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EFFECT OF PHASE-SENSITIVE RESERVOIR ON THE DECOHERENCE OF PAIR-CAT COHERENT STATES

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Abstract

We study the decoherence of a superposition of four coherent states under the action of a phase sensitive reservoir. We verify that the decoherence times $\tau_{k,l}$, $k, l = 1, 2, 3, 4$, between any two coherent states of the superposition can be controlled through the reservoir parameters. The decoherence time between two components of any pair, for instance $\tau_{1,2}$ or $\tau_{3,4}$, can be significantly increased, compared with the decoherence time when the state is acted by a thermal reservoir. However, this occurs at the expense of decreasing the decoherence time between the "cat states" (1,2) and (3,4). This can be useful in quantum computation.

1. Introduction

We consider the evolution of the special kind of finite superpositions of coherent states

$$|\Psi_N(\alpha_0)\rangle = \sum_{k=1}^{N} C_k \, |\alpha_k\rangle \, , \quad |\alpha_k\rangle = e^{-|\alpha_k|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_k^n}{\sqrt{n!}} \, |n\rangle \ , \quad \alpha_k = \alpha_0 e^{i\frac{2\pi k}{N}} , \tag{1}$$

caused by an interaction with a phase-sensitive squeezed reservoir. For the first time, a superposition such as (1) has been considered in [1]. It has been noticed in [2] that any sum of the form (1) is an eigenstate of the operator $\hat{a}^N$ with the eigenvalue α_0^N, and therefore it can be represented as a superposition of N orthogonal states, each one being a certain combination of N coherent states $|\alpha_0 \exp(i\phi_k)\rangle$. The case of $N = 2$ with $C_2 = \pm C_1$ was studied in [3], where the name "even and odd coherent states" has been introduced for the combinations

$$|\alpha\rangle = \mathcal{N}\left(|\alpha\rangle \pm |-\alpha\rangle\right) , \qquad \mathcal{N} = \left(2\left[1 \pm \exp(-2\alpha^2)\right]\right)^{-1/2} . \tag{2}$$

The superposition of two coherent states resulting in the exact Poissonian statistics has been considered in [4]. Different kinds of multiphoton states have been studied through the last two decades [5]. Here we consider the four-photon states which we call "pair-cat" states for the reasons explained in the text.

It is known that quantum superpositions are rapidly destroyed under the influence of the thermal environment [6]. Therefore, a lot of effort was devoted to engineering specific reservoirs which would permit one to increase the decoherence time. In particular, it has been shown that using "phase sensitive"

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(e.g., "squeezed") reservoirs [7, 8] one could increase the decoherence time for the usual (two-component) "cat" states [9]. We show that applying these techniques to the four-photon quantum states results in the splitting of the initial homogeneous superpositions into a pair of two usual cat states whose phases are shifted by $\pi/2$. The decoherence inside each pair can be made much more slow than in the case of the usual thermal environment, at the expense of a rapid loss of coherence between the pairs.

The plan of the paper is as follows. In Sec. 2, we discuss the properties of multiphoton and circular quantum states and their special case — four-photon states. In Sec. 3, we solve the master equation in the case of a squeezed reservoir and find explicit expressions describing the evolution of the Wigner function. In Sec. 4, we introduce the decoherence times and calculate their dependence on the parameters characterizing the state and the reservoir. Section 5 is devoted to brief conclusions and discussions.

2. Multiphoton and Circular States

The name "circular states" became popular after paper [10] where the coefficients C_k were chosen as

$$C_k^{(J)} = \exp\left(-\frac{2i\pi kJ}{N}\right).$$

The states proposed in [10] can be represented in two equivalent normalized forms:

$$|\tilde{\Psi}_{N,J}(b)\rangle = A_{N,J}^{-1/2}\sum_{k=1}^{N} e^{-2i\varphi_k J}|\beta_k\rangle = Z_{N,J}^{-1/2}\sum_{l=0}^{\infty}\frac{a^{(Nl+J)/2}}{\sqrt{(Nl+J)!}}|Nl+J\rangle\,, \tag{3}$$

where

$$\beta_k = \sqrt{b}\exp\left(\frac{2i\pi k}{N}\right), \quad Z_{N,J} = \sum_{k=0}^{\infty}\frac{b^{Nk+J}}{(Nk+J)!}\,, \quad Z_{N,J} = N^{-2}e^{b}A_{N,J}, \tag{4}$$

$$A_{N,J} = N + 2\sum_{k=0}^{N-1} k\exp\left[-2b\sin^2\left(\frac{\pi k}{N}\right)\right]\cos\left[b\sin\left(\frac{2\pi k}{N}\right)+\frac{2\pi k}{NJ}\right]. \tag{5}$$

