\varepsilon(x) \equiv 1$ for $-L < x < 0$ and $\varepsilon(x) \neq 1$ for $0 < x < D$, where D is the thickness of the slab and L is the cavity length. We assume that $D \ll L$.

For TE modes, the set of Maxwell's equations can be reduced [39] to the one-dimensional Helmholtz equation

$$\psi'' + \left[(\Omega/c)^2\varepsilon(x) - k_\perp^2\right]\psi = 0 \tag{58}$$

for any component of the electric field $E(x, \mathbf{r}_\perp) = \psi(x)\Phi(\mathbf{r}_\perp)$ (which is parallel to plane surfaces of the cylinder and slab), where $\Phi(\mathbf{r}_\perp)$ obeys the two-dimensional Helmholtz equation $\Delta_\perp\Phi + k_\perp^2\Phi = 0$. The solution of (58) in the domain $-L < x < 0$, satisfying the condition $\psi(-L) = 0$, is $\psi(x) = F_1\sin[k(x+L)]$, where the constant coefficient k is related to the field eigenfrequency Ω and the corresponding wavelength in vacuum λ as $\Omega = c\left(k^2 + k_\perp^2\right)^{1/2}$, $\lambda = 2\pi\left(k^2 + k_\perp^2\right)^{-1/2}$. The conditions of continuity of the function $\psi(x)$ and its derivative at $x = 0$ result in the transcendental equation for the wave number k

$$\tan(kL) = k\psi_+(0;k)/\psi_+'(0;k), \tag{59}$$

where $\psi_+(x;k)$ is the solution of equation (58) in the domain $0 < x < D$ satisfying the boundary condition $\psi_+(D) = 0$. In the case of thin slab with $D \ll \lambda \sim L$, the value of k must be close to π/L (we consider the lowest mode of the cavity). Thus we can write $k = (1+\xi)\pi/L$ with

$|\xi| \ll 1$ and replace $\tan(\pi\xi)$ in the left-hand side of (59) simply by $\pi\xi$. Moreover, with the same accuracy we can identify k with π/L in the right-hand side. Thus we arrive at the formula $\xi = \eta\Delta R(0)$, where the function $R(\tilde{x}) = \tilde{\psi}_+(\tilde{x})/\tilde{\psi}'_+(\tilde{x})$ of the dimensionless variable $\tilde{x} = x/D$ satisfies the first-order nonlinear *generalized Riccati equation*

$$\frac{dR}{d\tilde{x}} = 1 + \pi^2\Delta^2 \left[\varepsilon(\tilde{x}) - 1 + \eta^2\right] R^2 \tag{60}$$

in the domain $0 \le \tilde{x} \le 1$ and the boundary condition $R(1) = 0$. Here

$$\eta = \frac{\lambda}{2L} < 1, \qquad \Delta = \frac{2D}{\lambda} \ll 1. \tag{61}$$

The small relative shift of the resonance frequency can be expressed as

$$\chi_\Omega \equiv [\Omega - \omega_0]/\omega_0 = \eta^2(\xi - \xi_0) = \eta^3\Delta\left[R(0) - R_0(0)\right], \tag{62}$$

where ξ_0 or $R_0(0)$ correspond to the non-excited semiconductor slab with $\varepsilon(\tilde{x}) = \varepsilon_1 = const$. In this case equation (60) has an exact solution, which shows that for $\varepsilon_1 \sim 10$ (typical values for semiconductors) $R_0(0) \approx -1$ with the accuracy of the order of 0.01 or even better. When the slab is illuminated by the laser pulse, the absolute value of the dielectric function $\varepsilon(\tilde{x}) = \varepsilon_1 + i\varepsilon_2(\tilde{x})$ can attain very big values, so that $\pi^2\Delta^2|\varepsilon(\tilde{x})| \gg 1$ in some region $0 < \tilde{x} < \tilde{x}_0$ near the surface of the slab. Obviously, to create an effective 'plasma mirror' one needs the material with $\tilde{x}_0 \sim (\alpha D)^{-1} \ll 1$, where α is the absorption coefficient of the laser radiation. In the region $0 < \tilde{x} < \tilde{x}_0$ we can neglect the first term 1 in the right-hand side of equation (60), as well as the term $1 - \eta^2$. The simplified equation can be integrated immediately, resulting in the formula

$$\frac{1}{R(0)} - \frac{1}{R(\tilde{x}_0)} = (\pi\Delta)^2 \int_0^{\tilde{x}_0} \varepsilon(\tilde{x})d\tilde{x}. \tag{63}$$

In the region $\tilde{x}_0 < \tilde{x} < 1$ the nonlinear term in (60) becomes insignificant, so that we can write $R(1) - R(\tilde{x}_0) = 1 - \tilde{x}_0$. Since $R(1) = 0$ and $\tilde{x}_0 \ll 1$, we can take $R(\tilde{x}_0) = -1$. After simple algebra we arrive at the following interpolation formula for the complex frequency shift:

$$\chi_\Omega = \eta^3\Delta\frac{E}{E-1}, \qquad E = (\pi\Delta)^2 \int_0^\infty \tilde{\varepsilon}(\tilde{x})d\tilde{x}. \tag{64}$$

Here $\tilde{\varepsilon}(\tilde{x})$ means the *change of dielectric function* caused by the laser excitation. The upper limit of integration in (64) is extended formally to infinity, because the function $\tilde{\varepsilon}(\tilde{x})$ quickly goes to zero outside the interval $(0, \tilde{x}_0)$. Introducing the dimensionless parameter $\mu = \tilde{\varepsilon}_1/\tilde{\varepsilon}_2$ we can write

$$E = (i + \mu)A, \qquad A(t) = (4\pi^2\Delta|eb|/c)\mathcal{K}(t), \qquad \mathcal{K}(t) = \int_0^\infty n(x,t)dx, \tag{65}$$

where $\mathcal{K}(t)$ can be interpreted as the 'effective surface concentration' of electron-hole pairs created inside the slab, n is the volume concentration of these pairs, $c = f_0\lambda$ is the velocity of light and b is the total mobility of carriers for each electron-hole pair in the photo-excited slab ($|e|$ is the absolute value of the electron charge). Thus we can express the real and imaginary parts of the complex frequency shift as

$$\chi = \frac{\eta^3\Delta\left[A^2(1+\mu^2) - \mu A\right]}{A^2(1+\mu^2) - 2\mu A + 1}, \qquad \gamma = \frac{\eta^3\Delta A}{A^2(1+\mu^2) - 2\mu A + 1}. \tag{66}$$

We see that big values of χ can be achieved for $A \gg 1$. For the Drude model we have

$$b = e\tau_c/m_{ef}, \qquad \mu = \omega_0\tau_c = \omega_0 b m_{ef}/|e|, \tag{67}$$

where τ_c is the time between collisions and m_{ef} an effective mass of carriers. For realistic values $b \sim 1\,\mathrm{m^2V^{-1}s^{-1}}$, $m_{ef} \sim m_0$ (where m_0 is the mass of free electron) and $\omega_0/(2\pi) = 2.5\,\mathrm{GHz}$ we obtain $\mu \sim 0.1 \ll 1$. In this case the maximal value of γ is achieved for $A = 1$, and γ_{max} is only twice smaller than χ_{max}, so that the influence of damping by no means can be neglected. In principle, the situation can be different for $\mu \gg 1$ (i.e., for very high mobilities of carriers), because in this case γ_{max} is suppressed as μ^{-3}, and one can expect that the negative contribution of coefficient Λ to the photon generation rate (51) can be reduced in the same proportion.

