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REVIEW ARTICLE

'Nonclassical' states in quantum optics: a 'squeezed' review of the first 75 years

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Abstract
Seventy five years ago, three remarkable papers by Schrödinger, Kennard and Darwin were published. They were devoted to the evolution of Gaussian wave packets for an oscillator, a free particle and a particle moving in uniform constant electric and magnetic fields. From the contemporary point of view, these packets can be considered as prototypes of the coherent and squeezed states, which are, in a sense, the cornerstones of modern quantum optics. Moreover, these states are frequently used in many other areas, from solid state physics to cosmology. This paper gives a review of studies performed in the field of so-called 'nonclassical states' (squeezed states are their simplest representatives) over the past seventy five years, both in quantum optics and in other branches of quantum physics.

My starting point is to elucidate who introduced different concepts, notions and terms, when, and what were the initial motivations of the authors. Many new references have been found which enlarge the 'standard citation package' used by some authors, recovering many undeservedly forgotten (or unnoticed) papers and names. Since it is practically impossible to cite several thousand publications, I have tried to include mainly references to papers introducing new types of quantum states and studying their properties, omitting many publications devoted to applications and to the methods of generation and experimental schemes, which can be found in other well known reviews. I also mainly concentrate on the initial period, which terminated approximately at the border between the end of the 1980s and the beginning of the 1990s, when several fundamental experiments on the generation of squeezed states were performed and the first conferences devoted to squeezed and 'nonclassical' states commenced. The 1990s are described in a more 'squeezed' manner: I have confined myself to references to papers where some new concepts have been introduced, and to the most recent reviews or papers with extensive bibliographical lists.

Keywords: Nonclassical states, squeezed states, coherent states, even and odd coherent states, quantum superpositions, minimum uncertainty states, intelligent states, Gaussian packets, non-Gaussian coherent states, phase states, group and algebraic coherent states, coherent states for general potentials, relativistic oscillator coherent states, supersymmetric states, para-coherent states, q-coherent states, binomial states, photon-added states, multiphoton states, circular states, nonlinear coherent states

1. Introduction

The terms 'coherent states', 'squeezed states' and 'nonclassical states' can be encountered in almost every modern paper on quantum optics. Moreover, they are frequently used in many other areas, from solid state physics to cosmology. In 2001 and 2002 it will be seventy five years since the publication of three papers by Schrödinger [1], Kennard [2] and Darwin [3], in which the evolutions of Gaussian wavepackets for an oscillator, a free particle and a particle moving in uniform constant

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electric and magnetic fields were considered. From the contemporary point of view, these packets can be considered as prototypes of the coherent and squeezed states, which are, in a sense, the cornerstones of modern quantum optics. Since the squeezed states are the simplest representatives of a wide family of 'nonclassical states' in quantum optics, it seems appropriate, bearing in mind the jubilee date, to give an updated review of studies performed in the field of nonclassical states over the past seventy five years.

Although extensive lists of publications can be found in many review papers and monographs (see, e.g., [4–7]), the subject has not been exhausted, because each of them highlighted the topic from its own specific angle. My starting point was to elucidate who introduced different concepts, notions and terms, when, and what were the initial motivations of the authors. In particular, while performing the search, I discovered that many papers and names have been undeservedly forgotten (or have gone unnoticed for a long time), so that 'the standard citation package' used by many authors presents a sometimes rather distorted historical picture. I hope that the present review will help many researchers, especially the young, to obtain a less deformed vision of the subject.

One of the most complicated problems for any author writing a review is an inability to supply the complete list of all publications in the area concerned, due to their immense number. In order to diminish the length of the present review, I tried to include only references to papers introducing *new types of quantum states* and studying their properties, omitting many publications devoted to *applications* and to the methods of generation and experimental schemes. The corresponding references can be found, e.g., in other reviews [4–6]. I also concentrate mainly on the initial period, which terminated approximately at the border between the end of the 1980s and the beginning of the 1990s, when several fundamental experiments on the generation of squeezed states were performed and the first conferences devoted to squeezed and 'nonclassical' states commenced. The 1990s are described in a more 'squeezed' manner, because the recent history will be familiar to the readers. For this reason, describing that period, I have confined myself to references to papers where some new concepts have been introduced, and to the most recent reviews or papers with extensive bibliographical lists, bearing in mind that in the Internet era it is easier to find recent publications (contrary to the case of forgotten or little known old publications).

2. Coherent states

It is well known that it was Schrödinger [1] who constructed for the first time in 1926 the 'nonspreading wavepackets' of the harmonic oscillator. In modern notation, these packets can be written as (in the units $\hbar = m = \omega = 1$):

$$\langle x|\alpha\rangle = \pi^{-1/4}\exp(-\tfrac{1}{2}x^2 + \sqrt{2}x\alpha - \tfrac{1}{2}\alpha^2 - \tfrac{1}{2}|\alpha|^2). \quad (1)$$

A complex parameter α determines the mean values of the coordinate and momentum according to the relations:

$$\langle\hat{x}\rangle = \sqrt{2}\,\mathrm{Re}\,\alpha, \qquad \langle\hat{p}\rangle = \sqrt{2}\,\mathrm{Im}\,\alpha.$$

The variances of the coordinate and momentum operators, $\sigma_x = \langle\hat{x}^2\rangle - \langle\hat{x}\rangle^2$ and $\sigma_p = \langle\hat{p}^2\rangle - \langle\hat{p}\rangle^2$, have equal values, $\sigma_x = \sigma_p = 1/2$, so their product assumes the minimal value permitted by the Heisenberg uncertainty relation,

$$(\sigma_x\sigma_p)_{\min} = 1/4$$

(in turn, this relation, which was formulated by Heisenberg [8] as an approximate inequality, was strictly proven by Kennard [2] and Weyl [9]).

The simplest way to arrive at formula (1) is to look for the eigenstates of the non-Hermitian annihilation operator

$$\hat{a} = \left(\hat{x} + \mathrm{i}\hat{p}\right)/\sqrt{2} \quad (2)$$

satisfying the commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 1, \quad (3)$$

so that:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle. \quad (4)$$

For example, this was done by Glauber in [10], where the name 'coherent states' appeared in the text for the first time. However, several authors did similar things before him. The annihilation operator possessing property (3) was introduced by Fock [11], together with the eigenstates $|n\rangle$ of the number operator $\hat{n} = \hat{a}^\dagger\hat{a}$, known nowadays as the 'Fock states' (the known eigenfunctions of the harmonic oscillator in the coordinate representation in terms of the Hermite polynomials were obtained by Schrödinger in [12]). And it was Iwata who considered for the first time in 1951 [13] the eigenstates of the non-Hermitian annihilation operator $\hat{a}$, having derived formula (1) and the now well known expansion over the Fock basis:

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle. \quad (5)$$

The states defined by means of equations (4) and (5) were used as some auxiliary states, permitting to simplify calculations, by Schwinger [14] in 1953. Later, their mathematical properties were studied independently by Rashevskiy [15], Klauder [16] and Bargmann [17], and these states were discussed briefly by Henley and Thirring [18].

The coherent state (5) can be obtained from the vacuum state $|0\rangle$ by means of the unitary *displacement operator*:

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle, \qquad \hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}) \quad (6)$$

which was actually used by Feynman and Glauber as far back as 1951 [19, 20] in their studies of quantum transitions caused by the classical currents (which are reduced to the problem of the forced harmonic oscillator, studied, in turn, in the 1940s–1950s by Bartlett and Moyal [21], Feynman [22], Ludwig [23], Husimi [24] and Kerner [25]).

However, only after the works by Glauber [10] and Sudarshan [26] (and especially Glauber's work [27]), did the coherent states became widely known and intensively used by many physicists. The first papers published in magazines, which had the combination of words 'coherent states' in their titles, were [27, 28].

Coherent states have always been considered as 'the most classical' ones (among the *pure* quantum states, of course): see, e.g., [29]. Moreover, they can serve [27] as a starting point to define the 'nonclassical states' in terms of the Glauber–Sudarshan P-function, which was introduced in [10] to represent thermal states and in [26] for arbitrary density matrices:

$$\hat{\rho} = \int P(\alpha)|\alpha\rangle\langle\alpha|\,\mathrm{d}\,\mathrm{Re}\,\alpha\,\mathrm{d}\,\mathrm{Im}\,\alpha, \qquad \int P(\alpha)\,\mathrm{d}\,\mathrm{Re}\,\alpha\,\mathrm{d}\,\mathrm{Im}\,\alpha = 1. \tag{7}$$

Indeed, the following definition was given in [27]: *'If the singularities of $P(\alpha)$ are of types stronger than those of delta function, e.g., derivatives of delta function, the field represented will have no classical analog'*.

In this sense, the thermal (chaotic) fields are classical, since their P-functions are usual positive probability distributions. The states of such fields, being quantum mixtures, are described in terms of the density matrices (introduced by Landau [30]). Also, the coherent states are 'classical', because the P-distribution for the state $|\beta\rangle$ is, obviously, the delta function, $P_\beta(\alpha) = \delta(\alpha - \beta)$. On the other hand, these states lie 'on the border' of the set of the 'classical' states, because the delta function is the most singular distribution admissible in the classical theory. Consequently, it is enough to make slight modifications in each of the definitions (1), (4), (5), or (6), to arrive at various families of states, which will be 'nonclassical'. For this reason, many classes of states which are labelled nowadays as 'nonclassical', appeared in the literature as some kinds of *'generalized coherent states'*. At least four different kinds of generalizations exist.

(I) One can generalize equations (1) or (5), choosing different sets of coefficients $\{c_n\}$ in the expansion

$$|\psi\rangle = \sum_n c_n|n\rangle \tag{8}$$

or writing different explicit wavefunctions in other (continuous) representations (coordinate, momentum, Wigner–Weyl, etc).

(II) One can replace the exponential form of the operator $\hat{D}(\alpha)$ in equation (6) by some other operator function, also using some other set of operators $\hat{A}_k$ instead of operator $\hat{a}$ (e.g., making some kinds of 'deformations' of the fundamental commutation relations like (3)), and taking the 'initial' state $|\varphi\rangle$ other than the vacuum state. This line leads to states of the form

$$|\psi_\lambda\rangle = f(\hat{A}_k, \lambda)|\varphi\rangle, \tag{9}$$

where λ is some continuous parameter.

(III) One can look for the 'continuous' families of eigenstates of the new operators $\hat{A}_k$, trying to find solutions to the equation:

$$\hat{A}_k|\psi_\mu\rangle = \mu|\psi_\mu\rangle. \tag{10}$$

(IV) One can try to 'minimize' the generalized Heisenberg uncertainty relation for Hermitian operators $\hat{A}$ and $\hat{B}$ different from $\hat{x}$ and $\hat{p}$,

$$\Delta A \Delta B \geqslant \tfrac{1}{2}\,|\langle[\hat{A}, \hat{B}]\rangle|, \tag{11}$$

looking, for instance, for the states for which (11) becomes the equality.

Actually, all these approaches have been used for several decades of studies of 'generalized coherent states'. Some concrete families of states found in the framework of each method will be described in the following sections.

The 'nonclassicality' of the Fock states and their finite superpositions was mentioned in [31]. A simple criterium of 'nonclassicality' can be established, if one considers a generic Gaussian wavepacket with *unequal* variances of two quadratures, whose P-function reads [32] (in the special case of the statistically uncorrelated quadrature components)

$$P_G(\alpha) = \mathcal{N} \exp\left[-\frac{(\mathrm{Re}\,\alpha - a)^2}{\sigma_x - 1/2} - \frac{(\mathrm{Im}\,\alpha - b)^2}{\sigma_p - 1/2}\right] \tag{12}$$

(a and b give the position of the centre of the distribution in the α-plane; the symbol $\mathcal{N}$ hereafter is used for the normalization factor). Since function (12) exists as a normalizable distribution only for $\sigma_x \geqslant 1/2$ and $\sigma_p \geqslant 1/2$, the states possessing one of the quadrature component variances less than $1/2$ are *nonclassical*. This statement holds for any (not only Gaussian) state. Indeed, one can easily express the quadrature variance in terms of the P-function:

$$\sigma_x = \tfrac{1}{2} \int P(\alpha)[(\alpha + \alpha^* - \langle\hat{a} + \hat{a}^\dagger\rangle)^2 + 1]\,\mathrm{d}\,\mathrm{Re}\,\alpha\,\mathrm{d}\,\mathrm{Im}\,\alpha.$$

If $\sigma_x < 1/2$, then function $P(\alpha)$ must assume negative values, thus it cannot be interpreted as a classical probability [32].

