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Phenomenological-operator approach to introduce damping effects on radiation field states

N G de Almeida, P B Ramos, R M Serra and M H Y Moussa

Departamento de Física, Universidade Federal de São Carlos, CP 676, São Carlos 13565-905, SP, Brazil

E-mail: miled@power.ufscar.br

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Abstract. In this paper we propose an approach to deal with radiation field states which incorporates damping effects at zero temperature. By using some well known results on dissipation of a cavity-field state, obtained by standard *ab initio* methods, it was possible to infer, through a phenomenological way, the explicit form for the evolution of the state vector for the whole system: the cavity field plus reservoir. This proposal turns out to be extremely convenient for accounting for the influence of the reservoir over the cavity field. To illustrate the universal applicability of our approach we consider the attenuation effects on cavity-field states engineering. The main concern of the present phenomenological approach consists in furnishing a straightforward technique to estimate the fidelity resulting from processes in cavity QED phenomena. A proposal to maximize the fidelity of the process is presented.

Keywords: Radiation field states, damping effects, cavity field

1. Introduction

After more than half a century since their proposition by Einstein, Podolsky and Rosen (EPR) [1], entangled states have become the cornerstone of a set of striking proposals in theoretical physics. The advent of Bell's theorem [2], which has permitted us to empirically test the phenomenon of nonlocality, and more than a decade of experiments, confirming such an astonishing character of quantum mechanics, provided entanglement with the necessary credibility to motivate such proposals. Ranging from quantum communication [3] and cryptography [4] to teleportation [5, 6] and quantum computation [7], the nonlocal character of entangled states has left its original realm of quantum metaphysics to inaugurate the possibility of a new technology. In fact, inspired by these theoretical implications, the experimental setups, formerly designed to investigate fundamental physics such as quantum coherence and nonlocality, have recently been enhanced for the realization of teleportation [6] and to demonstrate quantum-logic operations [8].

However, the above-mentioned processes on quantum mechanics come up against a crucial problem intrinsic to quantum nature: the *decoherence* coming from the inevitable coupling between quantum systems and the surrounding environment. Such a decoherence process, transforming a pure state $|\psi\rangle$ into a statistical mixture ρ, constitutes the main difficulty preventing the realization of the above-mentioned quantum processes. Since the transformation $|\psi\rangle \longrightarrow \rho$ becomes faster as the excitation of the initial quantum state increases, the realization of massive computation, for example, turns out to be prohibitive and even the realization of a single logic gate operation [8] leads to a degraded output. In this connection, the calculation of the fidelity of a given process becomes a crucial task when proposing its experimental realization. However, this task turns out to be as difficult to achieve as the implementation of the protocol for accomplishing a given quantum process. As an example, we cite the process of engineering a cavity-field state, which requires a series of steps to be pursued [9], since in most of these schemes the cavity field is built up photon by photon.

To overcome the difficulty in estimating the fidelity in cavity QED processes, in this paper we propose a phenomenological way to handle with dissipation of a cavity-field state under the influence of a reservoir at absolute zero temperature. Our strategy is to incorporate, in a concise algebraic approach, the main results obtained by standard *ab initio* techniques on dissipation of a cavity field. In short, we include the exponential decay law accounting for the damping of the cavity-field excitation explicitly in the evolution of the whole system comprising of the cavity field plus reservoir. Once we have calculated the evolved whole state vector, obtaining the total density matrix is immediate, and by tracing out the reservoir variables we finally get the desired reduced density matrix for the cavity field, which is the same as the one obtained by standard methods. This approach resembles the Monte Carlo wavefunction method [10] in the sense that we work directly with the wavefunction, providing an efficient

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computational tool to account for dissipation in quantum optics.

It is worth mentioning that our result for the evolved state vector of the radiation field plus reservoir can be applied to calculate dissipation effects in any physical process based on cavity QED. Here, as an application, we consider damping effects in engineering a quantum field state and present an alternative method to maximize the fidelity of the engineered state. The process of generation of an arbitrary radiation field state, as will be analysed here, is severely damaged due to the reservoir attenuation.

This paper is organized as follows. In section 2 we describe the essential features of the model considered here, a monochromatic cavity field trapped in a lossy cavity. In section 3 we present an algebraic way to introduce damping explicitly on the evolution of the whole state vector for the cavity field plus reservoir. In section 4 we apply the technique developed in section 3 to analyse the fidelity of an engineered cavity-field state. Section 5 is devoted to illustrating a technique to optimize the fidelity of the engineered field state and finally, in section 6 we present our conclusions.

2. Model

We are concerned with the standard model of a cavity field described by creation and annihilation central oscillator amplitude operators $a^\dagger$, a, coupled to a reservoir consisting of a set of oscillators described by amplitude operators $b_k^\dagger$, b_k. The reservoir simulates the cavity damping mechanism: photon losses occur either at the mirrors or by leaking out of the cavity. The cavity field is coupled to the reservoir through the usual rotating wave approximation, so that the Hamiltonian for the system is written as

$$H = \hbar\omega a^\dagger a + \sum_k \hbar\omega_k b_k^\dagger b_k + \sum_k \hbar(g_k a b_k^\dagger + g_k^* a^\dagger b_k) \quad (1)$$

where g_k are coupling parameters. The Heisenberg equations for both amplitude operators a and b are given by