The difference between the states introduced in [1] and in [10] is in the "weights" of each coherent state in the superposition. In [1] these weights were chosen different, in order to ensure the Poissonian photon statistics, whereas in the case of circular states considered in [10], all the coefficients have the same absolute value, resulting in non-Poissonian statistics. Among various properties of the circular states, we mention the following:

$$\hat{a}^J|\tilde{\Psi}_{N,J}(b)\rangle = \beta_0^J|\tilde{\Psi}_{N,0}(b)\rangle, \qquad \langle\tilde{\Psi}_{N,J}(b)|\tilde{\Psi}_{N,J'}(b)\rangle = \delta_{J,J'},$$

$$\sum_{J=1}^{N} A_{N,J}|\tilde{\Psi}_{N,J}(b)\rangle\langle\tilde{\Psi}_{N,J}(b)|A_{N,J} = N|\beta_k\rangle\langle\beta_k|.$$

During the last decade, superpositions of coherent states attracted much attention due to their interference in the phase space, leading to nonclassical properties as antibunching or sub-Poissonian statistics, squeezing, etc. In particular, the results of studies of different kinds of circular states and their generalizations have been reported in [11]. "Crystallized" Schrödinger cat states have been introduced in [12]. The state (3) with $J = N/2$ for N even (or with $J = (N \pm 1)/2$ for N odd) was called the odd coherent state of the Nth-order [13]. Experimental schemes permitting one to generate the superpositions (3) were proposed in [14] (for the electromagnetic field in superconducting cavities) and in [15] (for the vibrational states of the center-of-mass of a trapped ion).

The four-photon states ($N = 4$) appeared for the first time, perhaps, in paper [4], where it was shown that such states arise in a Kerr-like medium with the interaction Hamiltonian

$$\hat{H}_{\rm int} = \Omega \hat{n}^k$$

and k odd. Later, the properties of such states and other methods for their generation (e.g., in processes involving competition between four-photon parametric amplification and four-photon absorption) were studied in [16–18]. Such states can be represented as superpositions of two even/odd coherent states (2)

$$|\alpha\rangle_\pm \qquad \text{and} \qquad |i\alpha\rangle_\pm \,,$$

whose labels are rotated by $\pi/2$ in the parameter complex plane. For this reason, the state $|\alpha\rangle_+ + |i\alpha\rangle_+$ was called in [19] "the orthogonal-even state."

The photon number probability in the multiphoton state (3) is

$$P_n\left(N,J,b\right) = |\langle n|\tilde{\Psi}_{N,J}(b)\rangle|^2 = \frac{b^{Nl+J}}{Z_{N,J}(Nl+J)!}\delta_{n,Nl+J}\,, \qquad l = 0,1,2,\ldots \tag{6}$$

So, for given N, state (3) is a superposition of the Fock states $|Nl+J\rangle$ with $l = 0,1,2,\ldots$ In Fig. 1, one can see that for $J = 0, N = 4$ the multiphoton state is a superposition of the Fock states $|4\rangle$, $|8\rangle$, $|12\rangle$, and $|16\rangle$.

The mean value of any power of $\hat{n} \equiv \hat{a}^\dagger\hat{a}$ can be calculated as

$$\overline{n^k} = \sum_{n=0}^{\infty} n^k P_n = Z_{N,J}^{-2}\sum_{l=0}^{\infty}\frac{b^{Nl+J}(Nl+J)^k}{(Nl+J)!}. \tag{7}$$

3. Dynamics of the Statistical Operator and Wigner Function

The relaxation of the cavity mode coupled to a phase-sensitive squeezed broadband reservoir was studied by many authors [7–9, 20, 21]. We use the master equation for the reduced density operator in the form given in [22]

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{dt}} = -i\Delta\left[\hat{a}^\dagger\hat{a},\hat{\rho}\right] + \hat{\mathcal{L}}\hat{\rho}, \tag{8}$$

where $\Delta = \omega - \omega_0$ is the difference between the cavity field frequency ω and the carrier frequency ω_0 of the squeezed reservoir. The term $\hat{\mathcal{L}}\hat{\rho}$ describing the dissipation has the following structure:

$$\begin{aligned}\hat{\mathcal{L}}\hat{\rho} \;&=\; \gamma(\nu+1)(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + \gamma\nu(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger)\\ &\quad+\gamma M e^{i\phi}(2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}^\dagger) + \text{h.c.}\,,\end{aligned} \tag{9}$$

where $\gamma > 0$ is the damping coefficient and ν is the mean number of squeezed photons in the reservoir. The coefficient M determines the intensity of the correlation between two photons in the reservoir of broadband squeezed vacuum. It must satisfy the inequality $M \leq \sqrt{\nu(\nu+1)}$. In this work, we consider