Another example of a 'nonclassical' state is a superposition of two different coherent states [33]. As a matter of fact, *all pure states*, excepting the coherent states, are 'nonclassical', both from the point of view of their physical contents [34] and the formal definition in terms of the P-function given above [35]. However, speaking of 'nonclassical' states, people usually have in mind not an arbitrary pure state, but members of some families of quantum states possessing more or less useful or distinctive properties. One of the aims of this paper is to provide a brief review of the known families which have been introduced whilst trying to follow the historical order of their appearance.

According to the *Web of Science* electronic database of journal articles, the combination of words 'nonclassical states' appeared for the first time in the titles of the papers by Helstrom, Hillery and Mandel [36]. The first three papers containing the combination of words 'nonclassical effects' were published by Loudon [37], Zubairy, and Lugiato and Strini [38]. 'Nonclassical light' was the subject of the first three studies by Schubert, Janszky *et al* and Gea-Banacloche [39].

3. Squeezed states

3.1. Generic Gaussian wavepackets

Historically, the first example of the nonclassical (squeezed) states was presented as far back as in 1927 by Kennard [2] (see the story in [40]), who considered, in particular, the evolution in time of the *generic Gaussian wavepacket*

$$\psi(x) = \exp(-ax^2 + bx + c) \tag{13}$$

of the harmonic oscillator. In this case, the quadrature variances may be arbitrary (they are determined by the real and imaginary parts of the parameter a), but they satisfy the Heisenberg inequality $\sigma_x\sigma_p \geqslant 1/4$. Within twenty five years, Husimi [41] found the following family of solutions to the time-dependent Schrödinger equation for the harmonic oscillator (we assume here $\omega = m = \hbar = 1$)

$$\begin{aligned}\psi(x,t) &= [2\pi \sinh(\beta + \mathrm{i}t)]^{-1/2}\\ &\times \exp\{-\tfrac{1}{4}[\tanh[\tfrac{1}{2}(\beta+\mathrm{i}t)](x+x')^2\\ &+\coth\left[\tfrac{1}{2}(\beta+\mathrm{i}t)\right](x-x')^2]\},\end{aligned} \tag{14}$$

where x' and $\beta > 0$ are arbitrary real parameters. He showed that the quadrature variances oscillate with twice the oscillator frequency between the extreme values $\frac{1}{2}\tanh\beta$ and $\frac{1}{2}\coth\beta$. A detailed study of the Gaussian states of the harmonic oscillator was performed by Takahasi [42].

3.2. *The first appearance of the squeezing operator*

An important contribution to the theory of squeezed states was made by Infeld and Plebański [43–45], whose results were summarized in a short article [46]. Plebański introduced the following family of states

$$|\tilde{\psi}\rangle = \exp\left[\mathrm{i}\left(\eta\hat{x} - \xi\hat{p}\right)\right]\exp\left[\frac{\mathrm{i}}{2}\log a\left(\hat{x}\hat{p}+\hat{p}\hat{x}\right)\right]|\psi\rangle, \tag{15}$$

where ξ, η and $a > 0$ are real parameters, and $|\psi\rangle$ is an *arbitrary* initial state. Evidently, the first exponential in the right-hand side of (15) is nothing but the *displacement operator* (6) written in terms of the Hermitian quadrature operators. Its properties were studied in the first article [43]. The second exponential is the special case of the *squeezing operator* (see equation (21)). For the initial *vacuum* state, $|\psi_0\rangle = |0\rangle$, the state $|\tilde{\psi}_0\rangle$ (15) is exactly the squeezed state in modern terminology, whereas by choosing other initial states one can obtain various *generalized squeezed states*. In particular, the choice $|\psi_n\rangle = |n\rangle$ results in the family of the *squeezed number states*, which were considered in [45, 46]. In the case $a = 1$ (considered in [43]) we arrive at the states known nowadays by the name *displaced number states*. Plebański gave the explicit expressions describing the time evolution of the state (15) for the harmonic oscillator with a constant frequency and proved the completeness of the set of 'displaced' number states. Infeld and Plebański [44] performed a detailed study of the properties of the unitary operator $\exp(\mathrm{i}\hat{T})$, where $\hat{T}$ is a generic inhomogeneous quadratic form of the canonical operators $\hat{x}$ and $\hat{p}$ with constant c-number coefficients, giving some classification and analysing various special cases (some special cases of this operator were discussed briefly by Bargmann [17]). Unfortunately, the publications cited appeared to be practically unknown or forgotten for many years.

3.3. *From 'characteristic' to 'minimum uncertainty' states*

In 1966, Miller and Mishkin [47] deformed the defining equation (4), introducing the 'characteristic states' as the eigenstates of the operator

$$\hat{b} = u\hat{a} + v\hat{a}^\dagger \tag{16}$$

(for real u and v). In order to preserve the canonical commutation relation $[\hat{b}, \hat{b}^\dagger] = 1$ one should impose the constraint $|u|^2 - |v|^2 = 1$.

Similar states were considered by Lu [48], who called them 'new coherent states,' and by Bialynicki-Birula [49]. The general structure of the wavefunctions of these states in the coordinate representation is

$$\langle x|\beta\rangle = (\pi w_-^2)^{-1/4}\exp\left[\frac{\sqrt{2}\beta x}{w_-} - \frac{w_+x^2}{2w_-} - \frac{w_-^*\beta^2}{2w_-} - \frac{|\beta|^2}{2}\right], \tag{17}$$

where $w_\pm = u \pm v$ and $\hat{b}|\beta\rangle = \beta|\beta\rangle$.

Wavefunction (17) also arises if one looks for states in which the uncertainty product $\sigma_x\sigma_p$ assumes the minimal possible value $1/4$. This approach was used in [29, 50–52]. The variances in the state (17) are given by

$$\sigma_x = \tfrac{1}{2}|u-v|^2, \qquad \sigma_p = \tfrac{1}{2}|u+v|^2 \tag{18}$$

so that for *real* parameters u and v one has $\sigma_x\sigma_p = 1/4$ due to the constraint $|u|^2 - |v|^2 = 1$. In this case one can express the 'transformed' operator $\hat{b}$ in (16) in terms of the quadrature operators

$$\hat{x} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2} \qquad \hat{p} = (\hat{a} - \hat{a}^\dagger)/(\mathrm{i}\sqrt{2}), \tag{19}$$

as:

$$\hat{b} = \left(\lambda^{-1}\hat{x} + \lambda\hat{p}\right)/\sqrt{2}, \quad \lambda^{-1} = u + v, \quad \lambda = u - v. \tag{20}$$

The term 'minimum uncertainty states' (MUS) seems to be used for the first time in the paper by Mollow and Glauber [32], where it was applied to the special case of the Gaussian states (12) with $\sigma_x\sigma_p = 1/4$. Stoler [51] showed that the MUS can be obtained from the oscillator ground state by means of the unitary operator:

$$\hat{S}(z) = \exp[\tfrac{1}{2}(z\hat{a}^2 - z^*\hat{a}^{\dagger 2})]. \tag{21}$$

In the second paper of [51] the operator $\hat{S}(z)$ was written for real z in terms of the quadrature operators as $\exp[\mathrm{i}r(\hat{x}\hat{p}+\hat{p}\hat{x})]$, which is exactly the form given by Plebański [46]. The conditions under which the MUS preserve their forms were studied in detail in [51, 53].

3.4. *Coherent states of nonstationary oscillators and Gaussian packets*

The operators like (16) and the state (17) arise naturally in the process of the dynamical evolution governed by the Hamiltonian

$$\hat{H} = \omega\hat{a}^\dagger\hat{a} + \kappa\hat{a}^{\dagger 2}\mathrm{e}^{-2\mathrm{i}\omega t - \mathrm{i}\varphi} + \mathrm{h.c.}, \tag{22}$$

which describes the *degenerate parametric amplifier*. This problem was studied in detail by Takahasi in 1965 [42]. Similar *two-mode* states were considered implicitly in 1967 by Mollow and Glauber [32], who developed the quantum theory of the *nondegenerate parametric amplifier*, described by the interaction Hamiltonian between two modes of the form $\hat{H}_{\mathrm{int}} = \hat{a}^\dagger\hat{b}^\dagger\mathrm{e}^{-\mathrm{i}\omega t} + \mathrm{h.c.}$

In the case of an oscillator with arbitrary time-dependent frequency $\omega(t)$, the evolution of initially coherent states was considered in [54], where the operator $\exp[(\hat{a}^{\dagger 2} + \hat{a}^2)s]$ naturally appeared. The generalizations of the coherent state (4) as the eigenstates of the *linear time-dependent integral of motion operator*

$$\hat{A} = [\varepsilon(t)\hat{p} - \dot{\varepsilon}(t)\hat{x}]/\sqrt{2}, \qquad [\hat{A}, \hat{A}^{\dagger}] = 1, \tag{23}$$

where function $\varepsilon(t)$ is a specific complex solution of the *classical* equation of motion

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0, \qquad \mathrm{Im} = 1(\dot{\varepsilon}\varepsilon^*), \tag{24}$$

have been introduced by Malkin and Man'ko [55]. The integral of motion (23) satisfies the equation

$$\mathrm{i}\partial\hat{A}/\partial t = [\hat{H}, \hat{A}]$$

(where $\hat{H}$ is the Hamiltonian) and it has the structure (16), with *complex* coefficients $u(t)$ and $v(t)$, which are certain combinations of the functions $\varepsilon(t)$ and $\dot{\varepsilon}(t)$. The eigenstates of $\hat{A}$ have the form (17). They are coherent with respect to the time-dependent operator $\hat{A}$ (equivalent to $\hat{b}(t)$ (16) and generalizing (20)), but they are *squeezed* with respect to the quadrature components of the 'initial' operator $\hat{a}$ defined via (19). For the most general forms in the case of one degree of freedom see, e.g., [56] and the review [57].

Coherent states of a charged particle in a constant homogeneous magnetic field (generalizations of Darwin's packets [3]) have been introduced by Malkin and Man'ko in [58] (see also [59]). Generalizations of the nonstationary oscillator coherent states to systems with several degrees of freedom, such as a charged particle in nonstationary homogeneous magnetic and electric fields plus a harmonic potential, were studied in detail in [60–62]. Multidimensional time-dependent coherent states for arbitrary quadratic Hamiltonians have been introduced in [63]. Their wavefunctions were expressed in the form of generic Gaussian exponentials of N variables. From the modern point of view, all those states can be considered as squeezed states, since the variances of different canonically conjugated variables can assume values which are less than the ground state variances. Similar one-dimensional and multidimensional quantum Gaussian states were studied in connection with the problems of the theories of photodetection, measurements, and information transfer in [64, 65]. The method of linear time-dependent integrals of motion turned out to be very effective for treating both the problem of an oscillator with time-dependent frequency (other approaches are due to Fujiwara, Husimi and, especially, Lewis and Riesenfeld [24, 66]) and generic systems with multidimensional quadratic Hamiltonians. For detailed reviews see, e.g., [57, 67].

Approximate quasiclassical solutions to the Schrödinger equation with arbitrary potentials, in the form of the Gaussian packets whose centres move along the classical trajectories, have been extensively studied in many papers by Heller and his coauthors, beginning with [68]. The first coherent states for the *relativistic* particles obeying the Klein–Gordon or Dirac equations have been introduced in [69].