$$\dot{a}(t) = -\mathrm{i}\omega a - \mathrm{i}\sum_k g_k b_k \quad (2a)$$

$$\dot{b}_k(t) = -\mathrm{i}\omega_k b_k - \mathrm{i}g_k^* a. \quad (2b)$$

Considering the Wigner–Weisskopf approximation, in which the frequencies of the reservoir oscillators densely cover a range in the neighbourhood of the field frequency, we obtain the solutions of the equations corresponding to the systems (2a) and (2b). The solution for the cavity field is

$$a(t) = \mu(t)a(0) + \sum_k \vartheta_k(t) b_k(0). \quad (3)$$

Introducing the damping factor γ and the typical small frequency shift $\Delta\omega$, the function $\mu(t)$ is explicitly given by

$$\mu(t) = \exp\left\{-\left[\frac{\gamma}{2} + \mathrm{i}(\Delta\omega + \omega)\right]t\right\}.$$

For the purpose at hand it is not necessary to specify the functions $\vartheta_k(t)$.

A convenient way to calculate the reduced density matrix for the cavity field consists of using the standard characteristic function technique [12]. For this purpose, we must first calculate the characteristic function χ, then the Glauber–Sudarshan P-representation, and finally the reduced density operator. The characteristic function can be written in terms of the operators a, $a^\dagger$ in the normal order form as

$$\chi(\eta, \eta^*, t) = \mathrm{Tr}\{\rho(t)\exp[\eta a^\dagger(0)]\exp[-\eta^* a(0)]\} \quad (4a)$$

$$= \mathrm{Tr}\{\rho(0)\exp[\eta a^\dagger(t)]\exp[-\eta^* a(t)]\}. \quad (4b)$$

Equation (4a) refers to the Schrödinger picture and equation (4b) to the Heisenberg picture. Since the time evolution of operators a, $a^\dagger$ is known from equation (3) together with the initial state of the whole system, equation (4b) turns out to be more convenient to calculate the characteristic function. Considering that the cavity field is initially in an arbitrary superposition state $\sum_{n=0}^N C_n|n\rangle$, where $|n\rangle$ is a n-photon Fock state, and the reservoir (assumed at absolute zero temperature) is in the vacuum state, the total density matrix at $t = 0$ is given by

$$\rho(0) = \sum_{n,m=0}^{N} C_n C_m^* |n\rangle\langle m| \otimes |\{0\}\rangle\langle\{0\}| \quad (5)$$

where we have denoted $\prod_k |0_k\rangle \equiv |\{0\}\rangle$.

The Glauber–Sudarshan P representation of the density matrix can be written as a two-dimensional Fourier transform of the characteristic function χ:

$$P(\alpha, \alpha^*, t) = \int \frac{\mathrm{d}^2\eta}{\pi^2}\chi(\eta, \eta^*, t)\exp(-\eta\alpha^* + \eta^*\alpha) \quad (6)$$

while the reduced density operator for the cavity field is given by, in the general form,

$$\rho(t) = \int \mathrm{d}^2\alpha\, P(\alpha, \alpha^*, t)|\alpha\rangle\langle\alpha|. \quad (7)$$

Considering all these definitions it is possible to obtain, after a little algebra, the explicit form for the reduced density operator of the cavity field:

$$\rho(t) = \sum_{n,m=0}^{N}\sum_{j=0}^{m}\sum_{l=0}^{j} C_n C_m^* (-1)^l \frac{|\mu(t)|^{2j}\mu(t)^{n-m}}{l!(m-j)!} \times\sqrt{\frac{n!m!}{(j+n-l-m)!(j-l)!}}|j+n-l-m\rangle\langle j-l|. \quad (8)$$

3. Phenomenological approach

The aim of this section is to include explicitly the energy dissipation of the cavity field in the time evolution of the state vector for the whole system. In short, the exponential decay law accounting for the damping of the cavity-field excitation will be explicitly included into the evolution of the state vector for the whole system. Such a state vector must lead, naturally, to the result derived from formal techniques for the reduced density matrix equation (8). As usual, the reduced density matrix for the cavity field is obtained by tracing over the reservoir variables, i.e.

$$\rho_{\mathrm{S}}(t) = \mathrm{Tr}_{\mathrm{R}}[\rho_{\mathrm{S+R}}(t)] \quad (9)$$

where $\rho_{\mathrm{S+R}}(t)$ is given, for pure states at time t, by

$$\rho_{\mathrm{S+R}}(t) = |\Psi_{\mathrm{S+R}}(t)\rangle\langle\Psi_{\mathrm{S+R}}(t)| \tag{10}$$

and the subscripts S and R refer to the system and reservoir, respectively. What is thus necessary to know is precisely the evolution of the whole state vector $|\Psi_{\mathrm{S+R}}(t)\rangle$. For this intent, we begin by considering first some examples which will guide us to the form of the evolution of the generalized n-photon cavity-field state in the presence of the reservoir. For the simplest case, a single photon in the cavity, the evolution of the whole state vector can be phenomenologically inferred as

$$|1\rangle|R\rangle \longrightarrow \exp\left(-\frac{\gamma}{2}t\right)|1\rangle|R\rangle + |0\rangle\sum_k \alpha_k b_k^\dagger|R\rangle \tag{11}$$

where each α_k represents the probability amplitude that a photon has been absorbed by the kth mode of the reservoir and γ is the damping rate for the one-photon state in the cavity. Under the normalization condition on state (11), we arrive at the constraint