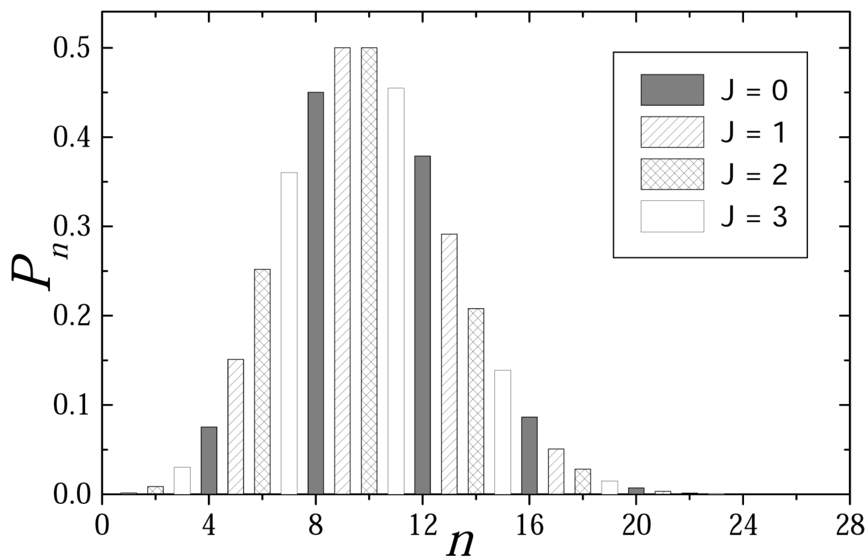

Fig. 1. Probability distribution P_n as a function of n for $N = 4$ and $b = 10$.

the case where $M = \sqrt{\nu(\nu+1)}$, i.e., we assume that the squeezed vacuum is injected into the cavity with the maximum correlation between two photons. Parameter ϕ is the phase of the squeezed vacuum.

To solve Eq. (8) we make the unitary transformation [23]

$$\tilde{\rho} = \hat{S}_c \hat{\rho} \hat{S}_c^\dagger, \tag{10}$$

where $\hat{S}_c$ is the usual squeezing operator that changes the annihilation operator for

$$\hat{S}_c \hat{a} \hat{S}_c^\dagger = u\hat{a} + v\hat{a}^\dagger, \quad u = \sqrt{\nu+1}, \quad v = \sqrt{\nu} e^{i\phi}. \tag{11}$$

After this transformation the master equation (8) becomes

$$\frac{\mathrm{d}\tilde{\rho}}{\mathrm{dt}} = -i\Delta\left[(2\nu+1)\hat{a}^\dagger\hat{a} + Me^{-i\phi}\hat{a}^2 + Me^{i\phi}\hat{a}^{\dagger 2}, \tilde{\rho}\right] + \hat{\mathcal{L}}_{\mathrm{vac}}\tilde{\rho}, \tag{12}$$

so that the relaxation term

$$\hat{\mathcal{L}}_{vac}\tilde{\rho} = \gamma(2\hat{a}\tilde{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\tilde{\rho} - \tilde{\rho}\hat{a}^\dagger\hat{a}) \tag{13}$$

looks formally like the standard dissipation operator describing a coupling with the usual reservoir at zero temperature (unsqueezed vacuum).

The solution of Eq. (12) can be formally written using the superoperators' techniques

$$\tilde{\rho}(t) = e^{\hat{\mathcal{L}}t}\tilde{\rho}(0), \tag{14}$$

where the corresponding Liouvillian is

$$\hat{\mathcal{L}}\cdot\ = -\beta\,\hat{K} - \beta^*\hat{P} - \lambda(\hat{a}^{\dagger 2}\bullet - \bullet\,\hat{a}^{\dagger 2}) + \lambda^*(\hat{a}^2\bullet - \bullet\,\hat{a}^2) + 2\gamma\hat{J}\,, \tag{15}$$

with

$$\beta = \gamma + i\Delta(2\nu + 1) \qquad \text{and} \qquad \lambda = i\Delta M e^{i\phi}.$$

The superoperators are

$$\hat{J} \equiv (\hat{a} \bullet \hat{a}^\dagger), \quad \hat{K} \equiv (\hat{a}^\dagger \hat{a} \bullet), \quad \hat{P} \equiv (\bullet \hat{a}^\dagger \hat{a}).$$