3.5. Possible applications and first experiments with 'two-photon' and 'squeezed' states

The first detailed review of the states defined by the relations (16) and (17) was given in 1976 by Yuen [70], who proposed the name 'two-photon states'. Up to that time, it was recognized that such states can be useful for solving various fundamental physical and technological problems. In particular, in 1978, Yuen and Shapiro [71] proposed to use the two-photon states in order to improve optical communications by reducing the quantum fluctuations in one (signal) quadrature component of the field at the expense of the amplified fluctuations in another (unobservable) component. Also, the states with the reduced quantum noise in one of the quadrature components appeared to be very important for the problems of measurement of weak forces and signals, in particular, for the detectors of gravitational waves and interferometers [72–76]. With the course of time, the terms 'squeezing', 'squeezed states' and 'squeezed operator', introduced in the papers by Hollenhorst and Caves [72, 75], became generally accepted, especially after the article by Walls [77], and they have replaced other names proposed earlier for such states. It is interesting to notice in this connection, that exactly at the same time, the same term 'squeezing' was proposed (apparently completely independently) by Brosa and Gross [78] in connection with the problem of nuclear collisions (they considered a simple model of an oscillator with time-dependent *mass* and fixed elastic constant, whereas in the simplest quantum optical oscillator models squeezed states usually appear in the case of time-dependent *frequency* and fixed mass). The words 'squeezed states' were used for the first time in the titles of papers [79–82], whereas the word 'squeezing' appeared in the titles of studies (in the field of quantum optics) [83–85], and 'squeezed light' was discussed in [86].

At the end of the 1970s and the beginning of the 1980s, many theoretical and experimental studies were devoted to the phenomena of *antibunching* or *sub-Poissonian photon statistics*, which are unequivocal features of the quantum nature of light. The relations between antibunching and squeezing were discussed, e.g., in [87], and the first experiment was performed in 1977 [88] (for the detailed story see, e.g., [37, 89]). Many different schemes of generating squeezed states were proposed, such as the four-wave mixing [90], the resonance fluorescence [91, 92], the use of the free-electron laser [80, 93], the Josephson junction [80, 94], the harmonic generation [84, 95], two- and multiphoton absorption [83, 96] and parametric amplification [79, 83, 97], etc. In 1985 and 1986 the results of the first successful experiments on the generation and detection of squeezed states were reported in [98] (backward four-wave mixing), [99] (forward four-wave mixing) and [100] (parametric down conversion). Details and other references can be found, e.g., in [101].

3.6. Correlated, multimode and thermal states

The states (17) with *complex* parameters u, v do not minimize the Heisenberg uncertainty product. But they are MUS for the *Schrödinger–Robertson uncertainty relation* [102]

$$\sigma_x\sigma_p - \sigma_{px}^2 \geqslant \hbar^2/4, \tag{25}$$

which can also be written in the form

$$\sigma_p\sigma_x \geqslant \hbar^2/[4(1 - r^2)], \tag{26}$$

where r is the *correlation coefficient* between the coordinate and momentum:

$$r = \sigma_{px}/\sqrt{\sigma_p \sigma_x} \qquad \sigma_{px} = \tfrac{1}{2}\langle \hat{p}\hat{x} + \hat{x}\hat{p}\rangle - \langle \hat{p}\rangle\langle \hat{x}\rangle.$$

For arbitrary Hermitian operators, inequality (25) should be replaced by [102]:

$$\sigma_A \sigma_B - \sigma_{AB}^2 \geqslant |\langle[\hat{A}, \hat{B}]\rangle|/4.$$

For further generalizations see, e.g., [103, 104].

Actually, the left-hand side of (26) takes on the minimal possible value $\hbar^2/4$ for *any pure* Gaussian state (this fact was known to Kennard [2]), which can be written as:

$$\psi(x) = \mathcal{N} \exp\left[-\frac{x^2}{4\sigma_{xx}}\left(1 - \frac{\mathrm{i}r}{\sqrt{1-r^2}}\right) + bx\right]. \qquad (27)$$

In order to emphasize the role of the correlation coefficient, state (27) was named the *correlated coherent state* [105]. It is unitary equivalent to the squeezed states with complex squeezing parameters, which were considered, e.g., in [106]. The dynamics of Gaussian wavepackets possessing a nonzero correlation coefficient were studied in [107]. Correlated states of the free particle were considered (under the name 'contractive states') in [108]. Gaussian correlated packets were applied to many physical problems: from neutron interferometry [109] to cosmology [110]. Reviews of properties of correlated states (including multidimensional systems, e.g., a charge in a magnetic field) can be found in [57, 111–113].

In the 1990s, various features of squeezed states (including phase properties and photon statistics) were studied and reviewed, e.g., in [114]. Different kinds of *two-mode squeezed states*, generated by the two-mode squeeze operator of the form $\exp(z\hat{a}\hat{b} - z^*\hat{a}^\dagger\hat{b}^\dagger + \cdots)$, were studied (sometimes implicitly or under different names, such as 'cranked oscillator states' or 'sheared states') in [115]. *Multimode squeezed states* were considered in [116]. The properties of generic (pure and mixed) *Gaussian states* (in one and many dimensions) were studied in [117–121]. Their special cases, sometimes named *mixed squeezed states* or *squeezed thermal states*, were discussed in [122–124]. Mixed analogues of different families of coherent and squeezed states were studied in [125]. *Grey-body states* were considered in [126]. 'Squashed states' have been introduced recently in [127].

Implicitly, the squeezed states of a charged particle moving in a homogeneous (stationary and nonstationary) magnetic field were considered in [60, 61]. In the explicit form they were introduced and studied in [111] and independently in [128]. For further studies see, e.g., [129–131]. In the case of an arbitrary (inhomogeneous) electromagnetic field or an arbitrary potential, the quasiclassical Gaussian packets centred on the classical trajectories (frequently called *trajectory-coherent states*) have been studied in [132].

The specific families of two- and multimode quantum states connected with the polarization degrees of freedom of the electromagnetic field (*biphotons*, *unpolarized light*) have been introduced by Karassiov [133]. For other studies see, e.g., [134].

3.7. Invariant squeezing

The instantaneous values of variances σ_x, σ_p and σ_{xp} cannot serve as true measures of squeezing in all cases, since they depend on time in the course of the free evolution of an oscillator. For example,

$$\sigma_x(t) = \sigma_x(0)\cos^2(t) + \sigma_p(0)\sin^2(t) + \sigma_{xp}(0)\sin(2t)$$

(in dimensionless units), and it can happen that both variances σ_x and σ_p are large, but nonetheless the state is highly squeezed due to large nonzero covariance σ_{xp}. It is reasonable to introduce some invariant characteristics which do not depend on time in the course of free evolution (or on phase angle in the definition of the field quadrature as $\hat{E}(\varphi) = \left[\hat{a}\exp(-\mathrm{i}\varphi) + \hat{a}^\dagger\exp(\mathrm{i}\varphi)\right]/\sqrt{2}$ [135]). They are related to the invariants of the total variance matrix or to the lengths of the principal axes of the ellipse of equal quasiprobabilities, which gives a graphical image of the squeezed state in the phase space [136] (this explains the name 'principal squeezing' used in [137]). The minimal σ_- and maximal σ_+ values of the variances σ_x or σ_p can be found by looking for extremal values of $\sigma_x(t)$ as a function of time [130]

$$\sigma_\pm = \mathcal{E} \pm \sqrt{\mathcal{E}^2 - d}\,, \qquad (28)$$

where $\mathcal{E} = \frac{1}{2}(\sigma_x + \sigma_p) \equiv \frac{1}{2} + \langle\hat{a}^\dagger\hat{a}\rangle - |\langle\hat{a}\rangle|^2$ is the *energy of quantum fluctuations* (which is conserved in the process of free evolution), whereas parameter $d = \sigma_x\sigma_p - \sigma_{xp}^2$, coinciding with the left-hand side of the *Schrödinger–Robertson uncertainty relation* (25), determines (for Gaussian states) the quantum 'purity' [118] $\mathrm{Tr}\hat{\rho}^2_{\mathrm{Gauss}} = 1/\sqrt{4d}$, being the simplest example of the so-called *universal quantum invariants* [138]. The expressions equivalent to (28) were obtained in [135–137]. Evidently, $\mathcal{E} \leqslant \frac{1}{2} + \bar{n}$, where $\bar{n}$ is the mean photon number. Then one can easily derive from (28) the inequalities:

$$\sigma_- \geqslant \bar{n} + 1/2 - \sqrt{(\bar{n} + 1/2)^2 - d} > \frac{d}{1 + 2\bar{n}}. \qquad (29)$$

For pure states ($d = 1/4$) these inequalities were found in [139]. For quantum mixtures, squeezing ($\sigma_- < 1/2$) is possible provided $\bar{n} \geqslant d^2 - 1/4$.

3.8. General concepts of squeezing

The first definition of squeezing for arbitrary Hermitian operators $\hat{A}$, $\hat{B}$ was given by Walls and Zoller [91]. Taking into account the uncertainty relation (11), they said that fluctuations of the observable A are 'reduced' if:

$$(\Delta A)^2 < \tfrac{1}{2}\,|\langle[\hat{A}, \hat{B}]\rangle|. \qquad (30)$$

This definition was extended to the case of several variables (whose operators are generators of some algebra) in [140] and specified in [141].

Hong and Mandel [142] introduced the concept of higher-order squeezing. The state $|\psi\rangle$ is squeezed to the $2n$th order in some quadrature component, say $\hat{x}$, if the mean value $\langle\psi|(\Delta\hat{x})^{2n}|\psi\rangle$ is less than the mean value of $(\Delta\hat{x})^{2n}$ in the coherent state. If $\hat{x}$ is defined as in (19), then the condition of squeezing reads:

$$\langle(\Delta\hat{x})^{2n}\rangle < 2^{-n}(2n-1)!!.$$

In particular, for $n = 2$ we have the requirement $\langle(\Delta\hat{x})^4\rangle < 3/4$. Hong and Mandel showed that the usual squeezed states are squeezed to any even order $2n$. The methods of producing such squeezing were proposed in [143].

Other definitions of the higher-order squeezing are usually based on the Walls–Zoller approach. Hillery [144] defined the second-order squeezing taking in (30):

$$\hat{A} = (\hat{a}^2 + \hat{a}^{\dagger 2})/2, \qquad \hat{B} = (\hat{a}^2 - \hat{a}^{\dagger 2})/(2\mathrm{i}).$$

A generalization to the kth-order squeezing, based on the operators $\hat{A} = (\hat{a}^k + \hat{a}^{\dagger k})/2$ and $\hat{B} = (\hat{a}^k - \hat{a}^{\dagger k})/(2\mathrm{i})$, was made in [145]. The uncertainty relations for higher-order moments were considered in [103]. For other studies on higher order squeezing see, e.g., [124, 146–149].

The concepts of the *sum squeezing* and *difference squeezing* were introduced by Hillery [150], who considered two-mode systems and used sums and differences of various bilinear combinations constructed from the creation and annihilation operators. These concepts were developed (including multimode generalizations) in [151]. A method of constructing squeezed states for general systems (different from the harmonic oscillator) was described in [152], where the eigenstates of the operator $\mu\hat{a}^2 + \nu\hat{a}^{\dagger 2}$ were considered as an example (see also [153]).

The concept of *amplitude squeezing* was introduced in [154]. It means the states possessing the property $\Delta n < \sqrt{\langle n\rangle}$ (for the coherent states, $\Delta n = \sqrt{\langle n\rangle}$, where n is the photon number). For further development see, e.g., [155]. Physical bounds on squeezing due to the finite energy of real systems were discussed in [156].

3.9. Oscillations of the distribution functions

While the photon distribution function $p_n = \langle n|\hat{\rho}|n\rangle$ is rather smooth for the 'classical' thermal and coherent states (being given by the Planck and Poisson distributions, respectively), it reveals strong oscillations for many 'nonclassical' states. The function p_n for a generic squeezed state was given in [70]

$$p_n = \frac{|v/(2u)|^n}{n!|u|}\exp\left[\mathrm{Re}\left(\frac{\beta^2 v^*}{u}\right) - |\beta|^2\right]\left|H_n\left(\frac{\beta}{\sqrt{2uv}}\right)\right|^2,$$

where $H_n(z)$ is the Hermite polynomial. The graphical analysis of this distribution made in [157] showed that for certain relations between the squeezing and displacement parameters, the photon distribution function exhibits strong irregular oscillations, whereas for other values of the parameters it remains rather regular. Ten years later, these oscillations were rediscovered in [158–162], and since that time they have attracted the attention of many researchers, mainly due to their interpretation [163] as the manifestation of the *interference in phase space* (a method of calculating quasiclassical distributions, based on the areas of overlapping in the phase plane, was used earlier in [164]).