In the case of TM modes the functions χ and γ are given by the same relations (66), with the only difference that the factor η^3 should be replaced by η in the first power [39]. This means that in the cavities with *the same* geometry the rate of generation is higher for the TM modes than for TE ones [31, 50, 51].

4. Influence of the slab properties on the photon generation rate

The dependence $n(x,t)$ in (65) can be found from equations which take into account, besides the photo-absorption, the effect of diffusion and different recombination processes. In the simplest case we have [52]

$$\partial n/\partial t = \nabla\cdot(Y\nabla n) + (\alpha\zeta/E_s)I(t)e^{-\alpha x} - \beta_1 n. \tag{68}$$

Here Y is the coefficient of ambipolar diffusion, α is the absorption coefficient of the laser radiation inside the layer, E_s is the energy gap of the semiconductor (which is close to the energy of laser photons), $I(t)$ is time-dependent intensity of the laser pulse which enters the slab (it can be less than the intensity of the pulse outside the slab, because the reflection coefficient from the semiconductor surface can be rather big, due to the big value of the dielectric constant $\varepsilon_1 \sim 10$; however, the reflection can be diminished if some quarter-wavelength film is put on the surface), $\zeta \le 1$ is the efficiency of the photo-electron conversion and β_1 is the trap-assisted recombination coefficient. We have disregarded nonlinear terms $-\beta_3 n^3 - \beta_2 n^2$ in the right-hand side of (68), because for modeling the DCE one needs very small recombination times, of the order of $T_r \sim 20 \div 30\,\mathrm{ps}$. Under these conditions, the contribution of neglected terms is several orders of magnitude smaller than that of the term $\beta_1 n$ [52]. In the most general case, one should use the function $I(t - x/v)$ instead of $I(t)$, where v is the group velocity. But for materials with high absorption coefficients the coordinate dependence can be neglected, if the duration of each pulse is of the order of a few picoseconds. The boundary condition to equation (68) is

$$Y\left.\frac{\partial n}{\partial x}\right|_{x=0} = Rn(0) \tag{69}$$

where R is the surface recombination velocity.

Since equation (68) is *linear*, it can be solved analytically [52]. If $n(x,t) = 0$ in the absence of the inhomogeneous term $I(t)$, then (for $Y = const$)

$$n(x,t) = \frac{\zeta\alpha}{E_s}\int_0^\infty dt' \exp\left(-\beta_1 t'\right) I(t-t')\Psi_g(x,t') \tag{70}$$

where

$$\Psi_g(x,t') = \frac{1}{\pi}\mathrm{Re}\left[\int_0^\infty d\kappa \exp\left(i\kappa\alpha x - \kappa^2\alpha^2 Y t'\right)\left(\frac{1}{1+i\kappa} + \frac{1}{1-i\kappa}\frac{\kappa - ig}{\kappa + ig}\right)\right], \tag{71}$$

$$g = R/(\alpha Y). \tag{72}$$

Journal of Physics: Conference Series **99** (2008) 012006 doi:10.1088/1742-6596/99/1/012006

We assume that $\alpha D \gg 1$, so that the presence of the right boundary of the semiconductor slab does not affect the carrier distribution near the irradiated surface. Integrating both sides of equation (68) over x from 0 to ∞ with account of the boundary condition and formulas (70) and (71) (with $x=0$) for the term $n(0)$, we arrive at the inhomogeneous ordinary differential equation of the first order

$$\frac{d\mathcal{K}}{dt} = -\beta_1 \mathcal{K} + \frac{\zeta}{E_s}\left[I(t) + \tilde{I}(t)\right], \tag{73}$$

where

$$\tilde{I}(t) = \frac{2}{\pi} R\alpha(1+g) \int_0^\infty dt' \exp\left(-\beta_1 t'\right) I(t-t') \int_0^\infty d\kappa \frac{\kappa^2 \exp\left(-\kappa^2\alpha^2 Y t'\right)}{\left(1+\kappa^2\right)\left(\kappa^2+g^2\right)}. \tag{74}$$

If the laser pulse starts at $t=0$, so that $I(t) = \mathcal{K}(t) = 0$ for $t<0$, then

$$\mathcal{K}(t) = (\zeta/E_s) \int_0^t \exp[-\beta_1(t-\tau)] \left[I(\tau) + \tilde{I}(\tau)\right] d\tau. \tag{75}$$

Supposing that the duration of laser pulse is much less than the recombination time, we can approximate the function $I(t)$ by the delta-function: $I(t) = (W/S)\delta(t)$, where W is the total energy of the laser pulse and S is the area of the surface of the semiconductor slab (we assume that the energy is distributed uniformly over this area). Then we obtain the following expression for the time-dependent function $A(t)$ in equation (66):

$$A(\tau) = A_0 e^{-\tau/Z} \mathcal{J}_g(\tau, h) \tag{76}$$

where

$$\mathcal{J}_g(\tau, h) = 1 + \frac{2}{\pi} g(1+g) \int_0^\infty \frac{d\kappa\left[1 - \exp\left(-h\kappa^2\tau\right)\right]}{\left(1+\kappa^2\right)\left(\kappa^2+g^2\right)} \tag{77}$$

and the new dimensionless variables and parameters are defined as follows:

$$\tau = \omega_0 t \qquad Z = \frac{\omega_0}{\beta_1} = \frac{2\pi T_r}{T_0} \qquad A_0 = \frac{4\pi^2 |eb| \zeta W \Delta}{(cE_s S)} \qquad h = \frac{\alpha^2 Y}{\omega_0} \tag{78}$$