$$\sum_k |\alpha_k|^2 = 1 - \exp(-\gamma t). \tag{12}$$

For the next case of a two-photon state, we infer the evolution of the whole system by noting that the interaction between the cavity mode and the reservoir is always characterized by one-photon exchange. In this way, the final form of the evolution of the whole state vector can be predicted to be

$$\begin{aligned}|2\rangle|R\rangle \longrightarrow{}& \exp\left(-2\frac{\gamma}{2}t\right)|2\rangle|R\rangle \\ &+\sqrt{2}\exp\left(-\frac{\gamma}{2}t\right)|1\rangle\sum_k \alpha_k b_k^\dagger|R\rangle \\ &+|0\rangle\sum_{kk'}\alpha_k\alpha_{k'}b_k^\dagger b_{k'}^\dagger|R\rangle.\end{aligned} \tag{13}$$

The first term on the RHS of equation (13) is associated with the probability that the cavity field remains in its original two-photon state. It is worth noting the difference between the relaxation rates for one- and two-photon field states: the damping rate for a two-photon state is increased by a factor of 2 compared to that of the one-photon state. In the second term on the RHS of equation (13), we have included the probability that the one-photon state has originated from the decay of the two-photon state and also the probability that the one-photon state remains as such. The last term accounts for the probability that all photons have been absorbed by the reservoir. We stress that the imposition of the normalization condition on state equation (13) gives the same constraint equation (12).

We note that states equations (11) and (13), as it should be, recover the same result as given by equation (8). To verify this assertion, it is necessary to build up the density operator for the system plus reservoir and after that, to get rid of the reservoir variables.

Finally it is possible, by induction, to generalize the above results to construct the evolution of an arbitrary n-photon state. This state vector for the whole system includes

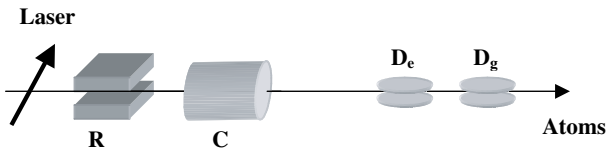

Figure 1. Sketch of the experimental setup for engineering a cavity-field state.

the ingredients discussed before and can be written as

$$\begin{aligned}|n\rangle|R\rangle \longrightarrow{}& \exp\left(-n\frac{\gamma}{2}t\right)|n\rangle|R\rangle \\ &+\sqrt{\frac{n!}{1!(n-1)!}}\exp\left[-(n-1)\frac{\gamma}{2}t\right]|n-1\rangle \\ &\times\sum_{k_1}\alpha_{k_1}b_{k_1}^\dagger|R\rangle \\ &+\sqrt{\frac{n!}{2!(n-2)!}}\exp\left[-(n-2)\frac{\gamma}{2}t\right]|n-2\rangle \\ &\times\sum_{k_1k_2}\alpha_{k_1}\alpha_{k_2}b_{k_1}^\dagger b_{k_2}^\dagger|R\rangle \\ &+\cdots+|0\rangle\sum_{k_1,\ldots,k_n}\alpha_{k_1},\ldots,\alpha_{k_n}b_{k_1}^\dagger,\ldots,b_{k_n}^\dagger|R\rangle\end{aligned} \tag{14}$$

where each term appearing in equation (14) includes the probability that a given state has originated from the initial n-photon state and also the probability that it will remain the same. Again, the desired reduced density matrix for the cavity field can be obtained from the time-dependent state vector equation (14) by using equations (9) and (10).

4. Noise effects on engineering quantum-field states

To illustrate the applicability of the method developed in the previous section, we restrict our analysis to the damping effects in the engineering process of a cavity-field state. To do that, we choose the Vogel *et al* scheme to engineer an arbitrary cavity-field state [11], which is based on successive resonant interactions of M two-level atoms with an initially empty cavity. The experimental setup for this scheme is depicted in figure 1. The Rydberg atoms are laser excited before entering into the Ramsey zone R, placed in the way of the atoms to the cavity C, preparing each atom in a particular superposition of excited $|\mathrm{e}\rangle$ and ground $|\mathrm{g}\rangle$ states, as is required to properly build up the cavity field, photon by photon. Each two-level atom, after interacting with a monochromatic field state in the cavity C, leaves its photon in the cavity, being necessarily detected in its ground state by detector chambers D (D_{e} and D_{g} for ionizing the states $|\mathrm{e}\rangle$ and $|\mathrm{g}\rangle$, respectively).

To exemplify an application of the present phenomenological approach, we consider the first step of this process. Initially, the cavity field C is in the vacuum state while the atom is in an arbitrary (unnormalized) superposition $|\mathrm{e}\rangle + \mathrm{i}\epsilon_1|\mathrm{g}\rangle$, where ϵ_1 is a complex parameter controlled by the Ramsey zone R. Once we consider the reservoir to be at absolute zero temperature, it will always be in the vacuum state $|R\rangle = |\{0\}\rangle$.

To simplify the calculations, we assume a stable excited atomic state and just take into account the errors introduced by the cavity dissipation mechanism. This is a good approximation, since on average eight out of ten atoms are able to travel the distance of the whole setup without decaying. In fact, since a Rydberg-atom excited state has a lifetime $(1/\gamma_a)$ of the order of 10^{-2} s [13], the probability of staying in this state is about 0.8 for an experiment duration of about 2×10^{-3} s. For high-Q superconducting cavities, the lifetime $(1/\gamma)$ is also of the order of 10^{-2} s [13]. Finally, we neglect the field dissipation during the time that the atoms interact with the cavity field.