It is worth mentioning that as far back as 1970, Walls and Barakat [165] discovered strong oscillations of the photon distribution functions, calculating eigenstates of the *trilinear* Hamiltonian $\hat{H}_{WB} = \sum_{k=1}^{3}\omega_k\hat{a}_k^\dagger\hat{a}_k + \kappa(\hat{a}_1\hat{a}_2^\dagger\hat{a}_3^\dagger + \mathrm{h.c.})$. The parametric amplifier time-dependent Hamiltonian (22), which 'produces' squeezed states, can be considered as the semiclassical approximation to $\hat{H}_{WB}$. For recent studies on trilinear Hamiltonians and references to other publications see [166].

The photocount distributions and oscillations in the two-mode nonclassical states were studied in [167]. The influence of thermal noise was studied in [123, 159, 168, 169]. The cumulants and factorial moments of the squeezed state photon distribution function were considered in [162, 168]. They exhibit strong oscillations even in the case of slightly squeezed states [170]. For other studies see, e.g., [171].

4. Non-Gaussian oscillator states

4.1. Displaced and squeezed number states

These states are obtained by applying the displacement operator $\hat{D}(\alpha)$ (6) or the squeezing operator $\hat{S}(z)$ (21) to the states different from the vacuum oscillator state $|0\rangle$. As a matter of fact, the first examples were given by Plebański [43], who studied the properties of the state $|n,\alpha\rangle = \hat{D}(\alpha)|n\rangle$. His results were rediscovered in [172], where the name *semicoherent state* was used. The general construction $\hat{D}(\alpha)|\psi_0\rangle$ for an arbitrary *fiducial state* $|\psi_0\rangle$ was considered by Klauder in [16]. The special case of the states $|n,z\rangle = \hat{S}(z)|n\rangle$ (known now under the name *squeezed number states*) for real z was considered in [45, 46]. These states were also briefly discussed by Yuen [70]. The *displaced number states* were considered in connection with the time-energy uncertainty relation in [173]. They can be expressed as [174]:

$$|n,\alpha\rangle = \hat{D}(\alpha)|n\rangle = \mathcal{N}(\hat{a}^\dagger - \alpha^*)^n|\alpha\rangle. \tag{31}$$

The detailed studies of different properties of displaced and squeezed number states were performed in [123, 146, 175].

4.2. First finite superpositions of coherent states

Titulaer and Glauber [176] introduced the 'generalized coherent states', multiplying each term of expansion (5) by arbitrary phase factors:

$$|\alpha\rangle_g = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\exp(\mathrm{i}\vartheta_n)|n\rangle. \tag{32}$$

These are the most general states satisfying Glauber's criterion of 'coherence' [27]. Their general properties and some special cases corresponding to the concrete dependences of the phases ϑ_n on n were studied in [177, 178].

In particular, Bialynicka-Birula [177] showed that in the *periodic case*, $\vartheta_{n+N} = \vartheta_n$ (with arbitrary values $\vartheta_0, \vartheta_1, \ldots, \vartheta_{N-1}$), state (32) is the superposition of N Glauber's coherent states, whose labels are uniformly distributed along the circle $|\alpha| = const$:

$$|\phi\rangle = \sum_{k=1}^{N} c_k|\alpha_0\exp(\mathrm{i}\phi_k)\rangle, \qquad \phi_k = 2\pi k/N. \tag{33}$$

The amplitudes c_k are determined from the system of N equations ($m = 0, 1, \ldots, N-1$)

$$\sum_{k=1}^{N} c_k \exp(\mathrm{i}m\phi_k) = \exp(\mathrm{i}\vartheta_m), \tag{34}$$

and they are different in the generic case. Stoler [178] has noticed that any sum (33) is an eigenstate of the operator $\hat{a}^N$ with the eigenvalue α_0^N, therefore it can be represented as a superposition of N orthogonal states, each one being a certain combination of N coherent states $|\alpha_0 \exp(\mathrm{i}\phi_k)\rangle$.

4.3. Even and odd thermal and coherent states

An example of 'nonclassical' *mixed* states was given by Cahill and Glauber [33], who discussed in detail the *displaced thermal states* (such states, which are obviously 'classical', were also studied in [32, 179]) and compared them with the following quantum mixture:

$$\hat{\rho}_{ev-th} = \frac{2}{2+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{2+\bar{n}}\right)^n |2n\rangle\langle 2n|. \tag{35}$$

The *even and odd coherent states*

$$|\alpha\rangle_{\pm} = \mathcal{N}_{\pm} \left(|\alpha\rangle \pm |-\alpha\rangle\right), \tag{36}$$

where $\mathcal{N}_{\pm} = (2[1 \pm \exp(-2|\alpha|^2)])^{-1/2}$, have been introduced by Dodonov *et al* [180]. They are not reduced to the superpositions given in (33), since coefficients $c_1 = \pm c_2$ can never satisfy equations (34) for real phases ϑ_0, ϑ_1. Besides, the photon statistics of the states (36) are quite different from the Poissonian statistics inherent to all states of the form (32). Even/odd states possess many remarkable properties: if $|\alpha| \gg 1$, they can be considered as the simplest examples of *macroscopic quantum superpositions* or *'Schrödinger cat states'*; being eigenstates of the operator $\hat{a}^2$ (cf equation (10)), the states $|\alpha\rangle_{\pm}$ are the simplest special cases of the *multiphoton states* (see later); they can be obtained from the vacuum by the action of *nonexponential* displacement operators

$$\hat{D}_{+}(\alpha) = \cosh(\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a}) \qquad \hat{D}_{-}(\alpha) = \sinh(\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a})$$

(cf equation (9)), etc. Moreover, from the modern point of view, the special cases of the time-dependent wavepacket solutions of the Schrödinger equation for the (singular) oscillator with a time-dependent frequency found in [180] are nothing but the *odd squeezed states*.

Parity-dependent squeezed states

$$[\hat{S}(z_1)\hat{\Pi}_{+} + \hat{S}(z_2)\hat{\Pi}_{-}]|\alpha\rangle$$

where $\hat{\Pi}_{\pm}$ are the projectors to the even/odd subspaces of the Hilbert space of Fock states, were studied in [181]. The actions of the squeezing and displacement operators on the superpositions of the form $|\alpha, \tau, \varphi\rangle = \mathcal{N}(|\alpha\rangle + \tau e^{\mathrm{i}\varphi}|-\alpha\rangle)$ (which contain as special cases even/odd, Yurke–Stoler, and coherent states) were studied in [182] (the special case $\tau = 1$ was considered in [183]). For other studies see, e.g., [6, 184–188]. Multidimensional generalizations have been studied in [189].

The name *shadowed states* was given in [190] to the mixed states whose statistical operators have the form $\hat{\rho}_{sh} = \sum p_n |2n\rangle\langle 2n|$ (i.e., generalizing the even thermal state (35)). Mixed analogues of the even and odd states were considered in [191].

5. Coherent phase states

The state of the classical oscillator can be described either in terms of its quadrature components x and p, or in terms of the amplitude and phase, so that $x + \mathrm{i}p = A\exp(\mathrm{i}\varphi)$. Moreover, in classical mechanics one can introduce the action and angle variables, which have the same Poisson brackets as the coordinate and momentum: $\{p, x\} = \{I, \varphi\} = 1$. However, in the quantum case we meet serious mathematical difficulties trying to define the phase operator in such a way that the commutation relation $[\hat{n}, \hat{\varphi}] = \mathrm{i}$ would be fulfilled, if the photon number operator is defined as $\hat{n} = \hat{a}^{\dagger}\hat{a}$. These difficulties originate in the fact that the spectrum of operator $\hat{n}$ is *bounded* from below.

The first solution to the problem was given by Susskind and Glogower [192], who introduced, instead of the phase operator itself, the *exponential phase operator*

$$\hat{E}_{-} \equiv \hat{C} + \mathrm{i}\hat{S} \equiv \sum_{n=1}^{\infty} |n-1\rangle\langle n| = (\hat{a}\hat{a}^{\dagger})^{-1/2}\hat{a}, \tag{37}$$

which can be considered, to a certain extent, as a quantum analogue of the classical phase $e^{\mathrm{i}\varphi}$ [29, 193]. Operator $\hat{E}_{-}$ and its Hermitian 'cosine' and 'sine' components satisfy the 'classical' commutation relations:

$$[\hat{C}, \hat{n}] = \mathrm{i}\hat{S}, \qquad [\hat{S}, \hat{n}] = -\mathrm{i}\hat{C}, \qquad [\hat{E}_{-}, \hat{n}] = \hat{E}_{-}. \tag{38}$$

However, operator $\hat{E}_{-}$ is *nonunitary*, since the commutator with its Hermitially conjugated partner $\hat{E}_{+} \equiv \hat{E}_{-}^{\dagger}$ is not equal to zero:

$$[\hat{E}_{-}, \hat{E}_{+}] = 1 - \hat{C}^2 - \hat{S}^2 = |0\rangle\langle 0|. \tag{39}$$

It is well known that the annihilation operator $\hat{a}$ has no inverse operator in the full meaning of this term. Nonetheless, it possesses the *right inverse operator*

$$\hat{a}^{-1} = \sum_{n=0}^{\infty} \frac{|n+1\rangle\langle n|}{\sqrt{n+1}} = \hat{a}^{\dagger}(\hat{a}\hat{a}^{\dagger})^{-1} = \hat{E}_{+}(\hat{a}\hat{a}^{\dagger})^{-1/2}, \tag{40}$$

which satisfies, among many others, the relations:

$$\hat{a}\hat{a}^{-1} = 1, \qquad [\hat{a}, \hat{a}^{-1}] = |0\rangle\langle 0|, \qquad [\hat{a}^{\dagger}, \hat{a}^{-1}] = \hat{a}^{-2}.$$

This operator was discussed briefly by Dirac [194], who noticed that Fock considered it long before. However, it has only found applications in quantum optics in the 1990s (see the paragraph on *photon-added states* later). Lerner [195] noticed that the commutation relations (38) do not determine the operators $\hat{C}, \hat{S}$ uniquely. Earlier, the same observation was made by Wigner [196] with respect to the triple $\{\hat{n}, \hat{a}, \hat{a}^{\dagger}\}$ (see the next section). In the general case, besides the 'polar decomposition' (37), which is equivalent to the relations

$$\hat{E}_{-}|n\rangle = (1 - \delta_{n0})|n-1\rangle, \qquad \hat{E}_{+}|n\rangle = |n+1\rangle, \tag{41}$$

one can define operator $\hat{U} = \hat{C} + \mathrm{i}\hat{S}$ via the relation $\hat{U}|n\rangle = f(n)|n-1\rangle$, where function $f(n)$ may be arbitrary enough, being restricted by the requirement $f(0) = 0$ and certain other constraints which ensure that the spectra of the 'cosine' and 'sine' operators belong to the interval $(-1, 1)$. The properties of Lerner's construction were studied in [197].

The states with the definite phase

$$|\varphi\rangle = \sum_{n=0}^{\infty} \exp(\mathrm{i}n\varphi)|n\rangle \qquad (42)$$

were considered in [29, 192]. However, they are not normalizable (like the coordinate or momentum eigenstates). The normalizable *coherent phase states* were introduced in [198] as the eigenstates of the operator $\hat{E}_-$:

$$|\varepsilon\rangle = \sqrt{1-\varepsilon\varepsilon^*}\sum_{n=0}^{\infty}\varepsilon^n|n\rangle, \qquad \hat{E}_-|\varepsilon\rangle = \varepsilon|\varepsilon\rangle, \quad |\varepsilon| < 1. \qquad (43)$$

The *pure* quantum state (43) has the same probability distribution $|\langle n|\varepsilon\rangle|^2$ as the mixed *thermal state* described by the statistical operator

$$\hat{\rho}_{th} = \frac{1}{1+\overline{n}}\sum_{n=0}^{\infty}\left(\frac{\overline{n}}{1+\overline{n}}\right)^n |n\rangle\langle n|, \qquad (44)$$

if one identifies the mean photon number $\overline{n}$ with $|\varepsilon|^2/(1-|\varepsilon|^2)$ [199].