If $g=0$ (an ideal surface with zero surface recombination velocity), then $\mathcal{J}_0(\tau, h) \equiv 1$, so that function $A(\tau)$ is reduced to the simple exponential function independently of the values of the diffusion and absorption coefficients. Also $\mathcal{J}_g(0, h) \equiv 1$ for any g and h. It can be shown [52] that the integral in (77) can be expressed in terms of the complementary error function

$$\mathrm{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp\left(-t^2\right) dt, \tag{79}$$

so that equivalent explicit expressions are (see also [53])

$$\mathcal{J}_g(\tau, h) = \frac{1}{g-1}\left[g e^{h\tau} \mathrm{Erfc}\left(\sqrt{h\tau}\right) - e^{h\tau g^2} \mathrm{Erfc}\left(g\sqrt{h\tau}\right)\right], \tag{80}$$

$$\mathcal{J}(t) = \frac{1}{\sqrt{T_d/T_s} - 1}\left[\sqrt{T_d/T_s}\, e^{t/T_d} \mathrm{Erfc}\left(\sqrt{t/T_d}\right) - e^{t/T_s} \mathrm{Erfc}\left(\sqrt{t/T_s}\right)\right]. \tag{81}$$

T_d can be called the characteristic diffusion time and T_s the characteristic surface recombination time. These times are related to the dimensionless parameters g and h as follows,

$$g = \sqrt{T_d/T_s}, \qquad h = T_r/T_d, \qquad hg^2 = T_r/T_s. \tag{82}$$

For $h\tau \ll 1$ and $g^2 h\tau \ll 1$ we have $\mathcal{J}_g(\tau,h) \approx 1 - gh\tau$, whereas for $h\tau \gg 1$ and $g^2 h\tau \gg 1$ we have $\mathcal{J}_g(\tau,h) \approx (g+1)/\sqrt{\pi g^2 h\tau}$. Consequently, the surface recombination causes a faster return of the functions $\chi(\tau)$ and $\gamma(\tau)$ to zero values after the excitation.

In the special case $g = 1$ (i.e. $T_d = T_s$) we have [52]

$$\mathcal{J}_1(\tau,h) = (1-2h\tau)e^{h\tau}\mathrm{Erfc}\left(\sqrt{h\tau}\right) + 2\sqrt{h\tau/\pi} \tag{83}$$

$$= (1-2t/T_d)\, e^{t/T_d}\mathrm{Erfc}\left(\sqrt{t/T_d}\right) + 2\sqrt{t/(\pi T_d)}, \tag{84}$$

whereas for $g = \infty$ (i.e., when the diffusion time is much bigger than the surface recombination time, $T_d \gg T_s$) we obtain

$$\mathcal{J}_\infty(\tau,h) = e^{h\tau}\mathrm{Erfc}\left(\sqrt{h\tau}\right) = e^{t/T_d}\mathrm{Erfc}\left(\sqrt{t/T_d}\right). \tag{85}$$

According to equation (53), the rate of the photon generation is determined by the difference

$$\nu - \Lambda = \eta^3 \Delta\, F, \qquad F = \tilde{\nu} - \tilde{\Lambda} \tag{86}$$

where coefficients $\tilde{\nu}$ and $\tilde{\Lambda}$ are given by some integrals (the Fourier transforms) following from formulas (56), (57) and (66). They can be expressed as follows,

$$\tilde{\nu} = Z\left|\int_0^\infty dx\, e^{-2iZx}\frac{\left[A_0\mathcal{J}_g(x)\exp(-x)\right]^2(1+\mu^2) - (\mu+iy)A_0\mathcal{J}_g(x)\exp(-x)}{1+\left[A_0\mathcal{J}_g(x)\exp(-x)\right]^2(1+\mu^2) - 2\mu A_0\mathcal{J}_g(x)\exp(-x)}\right|, \tag{87}$$

$$\tilde{\Lambda} = Z\int_0^\infty dx\frac{A_0\mathcal{J}_g(x)\exp(-x)}{1+\left[A_0\mathcal{J}_g(x)\exp(-x)\right]^2(1+\mu^2) - 2\mu A_0\mathcal{J}_g(x)\exp(-x)} \tag{88}$$

(the coefficient $\tilde{\Lambda}$ does not depend on the asymmetry parameter y).

The dependence of function F on A_0 and Z in the special case $g = \mu = y = 0$ was studied in [38]. The influence of surface recombination (the dependence on g and h for $\mu = y = 0$) was analyzed in detail in [52]. The role of the asymmetry parameter y was discussed in [54] (for $g = \mu = 0$) and the case of $\mu \neq 0$ was considered in [55] (for $g = y = 0$). In the most general case the integrals (87) and (88) can be calculated only numerically. However, for $g = 0$ (when $\mathcal{J}_g(x) \equiv 1$) explicit analytical expressions can be found. The integral (88) becomes obvious if one makes the change of variable $\exp(-x) = t$:

$$\tilde{\Lambda} = Z\left\{\tan^{-1}\left[A_0\left(1+\mu^2\right)\right] + \tan^{-1}(\mu)\right\}. \tag{89}$$

Making the same change of variable and using the known integral representation of the Gauss hypergeometric function [56]

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a}\,dt \tag{90}$$

we can express the integral in (87) as a linear combination of four such functions with different complex arguments and parameters (but $a = 1$ and $c = b+1$ in each function):

$$\begin{aligned}\tilde{\nu} = \frac{Z}{2}\Bigg|&\frac{A_0^2(1+\mu^2)}{2(1+iZ)}\Big[(\mu-i)\,{}_2F_1\left(1,2+2iZ;3+2iZ;A_0[\mu-i]\right)\\ &-(\mu+i)\,{}_2F_1\left(1,2+2iZ;3+2iZ;A_0[\mu+i]\right)\Big]\\ &-\frac{A_0(\mu+iy)}{1+2iZ}\Big[(\mu-i)\,{}_2F_1\left(1,1+2iZ;2+2iZ;A_0[\mu-i]\right)\\ &-(\mu+i)\,{}_2F_1\left(1,1+2iZ;2+2iZ;A_0[\mu+i]\right)\Big]\Bigg|.\end{aligned} \tag{91}$$

However, numerical calculations show that the amplification coefficient F can be positive only for $A_0 \gg 1$. Therefore it is convenient to rewrite (91), using one of Kummer's formulas, namely those relating Gauss hypergeometric functions with the arguments z and z^{-1}. In the special case concerned ($c = b+1$ and $a = 1$) the required relation is [56]

$$ {}_2F_1(1, b; b+1; z) = \frac{b}{z(1-b)}\, {}_2F_1\left(1, 1-b; 2-b; z^{-1}\right) + \frac{\pi b}{\sin(\pi b)}(-z)^{-b} \tag{92} $$