After the resonant atom–field interaction, when the atom has left the cavity and measured in the ground state, the state of the whole system can be written as

$$|\Psi^{(1)}\rangle = \sum_{n=0}^{1} \hat{Q}_n^{(1)} |n\rangle |R\rangle \tag{15}$$

where we have described the damping effects on the cavity field (given by equation (11)) through the operators $\hat{Q}_n^{(1)}$ appearing above, which are explicitly given by

$$\hat{Q}_0^{(1)} = \varphi_0^{(1)} + \varphi_1^{(1)} \sum_k \alpha_k^{(1)} b_k^\dagger \tag{16a}$$

$$\hat{Q}_1^{(1)} = \exp\left(-\frac{\gamma}{2} t'\right) \varphi_1^{(1)} \tag{16b}$$

where t' is the time in which the relaxation process occurs, i.e. the time interval in which the first atom left the cavity and is detected. At this point it is useful to define the parameters $C_n^{(k)} = \cos(g\tau_k\sqrt{n+1})$ and $S_n^{(k)} = \sin(g\tau_k\sqrt{n+1})$, where τ_k is the interaction time of the kth atom with the cavity field and g is the atom–field coupling constant, assumed to be the same for all the atoms. The constants $\varphi_i^{(1)}$, $i = 1, 2$ in equations (16a) and (16b) are the same as those in [11], i.e.

$$\varphi_1^{(1)} = S_0^{(1)} \tag{17}$$

$$\varphi_0^{(1)} = -\epsilon_1. \tag{18}$$

The next step consists of the analysis of the results after the passage of the second atom through the cavity and the subsequent relaxation of the radiation field then constructed. After the second atom (prepared in the superposition $|\mathrm{e}\rangle + \mathrm{i}\epsilon_2|\mathrm{g}\rangle$) has left the cavity and again been measured in the ground state, the two-photon cavity-field-reservoir state becomes

$$|\Psi^{(2)}\rangle = \sum_{n=0}^{2} \hat{Q}_n^{(2)} |n\rangle |R\rangle \tag{19}$$

where the operators $\hat{Q}_n^{(2)}$ turn out to be more complicated then those in equation (15) due to entanglement between the states of each atom and the cavity-field-reservoir states. These operators can be written as

$$\hat{Q}_0^{(2)} = \hat{T}_0^{(2)} + \hat{T}_1^{(2)} \sum_k \alpha_k^{(2)} b_k^\dagger + \hat{T}_2^{(2)} \sum_{k_1 k_2} \alpha_{k_1}^{(2)} \alpha_{k_2}^{(2)} b_{k_1}^\dagger b_{k_2}^\dagger \tag{20a}$$

$$\hat{Q}_1^{(2)} = \hat{T}_1^{(2)} \exp\left(-\frac{\gamma}{2} t\right) + \sqrt{2}\hat{T}_2^{(2)} \exp\left(-\frac{\gamma}{2} t'\right) \sum_k \alpha_k^{(2)} b_k^\dagger \tag{20b}$$

$$\hat{Q}_2^{(2)} = \hat{T}_2^{(2)} \exp\left(-2\frac{\gamma}{2} t\right) \tag{20c}$$

where the operators $\hat{T}_n^{(2)}$ appearing above are given by

$$\hat{T}_0^{(2)} = -\epsilon_2 \hat{Q}_0^{(1)} \tag{21a}$$

$$\hat{T}_1^{(2)} = S_0^{(2)} \hat{Q}_0^{(1)} - \epsilon_2 C_0^{(2)} \hat{Q}_1^{(1)} \tag{21b}$$

$$\hat{T}_2^{(2)} = S_1^{(2)} \hat{Q}_1^{(1)}. \tag{21c}$$

Here t is the time starting after the second atom left the cavity. For simplicity, we have assumed the time interval for the detection of the first and second atoms are equal, and that the second atom enters into the cavity immediately after the first atom being detected. Although here we are just concerned with the desired $|\psi_d\rangle = \sum_{n=0}^{2} d_n |n\rangle$ cavity-field state, the method described above can be generalized for generating an arbitrary cavity-field state ($|\psi_d\rangle = \sum_{n=0}^{M} d_n |n\rangle$) in a straightforward (if rather tedious) manner. Naturally, all the information we need is contained in $|\Psi^{(2)}\rangle$, according to equation (19). For instance, the reduced density operator can be obtained readily by using equation (19) and definition equations (9) and (10) as

$$\begin{aligned}\hat{\rho}_{\mathrm{S}}(t) = \mathcal{N}[&a|2\rangle\langle 2| + b|1\rangle\langle 1| + c|0\rangle\langle 0| \\ &+ (d|2\rangle\langle 1| + f|2\rangle\langle 0| + g|1\rangle\langle 0| + \mathrm{h.c.})]\end{aligned} \tag{22}$$

where the coefficients appearing above are given in the appendix and $\mathcal{N} = 1/(a+b+c)$ is the normalization constant.

From this reduced density operator, equation (22), which includes the inevitable coupling between the system and the reservoir, it is possible to calculate, in principle, any physical quantity (observable) of interest.