Using the standard algebraic techniques based, e.g., on the Baker–Campbell–Hausdorff formulas [24], Hausdorff similarity transformations [25], or some others [26, 27], one can factorize the operator (14) into the product of exponentials of the superoperators as follows:

$$\tilde{\rho}(t) = e^{-\hat{J}} e^{-\chi_2 \hat{T}} e^{-\chi_2^*(\nu+1)\hat{S}} e^{\chi_3^*(\nu+1)^{-2}(\hat{a}^{\dagger 2}\bullet)} e^{\chi_1 \hat{K}} e^{\chi_3(\bullet \hat{a}^2)} e^{\chi_1^* \hat{P}} e^{\chi_2^*(\nu+1)\hat{S}} e^{\chi_2 \hat{T}} e^{\hat{J}} \tilde{\rho}(0), \tag{16}$$

where

$$\hat{R} \equiv (\hat{a}^\dagger \bullet \hat{a}), \qquad \hat{T} \equiv (\hat{a}^\dagger \bullet \hat{a}^\dagger), \qquad \hat{S} \equiv (\hat{a} \bullet \hat{a}),$$

and

$$\chi_1(t) = -(\gamma + i\Delta)t, \quad \chi_2 = \frac{M e^{i\phi}}{\nu + 1}, \quad \chi_3(t) = i\frac{\Delta M(\nu+1)e^{-i\phi}\left(1 - e^{-2(\gamma - i\Delta)t}\right)}{2(\gamma - i\Delta)}.$$

The Wigner function is defined as the Fourier transform of the symmetric characteristic function

$$W(\xi, \xi^*, t) = \frac{1}{\pi}\int \exp\left(\xi\eta^* - \xi^*\eta\right) C^{(s)}\left(\eta, \eta^*, t\right) d^2\eta \tag{17}$$

defined as

$$C^{(s)}\left(\eta, \eta^*, t\right) = \mathrm{Tr}\left\{e^{\eta\hat{a}^\dagger - \eta^*\hat{a}}\hat{\rho}(t)\right\}. \tag{18}$$

Applying the unitary transformation (10), we have

$$C^{(s)}\left(\tilde{\eta}, \tilde{\eta}^*, t\right) = \mathrm{Tr}\left\{e^{\tilde{\eta}\,\hat{a}^\dagger - \tilde{\eta}^*\hat{a}}\tilde{\rho}(t)\right\} = \exp(-|\tilde{\eta}|^2/2)\mathrm{Tr}\left\{e^{\tilde{\eta}\,\hat{a}^\dagger} e^{-\tilde{\eta}^*\hat{a}}\tilde{\rho}(t)\right\},$$

where

$$\tilde{\eta} = \eta\sqrt{\nu + 1} - \eta^*\sqrt{\nu}e^{i\phi}. \tag{19}$$

Substituting the density operator (16) and using the cyclic property of the trace, after some calculations, we obtain

$$\mathrm{Tr}\left\{e^{\tilde{\eta}\hat{a}^\dagger - \tilde{\eta}^*\hat{a}}\tilde{\rho}(t)\right\} = \exp\left(\zeta_1(\tilde{\eta}, \tilde{\eta}^*, t\right) \mathrm{Tr}\left\{e^{\zeta(\tilde{\eta}, \tilde{\eta}^*, t)\hat{a}^\dagger} e^{-\zeta^*(\tilde{\eta}, \tilde{\eta}^*, t)\hat{a}}\tilde{\rho}(0)\right\}, \tag{20}$$

where

$$\zeta_1(\tilde{\eta}, \tilde{\eta}^*, t) = 2\mathrm{Re}\left\{\chi_3(t)(\tilde{\eta}^2 + 2\chi_2|\tilde{\eta}|^2 + \chi_2^2\tilde{\eta}^{*2})\right\}, \tag{21}$$

$$\zeta(\tilde{\eta}, \tilde{\eta}^*, t) = \left[e^{\chi_1^*(t)} - e^{\chi_1(t)}\right](\nu + 1)\chi_2\tilde{\eta}^* + \left[(\nu + 1)e^{\chi_1^*(t)} - \nu e^{\chi_1(t)}\right]\tilde{\eta}. \tag{22}$$

Expressing $\tilde{\rho}(0)$ in terms of $\hat{\rho}(0)$ by means of transformation (10) in Eq. (20) and replacing $\tilde{\eta}$ by η with the aid of Eq. (19), we obtain the symmetric characteristic function in the form

$$\begin{aligned} C^{(s)}\left(\eta, \eta^*, t)\right) &= \exp\left[-\frac{1}{2}A^*(t)\eta^2 - \frac{1}{2}A(t)\eta^{*2} - B(t)\eta\eta^*\right] \\ &\quad \times \mathrm{Tr}\left\{\exp\left[e^{-(\gamma - i\Delta)t}\eta\hat{a}^\dagger\right]\exp\left[-e^{-(\gamma + i\Delta)t}\eta^*\hat{a}\right]\hat{\rho}(0)\right\}, \end{aligned} \tag{23}$$

where

$$A(t)=Me^{i\phi}\frac{\gamma\left(e^{-2(\gamma-i\Delta)t}-1\right)}{\gamma-i\Delta},\quad B(t)=\frac{1}{2}-\nu\left(e^{-2\gamma t}-1\right).$$