The two-parameter set of states ($\kappa \neq -1$)

$$|z;\kappa\rangle = \mathcal{N}\sum_{n=0}^{\infty}\left[\frac{\Gamma(n+\kappa+1)}{n!\Gamma(\kappa+1)}\right]^{1/2}\left(\frac{z}{\sqrt{\kappa+1}}\right)^n|n\rangle \qquad (45)$$

has been introduced in [200] and studied in detail from different points of view in [180, 199, 201]. These states are eigenstates of the operator:

$$\hat{A}_\kappa = \hat{E}_-\left[\frac{(\kappa+1)\hat{n}}{\kappa+\hat{n}}\right]^{1/2} \equiv \left[\frac{(\kappa+1)}{\kappa+1+\hat{n}}\right]^{1/2}\hat{a}. \qquad (46)$$

If $\kappa = 0$, $\hat{A}_0 = \hat{E}_-$, and the state $|z;0\rangle$ coincides with (43). If $\kappa \to \infty$, then $\hat{A}_\infty = \hat{a}$, and the state (45) goes to the *coherent state* $|z\rangle$ (5). In the 1990s the state (45) appeared again under the name *negative binomial state* (see (67) in section 8.4). In the special case $\kappa = -p-1$, $p = 1, 2, \ldots$, the state (45) goes to the superposition of the first $p+1$ Fock states

$$|z;p\rangle = \mathcal{N}\sum_{n=0}^{p}\left[\frac{p!}{n!(p-n)!}\right]^{1/2}\left(\frac{z}{\sqrt{p}}\right)^n|n\rangle, \qquad (47)$$

which is nothing other than the *binomial state* introduced in 1985 (see section 8.4).

Sukumar [202] introduced the states ($\alpha = (r/m)\mathrm{e}^{\mathrm{i}\varphi}$, $\beta = \tanh r$, $m = 1, 2, \ldots$)

$$\exp(\alpha\hat{T}_m^\dagger - \alpha^*\hat{T}_m)|0\rangle = \sqrt{1-\beta^2}\sum_{n=0}^{\infty}(\beta\mathrm{e}^{\mathrm{i}\varphi})^n|mn\rangle,$$

where $\hat{T}_m = \hat{E}_-^m\hat{n} \equiv (\hat{n}+m)\hat{E}_-^m$, and operators $\hat{T}_\pm$ act on the Fock states as follows:

$$\hat{T}_m|n\rangle = \begin{cases} n|n-m\rangle, & n \geqslant m \\ 0, & n < m, \end{cases}$$

$$\hat{T}_m^\dagger|n\rangle = (n+m)|n+m\rangle.$$

For $m = 1$ one arrives again at the state (43).

The phase coherent state (43) was rediscovered in the beginning of the 1990s in [203] as a pure analogue of the thermal state. It was also noticed that this state yields a strong squeezing effect. By analogy with the usual squeezed states, which are eigenstates of the linear combination of the operators $\hat{a}, \hat{a}^\dagger$ (16), the *phase squeezed states* (PSS) were constructed in [204] as the eigenstates of the operator $\hat{B} = \mu\hat{E}_- + \nu\hat{E}_+$. The coefficients of the decomposition of PSS over the Fock basis are given by

$$c_n = \mathcal{N}(z_+^{n+1} - z_-^{n+1}), \qquad z_\pm = \left(\beta \pm \sqrt{\beta^2 - 4\mu\nu}\right)\Big/(2\mu),$$

where β is the complex eigenvalue of $\hat{B}$ and $\mathcal{N}$ is the normalization factor. It is worth noting, however, that PSS have actually been introduced as far back as 1974 by Mathews and Eswaran [205], who minimized the ratio $(\Delta C)^2(\Delta S)^2/|\langle[\hat{C},\hat{S}]\rangle|^2$, solving the equation ($v$ and λ are parameters):

$$[(1+v)\hat{E}_- + (1-v)\hat{E}_+]|\psi\rangle = \lambda|\psi\rangle. \qquad (48)$$

The continuous representation of arbitrary quantum states by means of the phase coherent states (43) (an analogue of the Klauder–Glauber–Sudarshan coherent state representation) was considered in [206] (where it was called the *analytic representation in the unit disk*), [207] (under the name *harmonious state representation*), and [208]. Eigenstates of operator $\hat{Z}(\sigma) = \hat{E}_-(\hat{n}+\sigma)$,

$$|z;\sigma\rangle = \sum_{n=0}^{\infty}\frac{z^n|n\rangle}{\Gamma(n+\sigma+1)}, \qquad \hat{Z}(\sigma)|z;\sigma\rangle = z|z;\sigma\rangle, \qquad (49)$$

were named as *philophase states* in [209]. If $\sigma = 0$, then (49) goes to the special case of the Barut–Girardello state (54) with $k = 1/2$.

The name 'pseudothermal state' was given to the state (43) in [210], where it was shown that this state arises naturally as an exact solution to certain nonlinear modifications of the Schrödinger equation. *Shifted thermal states*, which can be written as $\hat{\rho}_{th}^{(\mathrm{shift})} = \hat{E}_+\hat{\rho}_{th}\hat{E}_-$, have been considered in [211] (see also [212]).

For the most recent study on phase states see [213]. Comprehensive discussions of the problem of phase in quantum mechanics can be found, e.g., in [193, 214], and a detailed list of publications up to 1996 was given in the tutorial review [215].

6. Algebraic coherent states

6.1. Angular momentum and spin-coherent states

The first family of the coherent states for the angular momentum operators was constructed in [216], actually as some special two-dimensional oscillator coherent state, using the Schwinger representation of the angular momentum operators in terms of the auxiliary bosonic annihilation operators $\hat{a}_+$ and $\hat{a}_-$:

$$\hat{J}_+ = \hat{a}_+^\dagger\hat{a}_-, \qquad \hat{J}_- = \hat{a}_-^\dagger\hat{a}_+, \qquad \hat{J}_3 = \tfrac{1}{2}(\hat{a}_+^\dagger\hat{a}_+ - \hat{a}_-^\dagger\hat{a}_-). \qquad (50)$$

Such 'oscillator-like' angular momentum coherent states were studied in [217]. A possibility of constructing 'continuous' families of states using different modifications of the unitary displacement (6) and squeezing (21) operators was recognized in the beginning of the 1970s. The first explicit example is frequently related to the *coherent spin states* [218] (also named *atomic coherent states* [219] and *Bloch coherent states* [220])

$$|\vartheta, \varphi\rangle = \exp(\zeta \hat{J}_+ - \zeta^* \hat{J}_-)|j, -j\rangle \qquad \zeta = (\vartheta/2)\exp(-\mathrm{i}\varphi), \tag{51}$$

where $\hat{J}_\pm$ are the standard raising and lowering spin (angular momentum) operators, $|j, m\rangle$ are the standard eigenstates of the operators $\hat{\boldsymbol{J}}^2$ and $\hat{J}_z$, and ϑ, φ are the angles in the spherical coordinates. However, the special case of these states for spin-$\frac{1}{2}$ was considered much earlier by Klauder [16], who introduced the fermion coherent states $|\beta\rangle = \sqrt{1-|\beta|^2}\,|0\rangle + \beta\,|1\rangle$, where β could be an arbitrary complex number satisfying the inequality $|\beta| \leqslant 1$. Also, Klauder studied generic states of the form (51) in [221]. A detailed discussion of the *spin-coherent states* was given in [222], whereas *spin squeezed states* were discussed in [223]. For recent publications on the angular momentum coherent states see, e.g., [224].

6.2. Group coherent states

The operators $\hat{J}_\pm$, $\hat{J}_z$ are the generators of the algebra $su(2)$. A general construction looks like

$$|\xi_1, \xi_2, \ldots, \xi_n\rangle = \exp(\xi_1 \hat{A}_1 + \xi_2 \hat{A}_2 + \cdots + \xi_n \hat{A}_n)|\psi\rangle, \tag{52}$$

where $\{\xi_k\}$ is a continuous set of complex or real parameters, $\hat{A}_k$ are the generators of some algebra, and $|\psi\rangle$ is some 'basic' ('*fiducial*') state. This scheme was proposed by Klauder as far back as 1963 [221], and later it was rediscovered in [225]. A great amount of different families of 'generalized coherent states' can be obtained, choosing different algebras and basic states. One of the first examples was related to the $su(1,1)$ algebra [225]

$$|\zeta; k\rangle \sim \exp(\zeta \hat{K}_+)|0\rangle \sim \sum_{n=0}^{\infty} \left[\frac{\Gamma(n+2k)}{n!\Gamma(2k)}\right]^{1/2} \zeta^n |n\rangle, \tag{53}$$

where k is the so-called Bargmann index labelling the concrete representation of the algebra, and $\hat{K}_+$ is the corresponding rising operator. Evidently, the states (45) and (53) are the same, although their interpretation may be different. Some particular realizations of the state (53) connected with the problem of quantum 'singular oscillator' (described by the Hamiltonian $H = p^2 + x^2 + gx^{-2}$) were considered in [180, 201]. The 'generalized phase state' (45) and its special case (47) can also be considered as coherent states for the groups $O(2,1)$ or $O(3)$, with the 'displacement operator' of the form [199]:

$$\hat{D}_\kappa(z) = \exp\left(z\sqrt{\hat{n}(\hat{n}+\kappa)}\hat{E}_+ - z^*\hat{E}_-\sqrt{\hat{n}(\hat{n}+\kappa)}\right).$$

A comparison of the coherent states for the Heisenberg–Weyl and $su(2)$ algebras was made in [226] (see also [227]). Coherent states for the group $SU(n)$ were studied in [220,228], whereas the groups $E(n)$ and $SU(m,n)$ were considered in [229]. *Multilevel atomic coherent states* were introduced in [230].

The name *Barut–Girardello coherent states* is used in modern literature for the states which are eigenstates of some non-Hermitian *lowering operators*. The first example was given in [231], where the eigenstates of the lowering operator $\hat{K}_-$ of the $su(1,1)$ algebra were introduced in the form:

$$|z; k\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{z^n |n\rangle}{\sqrt{n!\Gamma(n+2k)}}. \tag{54}$$

The corresponding wavepackets in the coordinate representation, related to the problem of nonstationary 'singular oscillator', were expressed in terms of Bessel functions in [180].

The first two-dimensional analogues of the Barut–Girardello states, namely, eigenstates of the product of commuting boson annihilation operators $\hat{a}\hat{b}$,

$$\hat{a}\hat{b}|\xi, q\rangle = \xi|\xi, q\rangle, \qquad \hat{Q}|\xi, q\rangle = q|\xi, q\rangle, \qquad \hat{Q} = \hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}, \tag{55}$$

were introduced by Horn and Silver [232] in connection with the problem of pions production. In the simplified variant these states have the form (actually Horn and Silver considered the infinite-dimensional case related to the quantum field theory):

$$|\xi, q\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{\xi^n |n+q, n\rangle}{[n!(n+q)!]^{1/2}}. \tag{56}$$

Operator $\hat{Q}$ can be interpreted as the 'charge operator'; for this reason state (56) appeared in [233] under the name 'charged boson coherent state.' Joint eigenstates of the operators $\hat{a}_+\hat{a}_+$ and $\hat{a}_+\hat{a}_-$, which generate the angular momentum operators according to (50), were studied in [234]. The states (56) have found applications in quantum field theory [235]. Nonclassical properties of the states (56) (renamed as *pair coherent states*) were studied in [236]. An example of two-mode $SU(1,1)$ coherent states was given in [237]. The generalization of the 'charged boson' coherent states (56) in the form of the eigenstates of the operator $\mu\hat{a}\hat{b} + \nu\hat{a}^\dagger\hat{b}^\dagger$, satisfying the constraint $(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})|\psi\rangle = 0$, was studied in [238]. Nonclassical properties of the *even and odd charge coherent states* were studied in [239].