Ideal case. When damping effects are neglected, i.e. letting the damping constant $\gamma = 0$, equation (22) recovers the ideal case and $\hat{\rho}_{\mathrm{S}}$ can be written in terms of a pure state $|\Psi_{\mathrm{S}}\rangle\langle\Psi_{\mathrm{S}}|$, where $|\Psi_{\mathrm{S}}\rangle = \sum_{n=0}^{2} \psi_n^{(2)} |n\rangle$ and the coefficients $\psi_n^{(2)}$ are the same as those in [11], as can be checked from equations (16a)–(21c).

5. Optimizing the fidelity of an engineered cavity-field state

At this point this formulation is completely general, and can be applied to generate any arbitrary cavity-field state. However, we stress that there is a crucial difference between the procedure described here and the one in [11]. In fact, in the treatment proposed by Vogel *et al* it is necessary to solve polynomial equations for ϵ_1 and ϵ_2, which arise from the consistency of the recurrence equations relating the amplitudes $\varphi_k^{(n)}$. On the other hand, in the treatment described here, by virtue of the reservoir, the quantities $\hat{Q}_n^{(2)}$ in equation (19) are now operators, instead of simply being parameters. In this connection, there is no immediate way to extract useful information from the recurrence relation associated between the operators involved. Thus, a first question arising is, how to turn round this problem? We propose the following way to solve this problem: consider a specific desired cavity-field state to be engineered and calculate the fidelity of this ideal state (free from the reservoir

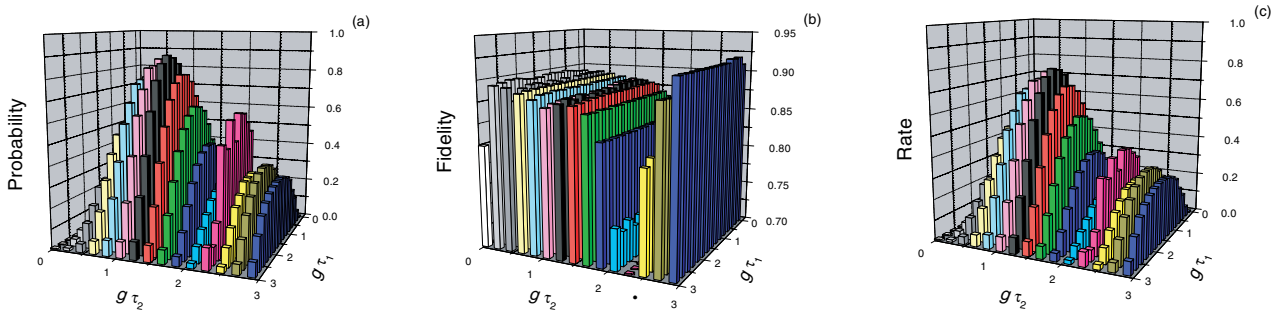

Figure 2. Histogram of the probability $\mathcal{P}$, fidelity $\mathcal{F}$ and rate $\mathcal{R}$, respectively, in the $g\tau_1 \times g\tau_2$ plane.

effects) with respect to the state $|\Psi^{(2)}\rangle$ in equation (19). By doing this procedure, the only unknown variables are ϵ_1 and ϵ_2, since all the others can be fixed. The trick here is to find numerically the values for ϵ_1 and ϵ_2 which maximize the resulting expression for the fidelity. This process can be even more optimized by an adequate choice of the interaction parameters $g\tau_k$.

Another important ingredient in this discussion is concerned with the probability for successfully achieving the engineering process. In fact, the success of the engineering process depends on detecting all the required atoms in their ground states. The expression for the probability to detect each atom in the ground state can be obtained in a usual way. The distinguished feature here is given by the presence of the reservoir. To do that, we have to consider the internal atomic states as well as the constructed radiation-field states being taken as normalized. We thus have to normalize the states given in equations (15) and (19) by following the recipe $|\Psi^{(k)}\rangle \to \mathcal{N}_k|\Psi^{(k)}\rangle$, where the normalization constants are

$$\mathcal{N}_1 = \left[\frac{1}{1+|\epsilon_1|^2}\langle R|\sum_{j=0}^{1}(\hat{Q}_j^{(1)})^\dagger \hat{Q}_j^{(1)}|R\rangle\right]^{-1/2} \qquad (23a)$$

$$\mathcal{N}_2 = \left[\frac{1}{1+|\epsilon_2|^2}\frac{\langle R|\sum_j(\hat{Q}_j^{(2)})^\dagger \hat{Q}_j^{(2)}|R\rangle}{\langle R|\sum_j(\hat{Q}_j^{(1)})^\dagger \hat{Q}_j^{(1)}|R\rangle}\right]^{-1/2}. \qquad (23b)$$

The total probability for successfully engineering the desired state is $\mathcal{P} = \prod_{k=1}^{2}\mathcal{P}_k$, where $\mathcal{P}_k = 1/(\mathcal{N}_k)^2$ accounts for the probability of detecting the kth atom in the ground state $|\mathrm{g}\rangle$.