For the initial circular state (3), Eqs. (17) and (23) result in the following Wigner function:

$$W^{(J)}(\xi,\xi^*,t)=\frac{A_{N,J}}{\pi}\sum_{l=1}^{N}\sum_{k=1}^{N}\exp\left(-i\phi_{lk}\right)\langle\beta_l|\beta_k\rangle$$
$$\times\int d^2\eta\exp\left\{-\frac{1}{2}A^*(t)\eta^2-\frac{1}{2}A(t)\eta^{*2}-B(t)\eta\eta^*+\left[\beta_l^*(t)-\xi^*\right]\eta-\left[\beta_k(t)-\xi\right]\eta^*\right\},\qquad(24)$$

where

$$\beta_k(t)=e^{-(\gamma+i\Delta)t}\beta_k,\quad \phi_{lk}=\frac{2\pi J}{N}(l-k),\quad \langle\beta_l|\beta_k\rangle=\exp\left[-\frac{1}{2}|\beta_l|^2-\frac{1}{2}|\beta_l|^2+\beta_l^*\beta_k\right].$$

After integration, we have

$$W^{(J)}(\xi,\xi^*,t)=\sum_{l=1}^{N}\sum_{k=1}^{N}W_{lk}^{(J)}(\xi,\xi^*,t),\qquad(25)$$

$$W_{lk}^{(J)}(\xi,\xi^*,t)=\frac{A_{N,J}\langle\beta_l|\beta_k\rangle}{\sqrt{F(t)}}\exp\left\{-\frac{1}{2F(t)}\Big(2B(t)\left[\beta_k(t)-\xi\right]\left[\beta_l^*(t)-\xi^*\right]\right.$$
$$\left.+A^*(t)\left[\beta_k(t)-\xi\right]^2+A(t)\left[\beta_l^*(t)-\xi^*\right]^2\Big)-i\phi_{lk}\right\},\qquad(26)$$

where

$$F(t)=B^2-|A|^2.\qquad(27)$$

For simplicity, we consider hereafter the special case of "resonance" between the cavity mode and the reservoir, $\Delta=0$. Then

$$F(t)=\frac{1}{4}+\nu e^{-2\gamma t}\left(1-e^{-2\gamma t}\right).\qquad(28)$$

4. Decoherence Times

Analizing the structure of Wigner function (25), one can verify that its "diagonal" components W_{kk} assume equal maximal values at the points of the complex phase plane $\xi=\beta_k(t)$:

$$W_{kk}^{(\max)}(t)=\frac{A_{N,J}}{\sqrt{F(t)}}.$$

It is interesting to note that asymptotically, as $t\to\infty$, these maximum values are the same as at the initial time moment $t=0$.

At the initial time moment each "off-diagonal" component W_{lk} takes maximum absolute value at the point $\beta_{lk}=\left(\beta_l+\beta_k\right)/2$, i.e., at the middle of the line connecting points β_k and β_l, and this maximum value is the same as for the initial diagonal components:

$$W_{lk}(\beta_{lk},\beta_{lk}^*,0)=2A_{N,J}\exp\left[\frac{1}{2}\left(\beta_k\beta_l^*-\beta_l\beta_k^*\right)-i\phi_{lk}\right],\quad |W_{lk}(\beta_{lk},\beta_{lk}^*,0)|=2A_{N,J}.$$

The logarithmic derivative of each component W_{lk} with respect to time at the initial time moment $t = 0$ equals

$$\begin{aligned}\Upsilon_{lk}(\xi) &\equiv -\frac{1}{2}\left.\frac{\partial \ln W_{lk}(\xi,\xi^*,t)}{\partial t}\right|_{t=0}\\ &= \gamma\Big[\beta_l^*(\beta_k-\xi)+\beta_k(\beta_l^*-\xi^*)+4\nu\Big((\beta_k-\xi)(\xi^*-\beta_l^*)+\frac{1}{2}\Big)\\ &\quad -2M\Big(e^{-i\phi}(\beta_k-\xi)^2+e^{i\phi}(\beta_l^*-\xi^*)^2\Big)\Big].\end{aligned}$$

For $k = l$ and $\xi = \beta_k$, Υ_{kk} depends only on γ and ν. On the contrary, for $k \neq l$ we have an extra strong dependence on ξ, β_k and β_l, which results in much greater values of $|\Upsilon_{lk}|$. Therefore, it seems reasonable to define the decoherence time between any two coherent-state components of the superposition (3) as the inverse real part of the initial logarithmic time derivative of the corresponding Wigner function component at the middle point between the initial positions of the coherent states in the phase plane (where this Wigner component has maximum initial absolute value, which gives rise to the maximum interference between the coherent states):

$$\tau_{lk} = \left.(\Upsilon_{lk}+\Upsilon_{lk}^*)^{-1}\right|_{\xi=\beta_{lk}}. \tag{29}$$

The explicit expression for τ_{lk} in the case of phase-sensitive reservoir considered is as follows:

$$\tau_{lk} = \left[2\gamma\left\{|\beta_k-\beta_l|^2+2\nu\left(|\beta_k-\beta_l|^2+1\right)-2M\mathrm{Re}\left(e^{-i\phi}(\beta_k-\beta_l)^2\right)\right\}\right]^{-1}. \tag{30}$$

So, the decoherence time is phase sensitive — for ν and b fixed, it can be significantly changed, according to the value of the reservoir phase ϕ.