The first explicit treatments of the squeezed states as the $SU(1,1)$ *coherent states* were given in studies [140, 240], whose authors considered, in particular, the realization of the $su(1,1)$ algebra in terms of the operators:

$$\hat{K}_+ = \hat{a}^{\dagger 2}/2, \qquad \hat{K}_- = \hat{a}^2/2, \qquad \hat{K}_3 = \left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right)/4. \tag{57}$$

If $(\Delta K_1)^2 < |\langle\hat{K}_3\rangle|/2$ or $(\Delta K_2)^2 < |\langle\hat{K}_3\rangle|/2$ (where $\hat{K}_\pm = \hat{K}_1 \pm \mathrm{i}\hat{K}_2$), then the state was named $SU(1,1)$ squeezed [140] (in [144], similar states were named *amplitude-squared squeezed states*). The relations between the squeezed states and the Bogolyubov transformations were considered in [241]. 'Maximally symmetric' coherent states have been considered in [242]. $SU(2)$ and $SU(1,1)$ phase states have been constructed in [243]. The algebraic approaches in studying the squeezing phenomenon have been used in [244]. Analytic representations based on the $SU(1,1)$ group coherent states and Barut–Girardello states were compared in [245]. $SU(3)$-coherent states were considered in [246]. Coherent states defined *on a circle* and *on a sphere* have been studied in [247]. For other studies on group and algebraic states and their applications see, e.g., [7,248,249].

7. 'Minimum uncertainty' and 'intelligent' states

For the operators $\hat{A}$ and $\hat{B}$ different from $\hat{x}$ and $\hat{p}$, the right-hand side of the uncertainty relation (11) depends on the quantum state. The problem of finding the states for which (11) becomes the *equality* was discussed by Jackiw [50], who showed that it is reduced essentially to solving the equation:

$$(\hat{A} - \langle\hat{A}\rangle)|\psi\rangle = \lambda(\hat{B} - \langle\hat{B}\rangle)|\psi\rangle. \tag{58}$$

Jackiw found the explicit form of the states minimizing one of several 'phase–number' uncertainty relations, namely

$$(\Delta\hat{n})^2(\Delta\hat{C})^2/\langle\hat{S}\rangle^2 \geqslant 1/4\,, \tag{59}$$

in the form $|\psi\rangle = \mathcal{N}\sum_{n=0}^{\infty}(-\mathrm{i})^n I_{n-\lambda}(\gamma)|n\rangle$, where $I_\nu(\gamma)$ is the modified Bessel function, and λ, γ are some parameters.

Eswaran considered the $\hat{n}$–$\hat{C}$ pair of operators and solved the equation $(\hat{n}+\mathrm{i}\gamma\hat{C})|\psi\rangle = \lambda|\psi\rangle$ [197]. If the product $\Delta n\Delta\varphi$ of the number and phase 'uncertainties' remains of the order of 1, then we have the *number–phase minimum uncertainty state* [250]. The methods of generating the MUS for the operators $\hat{N}$, $\hat{C}$, $\hat{S}$ (37) were discussed in [251]. For recent studies see, e.g., [252].

Multimode generalizations of the (Gaussian) MUS were discussed in [253], and their detailed study was given in [254]. *Noise minimum states*, which give the minimal value of the photon number operator fluctuation σ_n for the fixed values of the lower order moments, e.g., $\langle\hat{a}\rangle$, $\langle\hat{a}^2\rangle$, $\langle\hat{n}\rangle$, were considered in [255]. This study was continued in [256], where eigenstates of operator $\hat{a}^\dagger\hat{a} - \xi^*\hat{a} - \xi\hat{a}^\dagger$ were found in the form

$$|\xi; M\rangle = (\hat{a}^\dagger + \xi^*)^M|\xi\rangle_{\mathrm{coh}} \tag{60}$$

(these states differ from (31), due to the opposite sign of the term ξ^*). Different families of MUS related to the powers of the bosonic operators were considered in [257]. The eigenstates of the most general linear combination of the operators $\hat{a}$, $\hat{a}^\dagger$, $\hat{a}^2$, $\hat{a}^{\dagger 2}$, and $\hat{a}^\dagger\hat{a}$ were studied in [182], where the general concept of *algebra eigenstates* was introduced. These states were defined as eigenstates of the linear combination $\xi_1\hat{A}_1 + \xi_2\hat{A}_2 + \cdots + \xi_n\hat{A}_n$, where $\hat{A}_k$ are generators of some algebra. In the case of *two-photon* algebra these states are expressed in terms of the confluent hypergeometric function or the Bessel function, and they contain, as special cases, many other families of nonclassical states [258].

The case of the spin (angular momentum) operators was considered in [259], where the name *intelligent states* was introduced. The relations between coherent spin states, intelligent spin states, and minimum-uncertainty spin states were discussed in [260]. For recent publications see, e.g., [261]. Nowadays the 'intelligent' states are understood to be the states for which the Heisenberg uncertainty relation (11) becomes the equality, whereas the MUS are those for which the 'uncertainty product' $\Delta A\Delta B$ attains the minimal possible value (for arbitrary operators $\hat{A}$, $\hat{B}$ such states may not exist [50]). The relations between squeezing and 'intelligence' were discussed, e.g., in papers [104, 258, 262]. The properties and applications of the $SU(1,1)$ and $SU(2)$ intelligent states were considered in [263]. The intelligent states for the generators $\hat{K}_{1,2}$ of the $su(1,1)$ algebra were named *Hermite polynomial states* in [264], since they have the form $\hat{S}(z)H_m(\xi\hat{a}^\dagger)|0\rangle$, thus being finite superpositions of the *squeezed number states*. Such states were studied in [265]. The *minimum uncertainty state* for *sum squeezing* in the form $\hat{S}(\xi)H_{pq}(\mu\hat{a}_1^\dagger, \mu\hat{a}_2^\dagger)|00\rangle$ was found in [266]. Here H_{pq} is a special case of the family of *two-dimensional Hermite polynomials*, which are useful for many problems of quantum optics [67, 120, 267].

8. Non-Gaussian and 'coherent' states for nonoscillator systems

There exist different constructions of 'coherent states' for a particle moving in an arbitrary potential. MUS whose time evolution is as close as possible to the trajectory of a *classical* particle have been studied by Nieto and Simmons in a series of papers, beginning with [268, 269]. In [270] 'coherent' states were defined as eigenstates of operators like $\hat{A} = f(x) + \mathrm{i}\sigma(x)\mathrm{d}/\mathrm{d}x$. However, such packets do not preserve their forms in the process of evolution, losing the important property of the Schrödinger nonspreading wavepackets.

At least three anharmonic potentials are of special interest in quantum mechanics. The closest to the harmonic oscillator is the 'singular oscillator' potential $x^2 + gx^{-2}$ (also known as the 'isotonic,' 'pseudoharmonical,' 'centrifugal' oscillator, or 'oscillator with centripetal barrier'). Different coherent states for this potential have been constructed in [180, 201, 269, 271–273].

MUS for the Morse potential $U_0(1 - \mathrm{e}^{-ax})^2$ were constructed in [274]; the cases of the Pöschl–Teller and Rosen–Morse potentials, $U_0\tan^2(ax)$ and $U_0\tanh^2(ax)$, were considered in [269]. Algebraic coherent states for these potentials, based, in particular, on the algebras $su(1,1)$ or $so(2,1)$, have been proposed in [275], for recent constructions see, e.g., [272, 276]. Coherent states for the *reflectionless potentials* were constructed in [277]. The intelligent states for arbitrary potentials, with concrete applications to the Pöschl–Teller one, were considered recently in [278].

Klauder [279] has proposed a general construction of coherent states in the form

$$|z;\gamma\rangle = \sum_n \frac{z^n}{\sqrt{\rho_n}}\exp(-\mathrm{i}e_n\gamma)|n\rangle, \qquad \hat{H}|n\rangle = e_n|n\rangle, \tag{61}$$

where positive coefficients $\{\rho_n\}$ satisfy certain conditions, while the discrete energy spectrum $\{e_n\}$ may be quite arbitrary. This construction was applied to the hydrogen atom in [280]. Another special case of states (61) is the *Mittag–Leffler coherent state* [281]:

$$|z;\alpha,\beta\rangle = \mathcal{N}\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{\Gamma(\alpha n+\beta)}}|n\rangle. \tag{62}$$

Penson and Solomon [282] have introduced the state $|q,z\rangle = \varepsilon(q, z\hat{a}^\dagger)|0\rangle$, where function $\varepsilon(q,z)$ is a generalization of the exponential function given by the relations:

$$\mathrm{d}\varepsilon(q,z)/\mathrm{d}z = \varepsilon(q,qz), \qquad \varepsilon(q,z) = \sum_{n=0}^{\infty} q^{n(n-1)/2}z^n/n!.$$

The Gaussian exponential form of the coefficients ρ_n in (61) was used in [283] to construct localized wavepackets for the Coulomb problem, the planar rotor and the particle in a box. For the most recent studies see, e.g., [278, 284].

8.1. Coherent states and packets in the hydrogen atom

Introducing the nonspreading Gaussian wavepackets for the harmonic oscillator, Schrödinger wrote in the same paper [1][2]

'We can definitely foresee that, in a similar way, wave groups can be constructed which would round highly quantized Kepler ellipses and are the representation by wave mechanics of the hydrogen electron. But the technical difficulties in the calculation are greater than in the especially simple case which we have treated here.'

Indeed, this problem turned out to be much more complicated than the oscillator one. One of the first attempts to construct the quasiclassical packets moving along the Kepler orbits was made by Brown [285] in 1973. A similar approach was used in [286]. Different nonspreading and squeezed Rydberg packets were considered in [287].

The symmetry explaining degeneracy of hydrogen energy levels was found by Fock [288] (see also Bargmann [289]): it is $O(4)$ for the discrete spectrum and the Lorentz group (or $O(3,1)$) for the continuous one. The symmetry combining all the discrete levels into one irreducible representation (the dynamical group $O(4,2)$) was found in [290] (see also [291]). Different group related coherent states connected to these dynamical symmetries were discussed in [292]. The Kustaanheimo–Stiefel transformation [293], which reduces the three-dimensional Coulomb problem to the four-dimensional constrained harmonic oscillator [294], was applied to obtain various coherent states of the hydrogen atom in [295]. For the most recent publications see, e.g., [272, 296].

8.2. Relativistic oscillator coherent states

One of the first papers on the relativistic equations with internal degrees of freedom was published by Ginzburg and Tamm [297]. The model of 'covariant relativistic oscillator' obeying the modified Dirac equation

$$(\gamma_\mu \partial/\partial x_\mu + a[\xi_\mu\xi_\mu - \partial^2/\partial\xi_\mu\partial\xi_\mu] + m_0)\psi(x,\xi) = 0, \qquad (63)$$

where x is the 4-vector of the 'centre of mass', and 4-vector ξ is responsible for the 'internal' degrees of freedom of the 'extended' particle, was studied by many authors [298]. Coherent states for this model, related to the representations of the $SU(1,1)$ group and the 'singular oscillator' coherent states of [180], have been constructed in [299]. Coherent states of relativistic particles obeying the standard Dirac or Klein–Gordon equations were discussed in [300].

Different families of coherent states for several new models of the relativistic oscillator, different from the Yukawa–Markov type (63), have been studied during the 1990s. Mir-Kasimov [301] constructed *intelligent* states (in terms of the Macdonald function $K_\nu(z)$) for the coordinate and momentum operators obeying the 'deformed relativistic uncertainty relation':

$$[\hat{x}, \hat{p}] = \mathrm{i}\hbar \cosh[(\mathrm{i}\hbar/2mc)\,\mathrm{d}/\mathrm{d}x].$$

[2] Translation given in: Schrödinger E 1978 *Collected Papers on Wave Mechanics* (New York: Chelsea) pp 41–4.

Coherent states for another model, described by the equation

$$\mathrm{i}\partial\psi/\partial t = \left[\alpha_k\left(\hat{p}_k - \mathrm{i}m\omega\hat{x}_k\beta\right) + m\beta\right]\psi \qquad (64)$$

and named 'Dirac oscillator' in [302] (although similar equations were considered earlier, e.g., in [303]), have been studied in [304]. Aldaya and Guerrero [305] introduced the coherent states based on the modified 'relativistic' commutation relations:

$$[\hat{E}, \hat{x}] = -\mathrm{i}\hbar\hat{p}/m, \qquad [\hat{E}, \hat{p}] = \mathrm{i}m\omega^2\hbar\hat{x},$$
$$[\hat{x}, \hat{p}] = \mathrm{i}\hbar(1 + \hat{E}/mc^2).$$

These states were studied in [306].