(the formula $\Gamma(b)\Gamma(1-b) = \pi b/\sin(\pi b)$ was taken into account). Thus equation (91) can be simplified as follows:

$$ \begin{aligned} \tilde{\nu} = & \frac{Z}{2}\left| \frac{A_0\left(1+\mu^2\right)}{1+2iZ}\Big[{}_2F_1\left(1, -1-2iZ; -2iZ; [A_0(\mu+i)]^{-1}\right)\right. \\ & - {}_2F_1\left(1, -1-2iZ; -2iZ; [A_0(\mu-i)]^{-1}\right)\Big] \\ & + 2\pi\rho\frac{\cosh(\pi Z + 2Z\phi - i\phi)}{\sinh(2\pi Z)}\exp\left[-2iZ\ln\left(A_0\rho\right)\right] \\ & + (y - i\mu)\Bigg\{ \frac{1}{2Z}\Big[{}_2F_1\left(1, -2iZ; 1-2iZ; [A_0(\mu-i)]^{-1}\right) \\ & - {}_2F_1\left(1, -2iZ; 1-2iZ; [A_0(\mu+i)]^{-1}\right)\Big] \\ & \left. -2\pi\frac{\sinh(\pi Z + 2Z\phi)}{A_0\sinh(2\pi Z)}\exp\left[-2iZ\ln\left(A_0\rho\right)\right]\Bigg\}\right|, \end{aligned} \tag{93} $$

where

$$ \rho\exp(i\phi) \equiv 1 + i\mu, \qquad \rho = \sqrt{1+\mu^2}, \qquad \phi = \tan^{-1}(\mu). \tag{94} $$

The hypergeometric functions in the r.h.s. of (93) have rather simple explicit forms:

$$ {}_2F_1(1, b; b+1; z) = \sum_{k=0}^{\infty}\frac{bz^k}{b+k} \equiv R(z; b). \tag{95} $$

Equation (93) assumes rather simple form in terms of function $R(z; b)$ in the case $\mu = 0$:

$$ \begin{aligned} \tilde{\nu} = & \frac{1}{2}\left| R\left(-A_0^{-2}; -iZ\right) - \frac{\pi Z}{\sinh(\pi Z)}\exp\left[-2iZ\ln(A_0)\right]\right. \\ + & \left. y\left\{\frac{Z}{A_0(iZ-1/2)}R\left(-A_0^{-2}; 1/2 - iZ\right) + \frac{\pi Z}{\cosh(\pi Z)}\exp\left[-2iZ\ln(A_0)\right]\right\}\right| \end{aligned} \tag{96} $$

We have checked that numerical values of the amplification coefficient F (86) calculated with the aid of formulas (89) and (96) coincide with the results of numerical calculations of the integrals (87) and (88) performed in [38] for $g = \mu = 0$ in the wide range of parameters A_0 and Z. This fact convinces us in the reliability of numerical schemes used in our previous studies.

The behavior of function $F(A_0, Z)$ is shown in figure 1, where the plots are taken from [38]. In two upper plots we show the dependence of the amplification coefficient F on the parameters Z and A_0. In the lower left plot of Figure 1 we show the boundary between the regions of the positive and negative values of the amplification coefficient F in the plane of parameters A_0 and Z (the generation is possible for the parameters belonging to the domain below the upper line), as well as the 'optimal trajectory', corresponding to the sets of parameters for which the

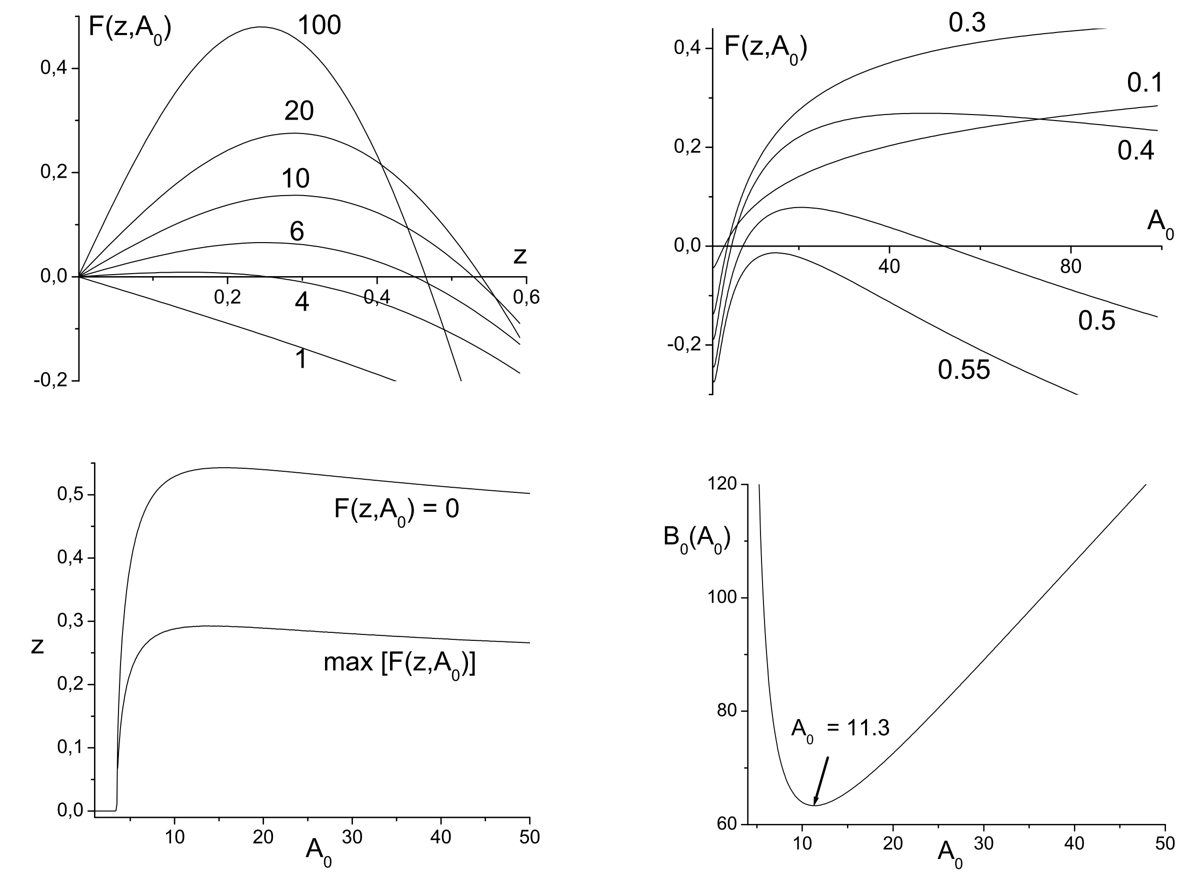

Figure 1. Upper left and upper right: The dependence of the amplification coefficient F on the parameter Z for fixed values of the parameter A_0 (given for each line) and on the parameter A_0 for fixed values of the parameter Z (also given for each line). Lower left: The boundary between the regions of the positive and negative values of the amplification coefficient F (the upper line) and the 'optimal trajectory' in the plane of parameters A_0 and Z (where the function $F(Z, A_0)$ takes maximal values). Lower right: The function $B_0(A_0, Z) = A_0/F(Z, A_0)$, which is proportional to the total energy of all laser pulses necessary to generate the fixed (big) number of photons, versus the parameter A_0 (for optimal values of parameter Z).

function $F(Z, A_0)$ takes maximal values. We see that the generation is impossible if $Z > 0.54$ or $A_0 < 4$. For moderate values of parameter A_0, the maximal values of F are achieved for $Z \approx 0.3$. Analyzing the figures, we conclude that the optimal value of Z is close to 0.3. The corresponding recombination time is close to $T_r^{opt} = T_0/(2\pi^2)$. For $T_0 = 400\,\text{ps}$ (or $f_0 = 2.5\,\text{GHz}$) we obtain $T_r^{opt} \approx 20\,\text{ps}$, and in no case it can be more than 35 ps. The value $T_r^{opt} \approx 20\,\text{ps}$ is quite realistic from the point of view of the available technology (and it was confirmed in recent preliminary measurements [57]).