To exemplify the whole procedure, we restrict our analysis to the generation of the following radiation-field truncated phase state:

$$|\Psi_d\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle).$$

The fidelity of the density matrix $\rho_s(t)$ relative to $|\Psi_d\rangle$ is given by

$$\mathcal{F} = \langle\Psi_d|\rho_s(t)|\Psi_d\rangle \qquad (24)$$

where $\rho_s(t)$ is the normalized density matrix. The resulting expression is a little bit involved and must be numerically maximized regarding the variables ϵ_1 and ϵ_2. To optimize the engineering process (24), we consider different interaction parameters $g\tau_k$ to each atom, which depend on experimental capabilities. In the present realistic scheme for engineering a cavity-field state two important features arise: the fidelity $\mathcal{F}$ of the engineered state and the probability $\mathcal{P}$ to successfully achieve the process. Figures 2(a)–(c) show the histogram of the probability $\mathcal{P}$, the fidelity $\mathcal{F}$ and the rate $\mathcal{R} \equiv \mathcal{P}\mathcal{F}$, which serves as a cost–benefit estimate to choose the best parameters, in terms of the parameters $g\tau_1$ and $g\tau_2$. For a definite value of the interaction parameters $g\tau_k$ is calculated the corresponding values ϵ_1 and ϵ_2 which maximize the fidelity. From this figure we note that a larger probability does not necessarily imply a better fidelity. This is an important point since the convenient probability $\mathcal{P}$ and the fidelity $\mathcal{F}$ to be chosen depend on the engineering necessities. In table 1 some values of $g\tau_k$ and the solutions for ϵ_1 and ϵ_2, as well as the corresponding quantities $\mathcal{P}$, $\mathcal{F}$ and $\mathcal{R}$, are exhibited. Figures 3(a)–(f) show both the histogram of the elements of the reduced density matrix and the associated Wigner functions. Figures 3(a) and (b) show the ideal case ($\gamma = 0$); figures 3(c) and (d) show the realistic engineered state by using the parameters ϵ_1 and ϵ_2 obtained from the scheme of Vogel *et al* , taking into account the reservoir; and figures 3(e) and (f) show the engineered state obtained by our maximization procedure. As can be viewed in figures 3(e) and (f), this procedure improves the quantum nature of the engineered state, which is revealed by the negative portions of the Wigner function. To plot figures 2 and 3 we used the realistic experimental parameters $\gamma = 10^2\ \mathrm{s}^{-1}$ [13] for the decay rate and the estimated time interval $t = t' = 10/\gamma$ [14] for the first and second atoms, respectively, to reach the detection chambers after leaving the cavity.

6. Conclusion

In this paper we have proposed an alternative way to treat damping effects at zero temperature in cavity QED processes. Our phenomenological approach consists of incorporating the main results obtained by standard *ab initio* techniques on dissipation of a cavity field directly into the evolution of the state vector of the whole system: cavity field plus reservoir. In summary, we have included the exponential decay law accounting for the damping of the cavity-field excitation explicitly in the evolution of the whole state vector of the system.

By considering a standard model of a given mode of the radiation field coupled to a collection of N modes representing the reservoir, it was possible to infer the time evolution for the whole state vector in a straightforward