8.3. Supersymmetric states

The concept of supersymmetry was introduced by Gol'fand and Likhtman [307]. The nonrelativistic supersymmetric quantum mechanics was proposed by Witten [308] and studied in [309]. Its super-simplified model can be described in terms of the Hamiltonian which is a sum of the free oscillator and spin parts, so that the lowering operator can be conceived as a matrix

$$\hat{H} = \hat{a}^\dagger\hat{a} - \tfrac{1}{2}\sigma_3, \qquad \hat{A} = \begin{Vmatrix} \hat{a} & 1 \\ 0 & \hat{a} \end{Vmatrix}$$

(σ_3 is the Pauli matrix). Then one can try to construct various families of states applying the operators like $\exp(\alpha\hat{A}^\dagger)$ to the ground (or another) state, looking for eigenstates of $\hat{A}$ or some functions of this operator, and so on. The first scheme was applied by Bars and Günaydin [310], who constructed group *supercoherent states*. The same (displacement operator) approach was used in [311]. The second way was chosen by Aragone and Zypman [312], who constructed the eigenstates of some 'supersymmetric' non-Hermitian operators.

Different coherent states for the Hamiltonians obtained from the harmonic oscillator Hamiltonian through various deformations of the potential (by the Darboux transformation, for example) have been studied in [313]. *'Supercoherent'* and *'supersqueezed'* states were studied in [314].

8.4. Binomial states

The finite combinations of the first $M+1$ Fock states in the form [315, 316]

$$|p, M; \theta_n\rangle = \sum_{n=0}^{M} \mathrm{e}^{\mathrm{i}\theta_n}\left[\frac{M!}{n!\,(M-n)!}\,p^n(1-p)^{M-n}\right]^{1/2} |n\rangle \qquad (65)$$

were named 'binomial states' in [315] (see also [317]). As a matter of fact, they were studied much earlier in the paper [199]: see equation (47). Binomial states go to the coherent states in the limit $p \to 0$ and $M \to \infty$, provided $pM = \text{const}$, so they are representatives of a wider class of intermediate, or interpolating, states. The special case of $M = 1$ (i.e., combinations of the ground and the first excited states) was named *Bernoulli states* in [315]. Other 'intermediate' states are (they are superpositions of an *infinite* number of Fock states) *logarithmic states* [318]

$$|\psi\rangle_{\log} = c|0\rangle + \mathcal{N}\sum_{n=1}^{\infty}\frac{z^n}{\sqrt{n}}\,|n\rangle \qquad (66)$$

and *negative binomial states* [319]

$$|\xi,\mu;\theta_n\rangle = \sum_{n=0}^{\infty} e^{i\theta_n}\left[(1-\xi)^{\mu}\frac{\Gamma(\mu+n)}{\Gamma(\mu)n!}\xi^n\right]^{1/2}|n\rangle, \quad (67)$$

where $\mu > 0$, $0 \leqslant \xi < 1$. One can check that for $\theta_n = n\theta_0$ the state (67) coincides with the generalized phase state (45) introduced in [199,200]. Mixed quantum states with *negative binomial distributions* of the diagonal elements of the density matrix in the Fock basis appeared in the theory of photodetectors and amplifiers in [320].

Reciprocal binomial states

$$|\Phi;M\rangle = \mathcal{N}\sum_{n=0}^{M} e^{in\phi}\sqrt{n!(M-n)!}\,|n\rangle \quad (68)$$

were introduced in [321], and schemes of their generation were discussed in [322]. *Intermediate number squeezed states* $\hat{S}(z)|\eta,M\rangle$ (where $|\eta,M\rangle$ is the binomial state) were introduced in [323] and generalized in [324]. *Multinomial states* have been introduced in [325]. Barnett [326] has introduced the modification of the *negative binomial states* of the form:

$$|\eta,-(M+1)\rangle \sim \sum_{n=M}^{\infty}\left[\frac{n!}{(n-M)!}\eta^{M+1}(1-\eta)^{n-M}\right]^{1/2}|n\rangle.$$

The *binomial coherent states*

$$|\lambda;M\rangle \sim (\hat{a}^{\dagger}+\lambda)^M|0\rangle \sim \sum_{n=0}^{M}\frac{\lambda^{M-n}|n\rangle}{(M-n)!\sqrt{n!}}$$
$$\sim \hat{D}(-\lambda)\hat{a}^{\dagger M}\hat{D}(\lambda)|0\rangle \quad (69)$$

(here λ is real and $\hat{D}(\lambda)$ is the displacement operator) were studied in [327,328]. State (69) is the eigenstate of operator $\hat{a}^{\dagger}\hat{a}+\lambda\hat{a}$ with the eigenvalue M. The superposition states $|\lambda;M\rangle + \exp(i\varphi)|-\lambda;M\rangle$ were studied in [329]. Replacing operator $\hat{a}^{\dagger}$ in the right-hand side of (69) by the spin raising or lowering operators $\hat{S}_{\pm}$ one arrives at the *binomial spin-coherent state* [328]. For other studies on binomial, negative binomial states and their generalizations see, e.g., [206,330].

8.5. Kerr states and 'macroscopic superpositions'

The main suppliers of nonclassical states are the media with *nonlinear* optical characteristics. One of the simplest examples is the so-called *Kerr nonlinearity*, which can be modelled in the single-mode case by means of the Hamiltonian $\hat{H} = \omega\hat{n} + \chi\hat{n}^2$ (where $\hat{n} = \hat{a}^{\dagger}\hat{a}$). In 1984, Tanaš [331] demonstrated a possibility of obtaining squeezing using this kind of nonlinearity. In 1986, considering the time evolution of the initial coherent state under the action of the 'Kerr Hamiltonian', Kitagawa and Yamamoto [250] showed that the states whose initial shapes in the complex phase plane α were circles (these shapes are determined by the equation $Q(\alpha) = \text{const}$, where the *Q-function* is defined as $Q(\alpha) = \langle\alpha|\hat{\rho}|\alpha\rangle$), are transformed to some 'crescent states', with essentially reduced fluctuations of the number of photons: $\Delta n \sim \langle n\rangle^{1/6}$ (whereas in the case of the squeezed states one has $\Delta n \geqslant \langle n\rangle^{1/3}$).

The behaviour of the Q-function was also studied by Milburn [332], who (besides confirming the squeezing effect) discovered that under certain conditions, the initial single Gaussian function is split into several well-separated Gaussian peaks. Yurke and Stoler [333] gave the analytical treatment to this problem, generalizing the nonlinear term in the Hamiltonian as $\hat{n}^k$ (k being an integer). The initial coherent state is transformed under the action of the Kerr Hamiltonian to the Titulaer–Glauber state (32)

$$|\alpha;t\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha_t^n}{\sqrt{n!}}\exp(-i\chi n^k t)|n\rangle, \quad (70)$$

where $\alpha_t = \alpha\exp(-i\omega t)$. Yurke and Stoler noticed that for the special value of time $t_* = \pi/2\chi$, state (70) becomes the superposition of two or four coherent states, depending on the parity of the exponent k:

$$|\alpha;t_*\rangle = \begin{cases} [e^{-i\pi/4}|\alpha_t\rangle + e^{i\pi/4}|-\alpha_t\rangle]/\sqrt{2}, & k \text{ even} \\ [|\alpha_t\rangle - |i\alpha_t\rangle + |-\alpha_t\rangle + |-i\alpha_t\rangle]/2, & k \text{ odd.} \end{cases} \quad (71)$$

Notice that the first superposition (for k even) is different from the even or odd states (36). The even and odd states arise in the case of the two-mode nonlinear interaction $\hat{H} = \omega(\hat{a}^{\dagger}\hat{a} + \hat{b}^{\dagger}\hat{b}) + \chi(\hat{a}^{\dagger}\hat{b} + \hat{b}^{\dagger}\hat{a})^2$ considered by Mecozzi and Tombesi [334]. In this case, the initial state $|0\rangle_a|\beta\rangle_b$ is transformed at the moment $t_* = \pi/4\chi$ to the superposition:

$$|\text{out},t_*\rangle = \tfrac{1}{2}e^{-i\pi/4}(|\beta_t\rangle_b - |-\beta_t\rangle_b)|0\rangle_a$$
$$+\tfrac{1}{2}|0\rangle_b(|-i\beta_t\rangle_a + |i\beta_t\rangle_a).$$

In the course of time, such 'macroscopic superpositions' of quantum states attracted great attention, being considered as simple models of the 'Schrödinger cat states' [335]. In particular, they were studied in detail in [336]. Other superpositions of quantum states of the electromagnetic field, which can be created in a cavity due to the interaction with a beam of two-level atoms passing one after another, were considered in [337]. Some of them, described in terms of specific solutions of the Jaynes–Cummings model [338], were named *tangent and cotangent states* in [339]. The *squeezed Kerr states* were analysed in [340].

Searching for the states giving maximal squeezing, various superpositions of states were considered. In particular, the superpositions of the Fock states $|0\rangle$ and $|1\rangle$ (*Bernoulli states*) were studied in [315, 341], similar superpositions (plus state $|2\rangle$) were studied in [342]. The superposition of two squeezed vacuum states was considered in [343]. The superpositions of the coherent states $|\alpha e^{i\varphi}\rangle$ and $|\alpha e^{-i\varphi}\rangle$ were considered in [344]. *Entangled coherent states* have been introduced by Sanders in [345]. Their generalizations and physical applications have been studied in [346]. Various superpositions of coherent, squeezed, Fock, and other states, as well as methods of their generation, were considered in [347].

Representations of nonclassical states (including quadrature squeezed and amplitude squeezed) via linear integrals over some curve $\mathcal{C}$ (closed or open, infinite or finite) in the complex plane of parameters,

$$|g\rangle = \int_{\mathcal{C}} g(z)|z\rangle\,dz, \quad (72)$$

were studied in [348]. Originally, $|z\rangle$ was the coherent state, but it is clear that one may choose other families of states, as well.

8.6. Photon-added states

An interesting class of nonclassical states consists of the *photon-added* states

$$|\psi, m\rangle_{\rm add} = \mathcal{N}_m \hat{a}^{\dagger m}|\psi\rangle, \tag{73}$$

where $|\psi\rangle$ may be an arbitrary quantum state, m is a positive integer—the number of added quanta (photons or phonons), and $\mathcal{N}_m$ is a normalization constant (which depends on the basic state $|\psi\rangle$). Agarwal and Tara [349] introduced these states for the first time as the *photon-added coherent states* (PACS) $|\alpha, m\rangle$, identifying $|\psi\rangle$ with Glauber's coherent state $|\alpha\rangle$ (5). These states differ from the displaced number state $|n, \alpha\rangle$ (31). Taking the initial state $|\psi\rangle$ in the form of a squeezed state, one obtains *photon-added squeezed states* [350]. The *even/odd photon-added states* were studied in [351]. One can easily generalize the definition (73) to the case of mixed quantum states described in terms of the statistical operator $\hat{\rho}$: the statistical operator of the mixed photon-added state has the form (up to the normalization factor) $\hat{\rho}_m = \hat{a}^{\dagger m}\hat{\rho}\hat{a}^m$. A concrete example is the *photon-added thermal state* [352]. A distinguishing feature of all photon-added states is that the probability of detecting n quanta (photons) in these states equals exactly zero for $n < m$. These states arise in a natural way in the processes of the field–atom interaction in a cavity [349]. Replacing the creation operator $\hat{a}^\dagger$ in the definition (73) by the annihilation operator $\hat{a}$ one arrives at the *photon-subtracted state* $|\psi, m\rangle_{\rm subt} = \mathcal{N}_m\hat{a}^m|\psi\rangle$. The methods of creating photon-added/subtracted states via conditional measurements on a beamsplitter were discussed in [353,354].

Photon-added states are closely related to the *boson inverse operator* (40), whose properties were studied in [355]. It was shown in [356] that PACS $|\alpha, m\rangle_{\rm add}$ is an eigenstate of the operator $\hat{a} - m\hat{a}^{\dagger -1}$ with eigenvalue α. The *photon-depleted coherent states* $|\alpha, m\rangle_{\rm depl} = \mathcal{N}_m\hat{a}^{\dagger -m}|\alpha\rangle$ have been introduced in [356]. For the most recent studies on photon-added states see [357].