The optimal value of the parameter A_0 can be found in the following way. According to (78), this parameter is proportional to the energy of the laser pulse. On the other hand, if we fix the number of photons which can be created after n pulses, then formula (51) shows that $n \approx const/F$. Consequently, the function $B_0(A_0, Z) = A_0/F(Z, A_0)$ is proportional to the *total energy* of all necessary laser pulses. Choosing for each value of A_0 the optimal value of parameter

Z, we obtain the lower right plot given in Figure 1. It shows that the best choice (corresponding to the minimal total energy) is

$$A_{0*} = 11.3, \qquad Z_* = 0.29, \qquad F_* = 0.18, \qquad \tilde{\nu}_* = 0.61, \qquad \tilde{\Lambda}_* = 0.43. \tag{97}$$

4.1. Optimization of geometry and estimations of possible photon generation rates
Simple numerical formulas for the number of photons, created *from vacuum* after $n \gg 1$ pulses, read as (see formula (53) with $G = G_0 = 1$ and $n\Lambda \gg 1$)

$$\mathcal{N}_n \approx \frac{\tilde{\nu}}{4F} \exp\left(2\eta^3 F \Delta\, n\right), \qquad \mathcal{N}_{*n} \approx 0.85\, \exp\left(0.36\, \eta^3 \Delta\, n\right). \tag{98}$$

The optimal value of the geometrical factor η can be determined with the aid of the following reasonings. The resonance wavelength λ, corresponding to the mode TE_{101} with the lowest eigenfrequency of the rectangular cavity, is related to the cavity length L and the biggest transverse dimension B as $\lambda = 2LB/\sqrt{L^2 + B^2}$. [1] Consequently, $B = \lambda/(2\sqrt{1-\eta^2})$. For the fixed values of parameter A_0 and the smallest transverse dimension, the energy of the pulse is proportional to the area of the surface, i.e., B, whereas the necessary number of pulses depends on L as $\eta^{-3} \sim L^3$. Consequently, the total energy is proportional to the product BL^3. Minimizing this product for the fixed value of λ, which is equivalent to maximization of the function $f(\eta) = \eta^3\sqrt{1-\eta^2}$, we find the optimal value $\eta_{opt} = \sqrt{3}/2 = 0.866$, which corresponds to $L = \lambda/\sqrt{3} \approx 7\,\mathrm{cm}$ and $B = \lambda = 12\,\mathrm{cm}$ (for $\omega_0/(2\pi) = 2.5\,\mathrm{GHz}$). However, since it can be difficult to illuminate such a long plate, we should consider other reasonable values of η. Fortunately, the profile of function $f(\eta)$ is rather flat in the vicinity of point η_{opt}, therefore the main requirement can be avoiding resonances with other cavity eigenfrequencies for the given modulation depth Δ (we considered $D = 2\,\mathrm{mm}$ and $\Delta = 1/30$). These resonances can happen due to the highly anharmonical profile of the frequency shift function (66), and their presence can diminish significantly the photon generation rate in the fundamental mode under consideration [35, 58]. The analysis shows that the best choice, permitting to avoid at least 4 accidental resonances, corresponds to the values of η in the interval between 2/3 and 3/4. The choice $\eta = 3/4$ which corresponds to $L = 8\,\mathrm{cm}$ and $B = 9\,\mathrm{cm}$. In this case we have $\eta^3 = 0.42$ and $f(\eta_1) = 0.28$, which is not too small comparing with the maximal value $f(\eta_{opt}) = 0.325$. Taking $T_r = 20\,\mathrm{ps}$, $b = 0.7\,\mathrm{m^2V^{-1}s^{-1}}$ and $E_g = 1.4\,\mathrm{eV}$ (for GaAs), we evaluate that 10^4 photons can be created in the cavity with dimensions $8 \times 9 \times 1$ cm after about 2000 laser pulses with the energy in each pulse about 0.5 mJ. The total energy of all pulses in this case must be about 1 J, which seems to be rather big quantity. However, it can be diminished if one considers cavities with a more sophisticated geometry instead of a simple rectangular cavity.

A specific property of highly doped semiconductors to be used in experiments on the dynamical Casimir effect is a low mobility of carriers at zero absolute temperature (the scattering on charged impurities results in the contribution to the mobility $b \sim T^{3/2}$ for $T \to 0$). Therefore it seems that an optimal strategy could be to prepare somehow only the resonance field mode in the vacuum state, while maintaining the walls at some finite temperature T chosen according to the following requirements: a high mobility of carriers, an optimal recombination time and a high quality factor of the cavity. Note that for the frequency $\omega_0/(2\pi) = 2.5\,\mathrm{GHz}$ the temperature gain factor G defined in (29) has the numerical value $G \approx 17\,T$ if $T > 1$ is expressed in Kelvins. Then formula (53) with $G \gg G_0 = 1$ becomes

$$\mathcal{N}_n \approx \frac{G\tilde{\Lambda}}{4F} \exp\left(2\eta^3 F \Delta\, n\right), \qquad \mathcal{N}_{*n} \approx 10\,T \exp\left(0.36\, \eta^3 \Delta\, n\right) \tag{99}$$

[1] The third dimension should be taken much smaller than L and B in order to diminish the illuminated surface of the slab. This choice excludes automatically the TM mode for the lowest eigenfrequency chosen.

and the ratio of the number of created photons to the mean number of thermal photons at temperature T is

$$\frac{\mathcal{N}_{*n}}{\langle n\rangle_{th}(T)} \approx 1.2 \exp\left(0.36\,\eta^3\Delta\,n\right). \tag{100}$$

For example, at the liquid nitrogen temperature $T = 77\,\mathrm{K}$ we need only 500 pulses to create 10^4 photons (with $\langle n\rangle_{th}(T) \approx 650$). [2] If the mobility could be made several times higher than the value used in the calculations, then the total energy of all pulses could be reduced by an order of magnitude or even more.