Denoting the initial distance between two coherent state components as $r \equiv |\beta_k - \beta_l|$, we see that the decoherence time has the maximum value if the phase of complex number $e^{-i\phi}(\beta_k-\beta_l)^2$ equals 0,

$$\tau_s = \left[2\gamma\left(r^2+2\nu\left(r^2+1\right)-2Mr^2\right)\right]^{-1}. \tag{31}$$

Setting $M = \sqrt{\nu(\nu+1)}$, we can find the maximum of τ_s as a function of ν for the fixed r:

$$\tau_s^{(\max)} = \left[2\gamma\left(\sqrt{2r^2+1}-1\right)\right]^{-1}, \quad \nu_{\max} = \frac{r^2+1-\sqrt{2r^2+1}}{2\sqrt{2r^2+1}}. \tag{32}$$

For $r \gg 1$, the decoherence time falls as $(2\sqrt{2}\gamma r)^{-1}$ for a phase sensitive reservoir, while for a thermal one it goes as $\tau_v = (2\gamma r^2)^{-1}$. The "price" for the increase of the decoherence time for the optimally selected reservoir parameters for the chosen pair of coherent components is the fast decoherence between other components. Indeed, choosing the positive sign in front of M in (31) we obtain for the optimal value $\nu_{\max}$ and for $r \gg 1$ the value $\tau_s^{(\min)} \approx (2\sqrt{2}\gamma r^3)^{-1}$.

As an example, we consider the case of the superposition of four coherent states (Fig. 2). Figures 3a and 3b show symmetrical decoherence in the (phase-insensitive) vacuum and thermal reservoirs (in accordance with results obtained in [28]). However, for the phase sensitive reservoir, $M \neq 0$, the decoherence is asymmetrical — for $\phi = 0$ the quantum interference among the states (2 and 4) is preserved (Fig. 4c) and for $\phi = \pi$ the quantum interference among the states (1 and 3) is preserved (Fig. 4d).

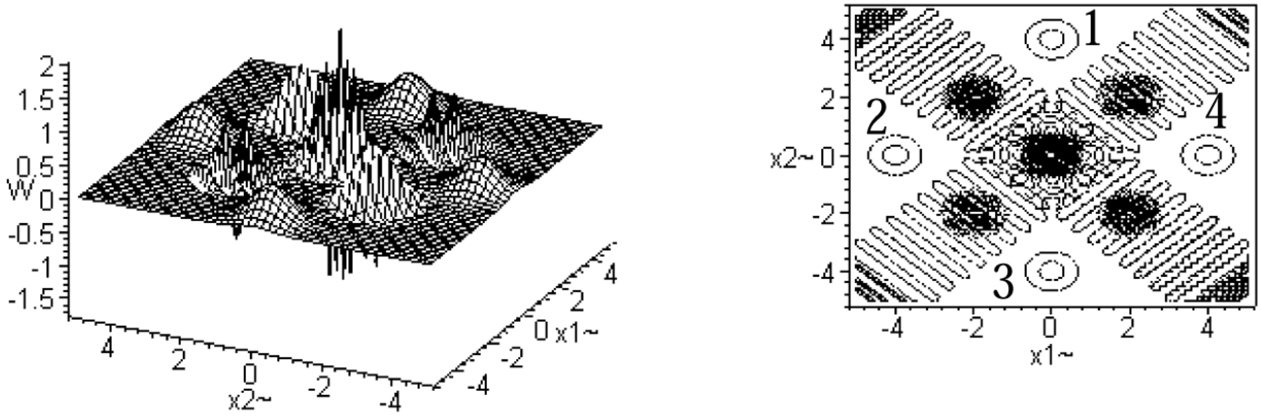

Fig. 2. The Wigner function and its contour plot for the initial multiphoton coherent state with $J = 0$, $N = 4$, and $b = 16$.