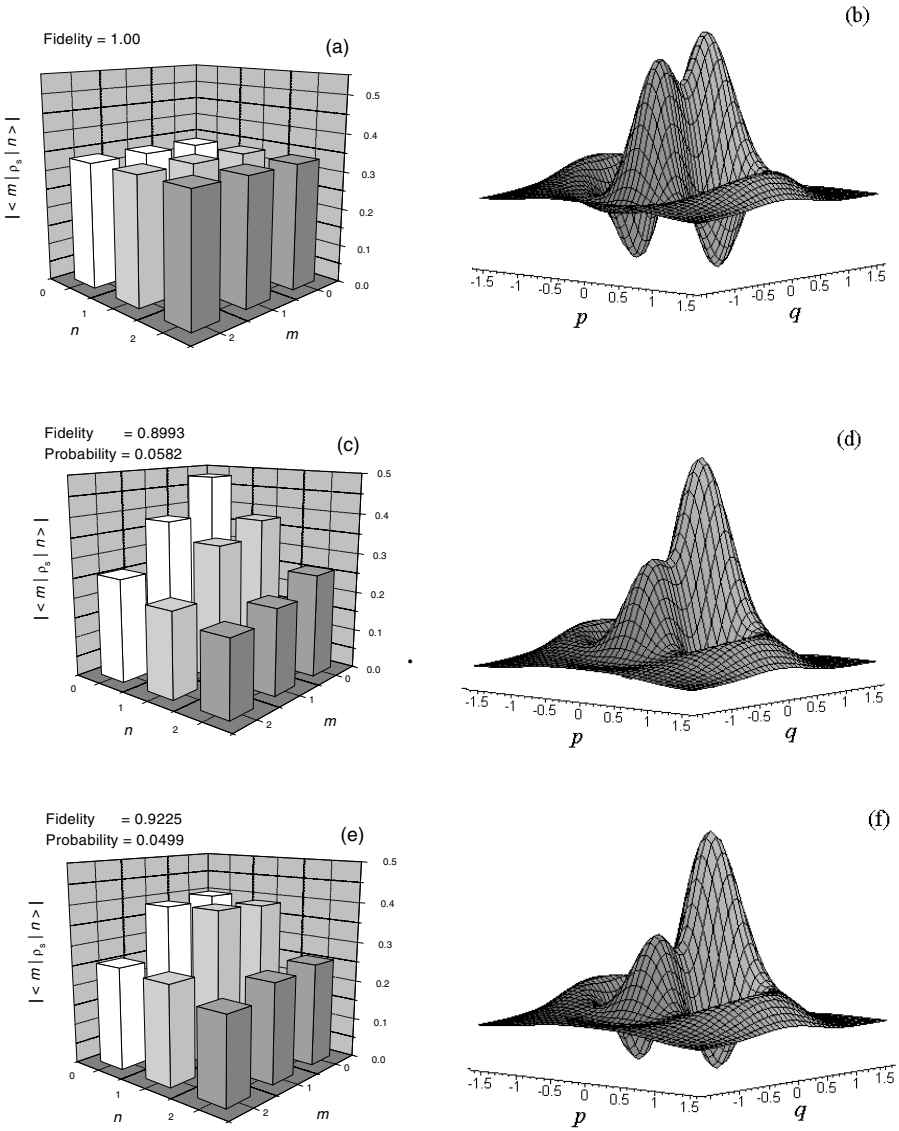

Figure 3. Elements of the reduced density matrix and the corresponding Wigner distribution functions: (*a*), (*b*) for the ideal state; (*c*), (*d*) for the realistic engineered state built with parameters ϵ_1 and ϵ_2 obtained from the scheme of Vogel *et al* , including the reservoir; and (*e*), (*f*) for the optimized engineered state by the maximization procedure. The Wigner $W(\alpha, \alpha^*)$ distribution functions are plotted as a function of $q = \mathrm{Re}\,(\alpha)$ and $p = \mathrm{Im}\,(\alpha)$.

Table 1. Some values of the interaction times $g\tau_1$ and $g\tau_2$, Ramsey zone parameters ϵ_1 and ϵ_2 (adjusted in order to maximize the fidelity), fidelity $\mathcal{F}$, probability $\mathcal{P}$ and rate $\mathcal{R}$.

$g\tau_1$	$g\tau_2$	ϵ_1	ϵ_2	$\mathcal{F}$	$\mathcal{P}$	$\mathcal{R}$
0.6	3.0	2.7693	−0.1583	0.9253	0.0697	0.0645
1.3	1.2	−0.8508 + i0.2874	−0.8838 − i0.1600	0.8789	0.8831	0.7761
1.4	2.8	1.2349 + i0.8632	−0.3583 + i0.2427	0.9087	0.3637	0.3305

manner. We stress that this result is rather general, and in principle can be applied to many quantum processes, such as, for instance, quantum communication, logic operations, teleportation and cavity-field state engineering, among others.

The phenomenological-operator approach to dissipation in cavity quantum electrodynamics presented here considerably simplifies the introduction of the inevitable errors due to the environmental degrees of freedom when describing processes involving atom–field interactions. The development of this technique became possible due to the previous formal work in quantum dissipation, from which we have recovered the main results concerning energy loss of a trapped radiation field. So, such a convenient approach precludes the necessity of performing the usually extensive *ab initio* calculations as the standard master equation, the characteristic functions, or even the path integral formalism. It is worth noting that the technique here developed at absolute zero can be extended

for a thermal reservoir.

As an application of the present technique, we have considered the process of engineering a cavity-field state in a lossy cavity. In order to estimate how far the generated state deviates from the idealized state, due to attenuation, we have analysed the fidelity of the engineering process. Also, we have proposed an alternative way for engineering a cavity-field state in the presence of a reservoir, which consists of maximizing the fidelity of the desired (ideal) state relative to the (damped) engineered state. A specific example has been given which takes into account realistic values for the parameters involved. To conclude, we should stress that damping effects seriously restrict such an engineering process. However, these noise effects can be minimized by the scheme proposed in this paper.

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Appendix

In this appendix we show explicitly the coefficients appearing in equation (22):