5. Influence of a nonzero duration of the laser pulse

The evaluations in the preceding section were done under the assumption of a very short laser pulse. To study the influence of the finite pulse duration we consider the case $g = \mu = 0$ (i.e., we neglect the surface recombination). Then the function $A(t)$ in (76) can be represented as $A(t) = A_0 \exp(-\beta_1 t) f(t)$,

$$f(t) = \frac{\int_0^t \exp(+\beta_1 x) I(x)dx}{\int_0^\infty I(x)dx}. \tag{101}$$

We tried different simple functions $I(t)$, such as

$$I_1(t) = \frac{\pi}{2a}\frac{W}{S}\sin(\pi t/a), \qquad I_2(t) = \frac{2}{a}\frac{W}{S}\sin^2(\pi t/a), \tag{102}$$

where a is the pulse duration (it is assumed that $I(t) \equiv 0$ for $t > a$), and the normalization is chosen in such a way that the total energy $\int_0^\infty I(x)dx = W/S$ is the same in all the cases. The dimensionless parameter characterizing the pulse duration in this case is

$$Y = a\omega_0/\pi = 2a/T_0. \tag{103}$$

The integrals (87) and (88) were calculated numerically (we do not bring here cumbersome explicit analytical expressions for the functions $f(x)$ in all examples). The results are shown in figures 2 and 3. We take $A_0 = 10$ (an optimal value for very short pulses) in all these plots. We see no significant decrease of the value of F for $Y \le 0.15$, if $Z = 0.3$ and $Z = 0.2$ (recombination time 13 ps). Note that $Y = 0.15$ corresponds to the pulse duration $a = 30\,\mathrm{ps}$, which equals approximately just the delay time

$$T_D = \left(\sqrt{L^2 + b^2} - L\right)/c \approx b^2/(2Lc) \tag{104}$$

between the instants when light can reach the center of the semiconductor slab at point$(L, 0)$ and its most remote (from the source of illumination) point (L, b), where $L \approx 9\,\mathrm{cm}$ is the cavity length and $b \approx 4\,\mathrm{cm}$ is a half of the perpendicular dimension $B = 2b \approx 8\,\mathrm{cm}$.

We also studied the dependence of the amplification factor F on the dimensionless parameter $V = a\xi$ (characterizing the speed of laser pulse formation) for the family of pulse shapes

$$I(t) = I_0\left[1 - e^{-\xi t}\right]\left[1 - e^{-\xi(a-t)}\right] \tag{105}$$

with different values of Y and Z (and fixed $A_0 = 10$). It appears that F is insensitive to the shape of laser pulse if $Y \le 0.15$.

To evaluate the effect of the nonuniform illumination we suppose that there is a point-like source of light at the point $(0, 0)$ with a time-dependent angular distribution of radiation $J(\phi, t)$.

[2] An idea to use the initial thermal state of the *field* to facilitate the observation of the dynamical Casimir effect was put forward for the first time in [22]. We emphasize that here we assume that the initial field state is *vacuum*.

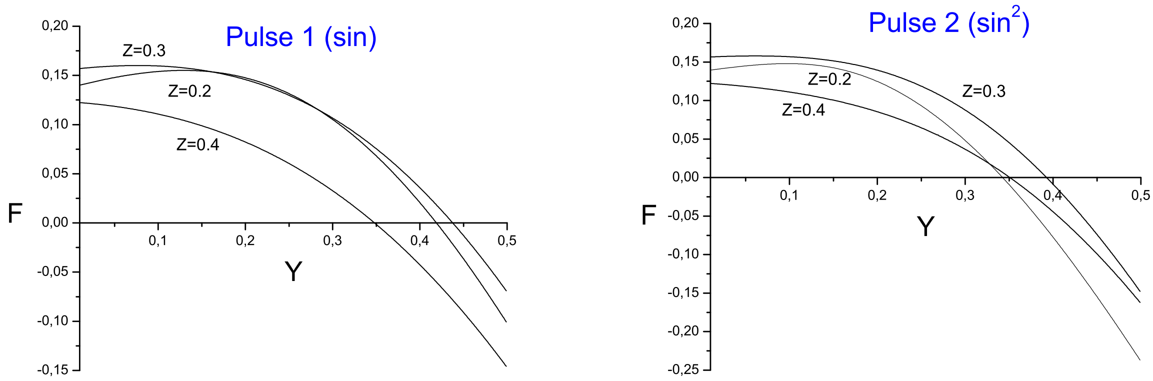

Figure 2. The dependence of the amplification coefficient F on the parameter Y for two effective shapes of the laser pulse (102) and the fixed value of parameter $A_0 = 10$.

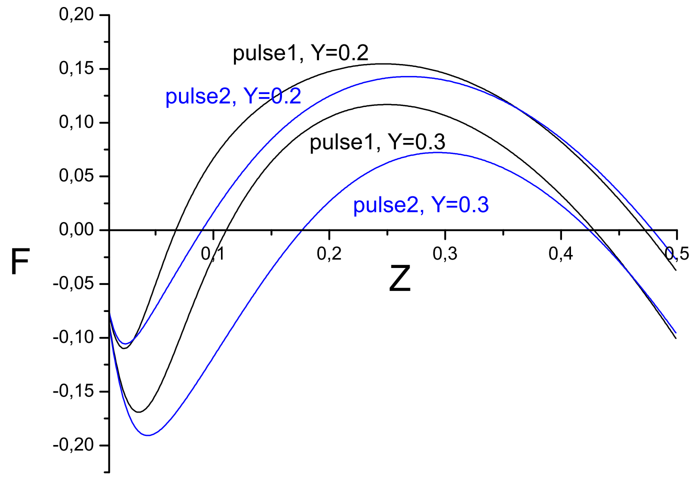

Figure 3. The dependence of the amplification coefficient F on the parameter Z for two effective shapes of the laser pulse (102) and the fixed value of parameter $A_0 = 10$.

The delay time between the arrival of light at points $(L, 0)$ and (L, y) equals $T(y) \approx y^2/(2Lc) \approx L\phi^2/(2c)$ (we suppose for simplicity that $y \ll L$, so that $\phi \approx y/L$). The intensity of light on the semiconductor surface at point y and time t equals $I(y, t) \approx J(y/L, t - T(y))$. It seems reasonable to suppose that the function $A(t)$ in the case of non-uniform illumination is proportional to the *effective average intensity*

$$I_{ef}(t) = \frac{1}{b}\int_0^b I(y,t)dy \approx \frac{1}{b}\int_0^b J(y/L, t - y^2/(2Lc))$$

Then, even if the initial laser pulse $J(\phi, t)$ is very short in time, the duration of the effective pulse $I_{ef}(t)$ will be not less than T_D (104). In the simplest case, when $J(\phi, t)$ does not depend on angle within the interval $(0, b/L)$ and has delta-shape with respect to time, $J(\phi, t) = J_0\delta(t)$, the effective intensity has very asymmetric form as function of time:

$$I_{ef}(t) = \frac{W}{2S}\sqrt{\frac{T_D}{t}}, \quad 0 < t < T_D, \qquad I_{ef}(t) \equiv 0, \quad t < 0, \quad t > T_D \tag{106}$$

A singularity at $t = 0$ and discontinuity at $t = T_D$ are the consequences of the delta-approximation; they disappear if one 'smoothes' the delta function, and they do not influence the physical results.