When the phase of squeezed light injected in the cavity is $\pi/2$, an interesting fact occurs — the initial circular state develops quickly to the state described by the Wigner function

$$W_{mc}(\xi,\xi^*,t) = \Big[4\sqrt{F}(1+e^{-2b})\Big]^{-1}\sum_{l=1}^{2}\sum_{k=1}^{2}\langle\beta_l|\beta_k\rangle\Bigg\{\exp\Bigg[-\frac{1}{2F}\Big(A(\xi^*-\beta_l^*(t))^2 + A^*(\xi-\beta_k(t))^2 + 2B(\xi^*-\beta_l^*(t))(\xi-\beta_k(t))\Big)\Bigg] + \exp\Bigg[-\frac{1}{2F}\Big(A(\xi^*+\beta_l^*(t))^2 + A^*(\xi+\beta_k(t))^2 + 2B(\xi^*+\beta_l^*(t))(\xi+\beta_k(t))\Big)\Bigg]\Bigg\} \tag{33}$$

shown in Fig. 4b. This state is almost indistinguishable from the classical statistical mixture of the two even coherent states (Fig. 4a)

$$\hat{\rho}_{mc}(0) = \frac{1}{4(1+e^{-2b})}\sum_{k=1}^{2}\sum_{l=1}^{2}\Big[|\beta_k\rangle\langle\beta_l| + |-\beta_k\rangle\langle-\beta_l|\Big]. \tag{34}$$

In this example (which is characterized by the relations $\tau_{14} = \tau_{23} \ll \tau_{12} = \tau_{34}$), the initial homogeneous four-photon quantum state quickly evolves into a mixture of two practically independent "cat" states, which can be named the "pair-cat" state.

5. Summary and Conclusions

Here we investigated the decoherence process of the superposition of four coherent states (circular state) when acted by a phase sensitive reservoir. We verified that the decoherence times between any two