$$
\begin{aligned}
a &= (S_0^{(1)} S_1^{(2)})^2 \mathrm{e}^{-\gamma(t'+2t)} \\
b &= 2(S_0^{(1)} S_1^{(2)})^2 \mathrm{e}^{-\gamma(t'+t)}(1-\mathrm{e}^{-\gamma t}) \\
&\quad -|\epsilon_2|^2 (S_0^{(1)} C_0^{(2)})^2 \mathrm{e}^{-\gamma(t'+t)} \\
&\quad +(S_0^{(1)} S_0^{(2)})^2 \mathrm{e}^{-\gamma t'}(1-\mathrm{e}^{-\gamma t}) \\
&\quad -|\epsilon_1|^2 (S_0^{(2)})^2 \mathrm{e}^{-\gamma t} \\
&\quad +(\epsilon_1\epsilon_2^* + \epsilon_1^*\epsilon_2) S_0^{(1)} C_0^{(2)} S_0^{(2)} \mathrm{e}^{-1/2\gamma(t'+2t)} \\
c &= -|\epsilon_2|^2 (S_0^{(1)})^2 (1-\mathrm{e}^{-\gamma t'}) + |\epsilon_1|^2 |\epsilon_2|^2 \\
&\quad +(S_0^{(1)} S_1^{(2)})^2 \mathrm{e}^{-\gamma t'}(1-\mathrm{e}^{-\gamma t})^2 \\
&\quad -|\epsilon_2|^2 (S_0^{(1)} C_0^{(2)})^2 \mathrm{e}^{-\gamma t'}(1-\mathrm{e}^{-\gamma t}) \\
&\quad +(S_0^{(1)} S_0^{(2)})^2 (1-\mathrm{e}^{-\gamma t'})(1-\mathrm{e}^{-\gamma t}) \\
&\quad -|\epsilon_1|^2 (S_0^{(2)})^2 (1-\mathrm{e}^{-\gamma t}) \\
&\quad +(\epsilon_1\epsilon_2^* + \epsilon_1^*\epsilon_2) S_0^{(1)} S_0^{(2)} C_0^{(2)} \mathrm{e}^{-1/2\gamma t'}(1-\mathrm{e}^{-\gamma t}) \\
d &= -\epsilon_2^* (S_0^{(1)})^2 S_1^{(2)} C_0^{(2)} \mathrm{e}^{-\gamma(t'+3/2t)} \\
&\quad -\epsilon_1^* S_0^{(1)} S_1^{(2)} S_0^{(2)} \mathrm{e}^{-\gamma/2(t'+3t)} \\
f &= \epsilon_1^*\epsilon_2^* S_0^{(1)} S_1^{(2)} \mathrm{e}^{-\gamma/2(t'+2t)} \\
g &= -\epsilon_2^* (S_0^{(1)})^2 S_0^{(2)} \mathrm{e}^{-1/2\gamma t}(1-\mathrm{e}^{-\gamma t'}) \\
&\quad -\epsilon_1^* |\epsilon_2|^2 S_0^{(1)} C_0^{(2)} \mathrm{e}^{-\gamma/2(t'+t)} \\
&\quad -\epsilon_2^* |\epsilon_1|^2 S_0^{(2)} \mathrm{e}^{-1/2\gamma t} \\
&\quad -\sqrt{2}\epsilon_2^* (S_0^{(1)})^2 S_1^{(2)} C_0^{(2)} \mathrm{e}^{-\gamma/2(2t'+t)}(1-\mathrm{e}^{-\gamma t}) \\
&\quad -\sqrt{2}\epsilon_1^* S_0^{(1)} S_0^{(2)} S_1^{(2)} \mathrm{e}^{-\gamma t'}(1-\mathrm{e}^{-\gamma t}).
\end{aligned}
$$

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[2] Bell J S 1964 *Ann. Phys., NY* **1** 195

[3] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 *Phys. Rev. Lett.* **78** 3221
Pellizzari T 1997 *Phys. Rev. Lett.* **79** 5242

[4] Lo H-K, Chau H and Shor P W 1997 *Phys. Rev. Lett.* **78** 3410
Mayers D 1996 *Preprint* quant-ph/9603015

[5] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W 1993 *Phys. Rev. Lett.* **70** 1895
Moussa M H Y 1996 *Phys. Rev.* A **54** 4661
Moussa M H Y 1997 *Phys. Rev.* A **55** R3287
Bouwmeester D, Pan J-W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575
Boschi D, Branca, De Martini F, Hardy L and Popescu S 1998 *Phys. Rev. Lett.* **80** 1121
Villas-Bôas C J, de Almeida N G and Moussa M H Y 1999 *Phys. Rev.* A **60** 2759

[6] Furusawa A, Sorensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706 and references therein

[7] Shor P W 1994 *Proc. 35th Ann. Symp. on the Foundations of Calculator Science* ed S Goldwasser (Los Alamitos, CA: IEEE Calculator Society Press) p 124

[8] Chuang I L, Vandersypen L M K, Zhou X, Leung D W and Lloyd S 1998 *Nature* **393** 143 and references therein

[9] Krause J, Scully M O and Walther H 1987 *Phys. Rev.* A **36** R4547
Brune M S, Haroche S, Raimond J M, Davidovich L and Zagury N 1992 *Phys. Rev.* A **45** 5193
Parkins A S, Marte P, Zoller P and Kimble H J 1993 *Phys. Rev. Lett.* **71** 3095
Janszky J, Domokos P, Szabó S and Adam P 1995 *Phys. Rev.* A **51** 4191
Bose S, Jacobs K and Knight P L 1997 *Phys. Rev.* A **56** 4175
Gheri K M, Saavedra C, Törmä P, Cirac J I and Zoller P 1998 *Phys. Rev.* A **58** R2627
Moussa M H Y and Baseia B 1998 *Phys. Lett.* A **238** 223
Shi-Biao Zheng 1998 *Opt. Commun.* **154** 290
Vidiella-Barranco A and Roversi J A 1998 *Phys. Rev.* A **58** 3349
Pegg D T, Phillips L S and Barnett S M 1998 *Phys. Rev. Lett.* **81** 1604
Dakna M, Clausen J, Knöll L and Welsch D G 1999 *Phys. Rev.* A **59** 1658
Branning D, Grice W P, Erdmann R and Walmsley I A 1999 *Phys. Rev. Lett.* **83** 955

[10] Dalibard J, Castin Y and Mølmer K 1992 *Phys. Rev. Lett.* **68** 580
Dum R, Zoller P and Ritsch H 1992 *Phys. Rev.* A **45** 4879

[11] Vogel K, Akulin V M and Schleich W P 1993 *Phys. Rev. Lett.* **71** 1816

[12] See, for example, Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)
Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)

[13] Weindinger M, Varcoe B T H, Heerlein R and Walther H 1999 *Phys. Rev. Lett.* **82** 3795
Brune M, Hagley E, Dreyer J, Maître X, Maali A, Wunderlich C, Raimond J M and Haroche S 1996 *Phys. Rev. Lett.* **77** 4887

[14] Nussenzveig P Private communication