For the time-dependent 'form-factor' (101) we obtain, after normalization, the following expression:

$$f(x) = \begin{cases} \frac{1}{2}\sqrt{\pi Z/R}\,\mathrm{erfi}(\sqrt{x}), & x \le R/Z \\ \frac{1}{2}\sqrt{\pi Z/R}\,\mathrm{erfi}(\sqrt{R/Z}) = const, & x \ge R/Z \end{cases} \qquad \mathrm{erfi}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x \exp(z^2)dz \tag{107}$$

where

$$R = \omega_0 T_D = 2\pi T_D/T_0 = \pi Y. \tag{108}$$

The dependence $F(R)$ is shown in Figure 4 for $A_0 = 10$. We see that a rapid decay of the

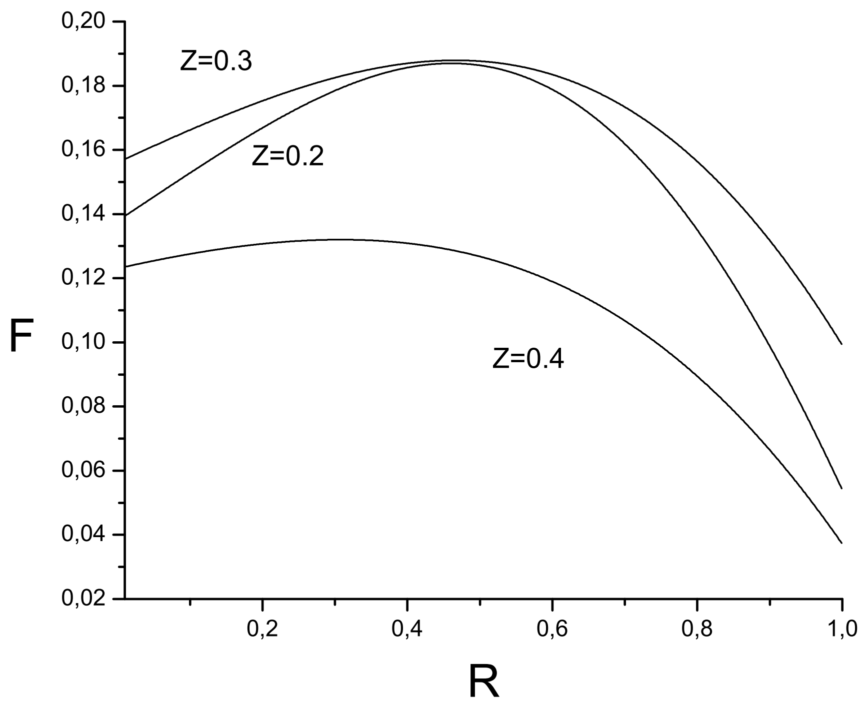

Figure 4. The dependence of the amplification coefficient F on the dimensionless time delay R (108) of the laser pulse between the central and boundary regions of the illuminated slab. The pulse shape is approximated by equation (107) and the dimensionless energy of pulse $A_0 = 10$.

amplification factor F is observed only when $R > 0.5$ (which is equivalent to $T_D = 30\,$ps) for different values of the recombination time (parameter Z). Therefore, all models lead to the same conclusion which is very favorable for the experiment: the non-uniformity of illumination or temporal spread of laser pulses will not deteriorate significantly the rate of photon generation if the recombination time is in the interval 10-20 ps.

6. Conclusions

We have shown interesting connections between such distant, at first sight, areas as the nonstationary Casimir effect, quantum nonstationary damped oscillator, physics of semiconductors and physics of ultrashort laser pulses. The first result is a consistent model of a damped nonstationary quantum oscillator with arbitrary time-dependent frequency and damping coefficients, based on the generalization of the Senitzky–Schwinger–Haus–Lax noise operator approach. We have derived the set of noise operators which enabled us to obtain a simple generalization of the Husimi solution to the case of a quantum damped oscillator. For zero temperature of the reservoir this set is practically unique if one uses the natural requirements of preservation of the positivity of the density matrix and the maximum closeness of the asymptotic stationary state to the thermodynamical-equilibrium state in the case of constant damping coefficients. For nonzero temperature other sets of coefficients (depending on the asymmetry parameter y) are possible in principle, although the choice $y = 0$ seems to be the best one from the point of view of simplicity and mathematical beauty. New solutions are exact and they can be applied to a variety of problems in quantum mechanics of nonstationary dissipative systems.

The second result is connected with the application of the model to the physical problem of Dynamical Casimir Effect. A consequence of the general theory is an approximate formula for the number of photons which can be produced after n periods of small perturbations of the oscillator frequency in the resonance case. It depends on two parameters: the absolute value ν of the amplitude 'reflection coefficient' from an effective single pulse barrier in the time domain, which is given by the Fourier transform of the time-dependent frequency $\omega(t)$ calculated at twice the resonant frequency, and the 'accumulated damping factor' Λ given by the integral of the periodical damping coefficient (the absolute value of the imaginary part of complex eigenfrequency) over the period of this function. The asymptotical rate of the photon generation is twice the difference $\nu - \Lambda$.

We have obtained simple approximate analytical expressions for the real and imaginary parts of the complex shift of the resonance frequency of the cavity $\Omega(t) = \omega(t) - i\gamma(t)$ in the case of a thin semiconductor slab irradiated by short laser pulses. Using these expressions, we have calculated the coefficients ν and Λ and found their dependence on several dimensionless parameters, which determine the dynamics of the process: normalized geometrical factors Δ and η, normalized recombination time Z and normalized energy of the pulse A_0. We have found the boundary of the region of photon generation in the plane of parameters $Z - A_0$, as well as the dependence $A_0(Z)$ which gives the maximal value of the photon generation rate $\nu - \Lambda$. Besides, using different criteria we have found the optimal values for the pulse energy, recombination time and the geometrical factors. In particular, we have shown that the recombination time should not exceed 35 ps, and the optimal choice is 20 ps. Our evaluations show that an experimental demonstration of the dynamical Casimir effect is a quite feasible task which can be achieved in the nearest future.

Acknowledgments

VVD acknowledges a partial support of the Brazilian agency CNPq. AVD acknowledges a full support of the Brazilian agency FAPESP, project 04/13705-3.