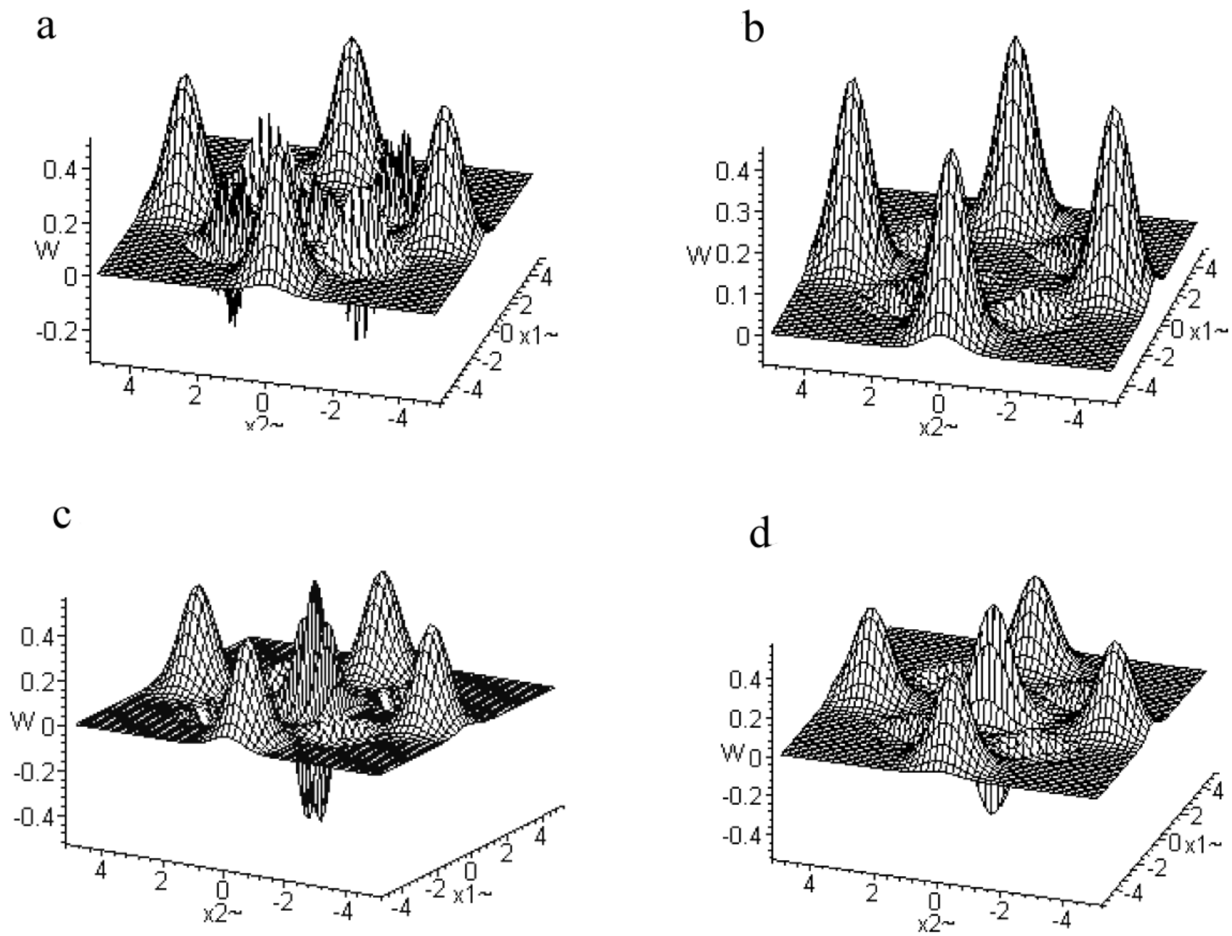

Fig. 3. The Wigner function of the initial circular state shown in Fig. 2 ($J = 0$, $N = 4$ and $b = 16$) at $2\gamma t = 0.075$; a) after the interaction with the vacuum reservoir; b) after the interaction with the thermal reservoir characterized by $\nu = 0.8$ and $M = 0$; c) after the interaction with the squeezed vacuum reservoir characterized by $\nu = 0.8$, $M = 1.2$, $\phi = 0$; d) after the interaction with the squeezed vacuum reservoir characterized by $\nu = 0.8$, $M = 1.2$, $\phi = \pi$.

coherent states of the superposition can be controlled through the reservoir parameters. The decoherence time between two components of any pair, for instance $\tau_{1,2}$ or $\tau_{3,4}$, can be significantly increased, compared with the decoherence time when the state is acted upon by a thermal reservoir. However, this occurs at the expense of decreasing the decoherence time between the "cat states" (1,2) and (3,4). By representing the circular state in the phase space, in terms of the Wigner function, calling r the distance between the centers of any two bumps standing for each coherent state, we verified that for $r \gg 1$, the decoherence time falls as $(2\sqrt{2}\gamma r)^{-1}$ for a phase sensitive reservoir while for a thermal one it goes as $\tau_v = (2\gamma r^2)^{-1}$. Thus, with conveniently chosen parameters, the squeezed reservoir delays the decoherence between a pair of coherent states; however, a price has to be payed, which is the decrease of the decoherence time between other components. We obtained for the optimal value $\nu_{\max}$ and for $r \gg 1$ the value $\tau_s^{(\min)} \approx (2\sqrt{2}\gamma r^3)^{-1}$.

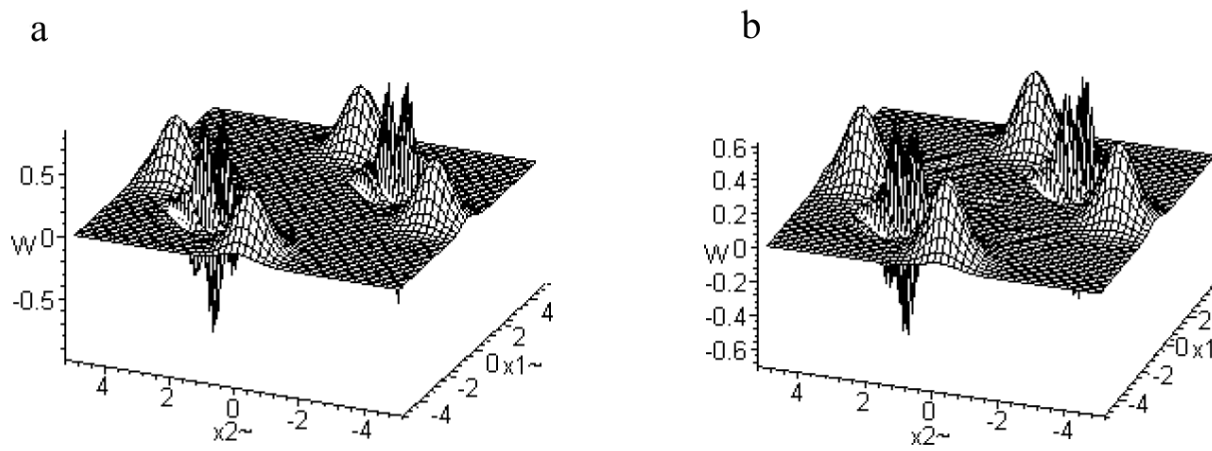

Fig. 4. a) The Wigner function of the mixed state (34). b) The Wigner function of the initial circular state shown in Fig. 2 ($J = 0$, $N = 4$ and $b = 16$) at $2\gamma t = 0.075$ after the interaction with the squeezed vacuum reservoir characterized by $\nu = 0.8$, $M = 1.2$ and $\phi = \pi/2$.

So, the initial homogeneous four-photon quantum state quickly evolves into a mixture of two practically independent "cat" states, which can be named "pair-cat" state. We believe that this separation of the initial state as the "pair-cat" state can be useful in quantum computations.

6. Acknowledgments

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