References

[1] Yablonovitch E 1989 *Phys. Rev. Lett.* **62** 1742–5
[2] Schwinger J 1993 *Proc. Nat. Acad. Sci. USA* **90** 958–9
[3] Dodonov V V, Klimov A B and Nikonov D E 1993 *Phys. Rev.* A **47** 4422–9
[4] Johnston H and Sarkar S 1995 *Phys. Rev.* A **51** 4109–15
[5] Okushima T and Shimizu A 1995 *Japan. J. Appl. Phys.* **34** 4508–10
[6] Saito H and Hyuga H 1996 *J. Phys. Soc. Japan* **65** 3513–23
[7] Barton G and Eberlein C 1993 *Ann. Phys.* **227** 222–74

[8] Gütig R and Eberlein C 1998 *J. Phys. A: Math. Gen.* **31** 6819–38
[9] Pendry J B 1998 *J. Mod. Opt.* **45** 2389–408
[10] Braggio C, Bressi G, Carugno G, Del Noce C, Galeazzi G, Lombardi A, Palmieri A, Ruoso G and Zanello D 2005 *Europhys. Lett.* **70** 754–60
[11] Segev E, Abdo B, Shtempluck O, Buks E and Yurke B 2007 *Phys. Lett.* A **370** 202–6
[12] Kim W-J, Brownell J H and Onofrio R 2006 *Phys. Rev. Lett.* **96** 200402
[13] Dodonov V V 1995 *Phys. Lett.* A **207** 126–32
[14] Dodonov V V and Klimov A B 1996 *Phys. Rev.* A **53** 2664–82
[15] Moore G T 1970 *J. Math. Phys.* **11** 2679–91
[16] Askar'yan G A 1962 *Sov. Phys. – JETP* **15** 1161–2
[17] Rivlin L A 1979 *Sov. J. Quant. Electron.* **9** 1320–1
[18] Sarkar S 1988 in *Photons and Quantum Fluctuations* ed E R Pike and H Walther (Bristol: Hilger) p 151–72
[19] Lambrecht A, Jaekel M T and Reynaud S 1996 *Phys. Rev. Lett.* **77** 615–18
[20] Dodonov V V 1998 *J. Phys. A: Math. Gen.* **31** 9835–54
[21] Dalvit D A R and Mazzitelli F D 1999 *Phys. Rev.* A **59** 3049–59
[22] Plunien G, Schützhold R and Soff G 2000 *Phys. Rev. Lett.* **84** 1882–5
[23] Husimi K 1953 *Prog. Theor. Phys.* **9** 381–402
[24] Man'ko V I 1992 in *Quantum Measurements in Optics* (*NATO ASI Series B: Physics* vol 282) ed P Tombesi and D F Walls (New York: Plenum) pp 239–46
[25] Grishchuk L P, Haus H A and Bergman K 1992 *Phys. Rev.* D **46** 1440–9
[26] Lozovik Y E, Tsvetus V G and Vinogradov E A 1995 *Phys. Scripta* **52** 184–90
[27] Braggio C, Bressi G, Carugno G, Lombardi A, Palmieri A, Ruoso G and Zanello D 2004 *Rev. Sci. Instrum.* **75** 4967–70
[28] Cirone M, Rzążewski K and Mostowski J 1997 *Phys. Rev.* A **55** 62–6
[29] Cirone M and Rzążewski K 1999 *Phys. Rev.* A **60** 886–92
[30] Crocce M, Dalvit D A R, Lombardo F C and Mazzitelli F D 2004 *Phys. Rev.* A **70** 033811
[31] Uhlmann M, Plunien G, Schützhold R and Soff G 2004 *Phys. Rev. Lett.* **93** 193601
[32] Man'ko V I 1991 *J. Sov. Laser Res.* **12** 383–5
[33] Sassaroli E, Srivastava Y N and Widom A 1994 *Phys. Rev.* A **50** 1027–34
[34] Schützhold R, Plunien G and Soff G 1998 *Phys. Rev.* A **57** 2311–8
[35] Crocce M, Dalvit D A R and Mazzitelli F D 2001 *Phys. Rev.* A **64** 013808
[36] Crocce M, Dalvit D A R, Lombardo F C and Mazzitelli F D 2005 *J. Opt. B: Quantum Semiclassical Opt.* **7** S32–9
[37] Dodonov V V 2005 *J. Opt. B: Quantum Semiclassical Opt.* **7** S445–51
[38] Dodonov V V and Dodonov A V 2005 *J. Russ. Laser Res.* **26** 445–83
[39] Dodonov V V and Dodonov A V 2006 *J. Phys. A: Math. Gen.* **39** 6271–81
[40] Senitzky I R 1960 *Phys. Rev.* **119** 670–9
[41] Schwinger J 1961 *J. Math. Phys.* **2** 407–32
[42] Haus H A and Mullen J A 1962 *Phys. Rev.* **128** 2407–13
[43] Lax M 1966 *Phys. Rev.* **145** 110–45
[44] Dodonov V V and Manko V I 1987 *Group Theory, Gravitation and Elementary Particle Physics (Proceedings of Lebedev Physics Institute, vol 167)* ed A A Komar (Commack: Nova Science) p 7–101
[45] Dodonov V V, Man'ko O V and Man'ko V I 1995 *J. Russ. Laser Res.* **16** 1–56
[46] Dodonov V V 2003 *Theory of Nonclassical States of Light* ed V V Dodonov and V I Man'ko (London: Taylor & Francis), pp 153–218
[47] Agarwal G S 1971 *Phys. Rev.* A **3** 828-31
[48] Dodonov V V and Man'ko O V 1985 *Physica* A **130** 353–66
[49] Ginzburg V L 1967 *Propagation of Electromagnetic Waves in Plasmas* (Moscow: Nauka) pp 292-4
[50] Crocce M, Dalvit D A R and Mazzitelli F D 2002 *Phys. Rev.* A **66** 033811
[51] Dodonov A V and Dodonov V V 2005 *J. Opt. B: Quantum Semiclassical Opt.* **7** S47–58
[52] Dodonov V V and Dodonov A V 2006 *J. Phys. B: At. Mol. Opt. Phys.* **39** S749–66
[53] Beard M C, Turner G M and Schmuttenmaer C A 2000 *Phys. Rev.* B **62** 15764–77
[54] Dodonov V V and Dodonov A V 2006 *J. Russ. Laser Res.* **27** 379–88
[55] Dodonov V V and Dodonov A V 2006 *Acta Phys. Hung. B - Quantum Electronics* **26** 29–36
[56] Erdélyi A (ed) 1953 *Bateman Manuscript Project: Higher Transcendental Functions* vol I (New York: McGraw-Hill)
[57] Ruoso G et al 2007 (unpublished) `http://www.physik.uni-leipzig.de/~bordag/Talks/talks_on_QFEXT07/Ruoso.pdf`
[58] Dodonov A V and Dodonov V V 2001 *Phys. Lett.* A **289** 291–300