8.7. Multiphoton and 'circular' coherent and squeezed states

Multiphoton states, defined as eigenstates of operator $\hat{a}^k$, were studied in [358, 359]. The case of $k = 3$ was considered in [360]. *Four-photon states* ($k = 4$) [359, 361] can be represented as superpositions of two even/odd coherent states (36) $|\alpha\rangle_\pm$ and $|{\rm i}\alpha\rangle_\pm$ whose labels are rotated by 90° in the parameter complex plane. For this reason, the state $|\alpha\rangle_+ + |{\rm i}\alpha\rangle_+$ was called the *orthogonal-even state* in [362]. General schemes of constructing multiphoton coherent and squeezed states of arbitrary orders were given in [363].

There exist k orthogonal multiphoton states of the form

$$|\alpha; k, j\rangle = \mathcal{N}\sum_{n=0}^{\infty}\frac{\alpha^{nk+j}}{\sqrt{(nk+j)!}}|nk+j\rangle, \qquad j = 0, 1, \ldots, k-1, \tag{74}$$

which can also be expressed as the superpositions of k coherent states uniformly distributed along the circle:

$$|\alpha; k, j\rangle = \mathcal{N}\sum_{n=0}^{k-1}\exp(-2\pi{\rm i}nj/k)\,|\alpha\exp(2\pi{\rm i}n/k)\rangle. \tag{75}$$

Such orthogonal 'circular' states were studied in [364]. The difference between the Bialynicki-Birula states (33) and states (75) is in the 'weights' of each coherent state in the superposition. In the case of (33) these weights are different, to ensure the *Poissonian* photon statistics, whereas in the 'circular' case all the coefficients have the same absolute value, resulting in non-Poissonian statistics. Eigenstates of the operator $(\mu\hat{a} + \nu\hat{a}^\dagger)^k$ were considered in [365] (with the emphasis on the case $k = 2$). Eigenstates of products of annihilation operators of different modes, $\hat{a}^k\hat{b}^l\cdots\hat{c}^m|\eta\rangle = \eta|\eta\rangle$, have been found in [366] in the form of finite superpositions of products of coherent states. For example, in the two-mode case one has (M is an integer):

$$|\eta\rangle = \mathcal{N}\sum_{n=0}^{Mkl-1} c_j\left|\alpha\exp\left[2\pi{\rm i}\,\frac{n(M+1)}{kM}\right]\right\rangle \otimes\left|\beta\exp\left(-2\pi{\rm i}\,\frac{n}{lM}\right)\right\rangle, \qquad \eta = \alpha^k\beta^l.$$

'Crystallized' Schrödinger cat states have been introduced in [367]. For the most recent studies on the 'multiphoton' or 'circular' states and their generalizations see [149,368].

8.8. 'Intermediate' and 'polynomial' states

After the papers by Pegg and Barnett [369], where the finite-dimensional 'truncated' Hilbert space was used to define the phase operator, various 'truncated' versions of nonclassical states have been studied. Properties of the state $\sum_{n=0}^M (1+n)^{-1}|n\rangle$ were discussed in [204,370]. *Quasiphoton phase states* $\sum_{n=0}^s \exp({\rm i}n\vartheta)|n\rangle_g$, where $|n\rangle_g$ is the squeezed number state, were considered in [371]. The 'finite-dimensional' and 'discrete' coherent states were considered in [372]. The *generalized geometric state* $|y; M\rangle = \sum_{n=0}^M y^{n/2}|n\rangle$ was introduced in [373], and its *even* variant in [374]. In the limit $M \to \infty$ this state goes to the *phase coherent states* (43) (called *geometric states* in [373, 374], due to the geometric progression form of the coefficients in the Fock basis). For other examples see [375].

The *Laguerre polynomial state* $L_M(-y\hat{R}^\dagger)|0\rangle$, where $\hat{R}^\dagger = \hat{a}^\dagger\sqrt{\hat{N}+1}$, was introduced in [376]. A more general definition $L_M(\xi\hat{J}_+)|k, 0\rangle$ (where $\hat{J}_+$ is one of the generators of the $su(1, 1)$ algebra, and $|k, n\rangle$ is the discrete basis state labelled by the representation index k) has been given in [377]. The properties of these states were studied in [378].

Jacobi polynomial states were introduced in [354]. Several modifications of the binomial distributions have been used to define *Pólya states* [379], *hypergeometric states* [380], and so on. For example, the states introduced in [381]

$$|N, \alpha, \beta\rangle \sim \sum_{n=0}^{N}\left[\frac{(\alpha+1)_n(\beta+1)_{N-n}}{n!(N-n)!}\right]^{1/2}|n\rangle$$

are related to the *Hahn polynomials*. They contain, as special or limit cases, usual binomial and negative binomial states.

9. 'Quantum deformations' and related states

9.1. Para-coherent states

In 1951, Wigner [196] pointed out that to obtain an equidistant spectrum of the harmonic oscillator, one could use, instead of the canonical bosonic commutation relation (3), more weak conditions:

$$[\hat{a}_\epsilon, \hat{H}] = \hat{a}_\epsilon, \qquad [\hat{a}_\epsilon^\dagger, \hat{H}] = -\hat{a}_\epsilon^\dagger, \\ \hat{H} = \left(\hat{a}_\epsilon^\dagger \hat{a}_\epsilon + \hat{a}_\epsilon \hat{a}_\epsilon^\dagger\right)/2. \tag{76}$$

Then the spectrum of the oscillator becomes (in the dimensionless units) $E_n = n + \epsilon$, $n = 0, 1, \ldots$, with an arbitrary possible lowest energy ϵ (for the usual oscillator $\epsilon = 1/2$). Wigner's observation is closely related to the problem of *parastatistics* (intermediate between the Bose and Fermi ones) [382], which was studied by many authors in the 1950s and 1960s; see [383, 384] and references therein. In 1978, Sharma *et al* [385] introduced *para-Bose coherent states* as the eigenstates of the operator $\hat{a}_\epsilon$ satisfying the relations (76)

$$|\alpha\rangle_\epsilon \sim \sum_{n=0}^{\infty} \left\{ \Gamma\left(\left[\left[\frac{n}{2}\right]\right] + 1\right) \Gamma\left(\left[\left[\frac{n+1}{2}\right]\right] + \epsilon\right) \right\}^{-1/2} \\ \times \left(\frac{\alpha}{\sqrt{2}}\right)^n |n\rangle_\epsilon, \tag{77}$$

where $|n\rangle_\epsilon \sim \hat{a}_\epsilon^{\dagger n}|0\rangle_\epsilon$, $|0\rangle_\epsilon$ is the ground state, and $[[u]]$ means the greatest integer less than or equal to u. Introducing the Hermitian 'quadrature' operators in the same manner as in (19) (but with a different physical meaning), one can check that $\Delta x_\epsilon \Delta p_\epsilon = |\langle[\hat{x}_\epsilon, \hat{p}_\epsilon]\rangle|/2$ in the state (77), i.e., this state is 'intelligent' for the operators $\hat{x}_\epsilon$ and $\hat{p}_\epsilon$ [386].

Another kind of 'para-Bose operator' was considered in [387] on the basis of a nonlinear transformation of the canonical bosonic operators:

$$\hat{A} = F^*(\hat{n}+1)\hat{a}, \qquad \hat{A}^\dagger = \hat{a}^\dagger F(\hat{n}+1), \qquad \hat{n} = \hat{a}^\dagger \hat{a}. \tag{78}$$

The transformed operators satisfy the relations:

$$[\hat{A}, \hat{n}] = \hat{A}, \qquad [\hat{A}^\dagger, \hat{n}] = -\hat{A}^\dagger, \qquad \hat{n} = \hat{a}^\dagger \hat{a}. \tag{79}$$

Defining the 'para-coherent state' $|\lambda\rangle$ as an eigenstate of the operator $\hat{A}$, $\hat{A}|\lambda\rangle = \lambda|\lambda\rangle$, one obtains:

$$|\lambda\rangle = \mathcal{N}\left(|0\rangle + \sum_{n=1}^{\infty} \frac{\lambda^n |n\rangle}{\sqrt{n!}F^*(1)\cdots F^*(n)}\right). \tag{80}$$

Evidently, the choice $F(n) = \sqrt{n+2k-1}$ reduces the state (80) to the Barut–Girardello state (54). Nowadays the states (80) are known mainly under the name *nonlinear coherent states* (NLCS): see section 9.4. Varios *'para-states'* were considered in [388–390]. In particular, the generalized 'para-commutator relations'

$$[\hat{n}, \hat{a}^\dagger] = \hat{a}^\dagger, \qquad [\hat{n}, \hat{a}] = -\hat{a}, \\ \hat{a}^\dagger \hat{a} = \phi(\hat{n}), \qquad \hat{a}\hat{a}^\dagger = \phi(\hat{n}+1)$$

have been resolved in [389] by means of the nonlinear transformation to the usual bosonic operators $\hat{b}$, $\hat{b}^\dagger$

$$\hat{a} = \sqrt{\frac{\phi(\hat{n}+1)}{\hat{n}+1}}\hat{b}, \qquad \hat{A} = \frac{\hat{n}+1}{\phi(\hat{n}+1)}\hat{a}, \qquad [\hat{A}, \hat{a}^\dagger] = 1, \tag{81}$$

and coherent states were defined as $|\alpha\rangle \sim \exp(\alpha\hat{A}^\dagger)|0\rangle$.

9.2. *q-coherent states*

One of the most popular directions in mathematical physics of the last decades of the 20th century was related to various *deformations* of the canonical commutation relations (3) (and others). Perhaps, the first study was performed in 1951 by Iwata [391], who found eigenstates of the operator $\hat{a}_q^\dagger \hat{a}_q$, assuming that $\hat{a}_q$ and $\hat{a}_q^\dagger$ satisfy the relation (he used letter ρ instead of q):

$$\hat{a}_q \hat{a}_q^\dagger - q\,\hat{a}_q^\dagger \hat{a}_q = 1, \qquad q = \text{const}. \tag{82}$$

Twenty five years later, the same relation (82) and its generalizations to the case of several dimensions were considered by Arik and Coon [392, 393], Kuryshkin [394], and by Jannussis *et al* [395]. A realization of the commutation relation (82) in terms of the usual bosonic operators $\hat{a}$, $\hat{a}^\dagger$ by means of the *nonlinear* transformation was found in [395]:

$$\hat{a}_q = F(\hat{n})\hat{a}, \qquad \hat{n} = \hat{a}^\dagger \hat{a}. \tag{83}$$

For *real* functions $F(n)$ equation (82) is equivalent to the recurrence relation $(n+1)[F(n)]^2 - qn[F(n-1)]^2 = 1$, whose solution is

$$F(n) = \left\{[n+1]_q/(n+1)\right\}^{1/2}, \tag{84}$$

where symbol $[n]_q$ means:

$$[n]_q = (q^n - 1)/(q-1) \equiv 1 + q + \cdots + q^{n-1}. \tag{85}$$

The operators given by (83) obey the relations (79). Using the transformation (83) one can obtain the realizations of more general relations than (82):

$$\hat{A}\hat{A}^\dagger = 1 + \sum_{k=1}^{K} q_k \hat{A}^{\dagger k}\hat{A}^k.$$

The corresponding recurrence relations for $F(n)$ were given in [395].

Coherent states of the *pseudo-oscillator*, defined by the 'inverse' commutation relation $[\hat{a}, \hat{a}^\dagger] = -1$, were studied in [396]. These states satisfy relations (5) and (4), but in the right-hand side of (5) one should write $-\alpha$ instead of α.

The *q-coherent state* was introduced in [393, 395] as

$$|\alpha\rangle_q = \exp_q(-|\alpha|^2/2)\exp_q(\alpha\hat{a}_q^\dagger)|0\rangle_q, \tag{86}$$

where:

$$\exp_q(x) \equiv \sum_{n=0}^{\infty} \frac{x^n}{[n]_q!}, \qquad [n]_q! \equiv [n]_q[n-1]_q \cdots [1]_q.$$

Assuming other definitions of the symbol $[n]_q$, but the same equations (83) and (84), one can construct other 'deformations' of the canonical commutation relations. For example, making the choice

$$[x]_q \equiv (q^x - q^{-x})/(q - q^{-1}) \tag{87}$$

one arrives at the relation

$$\hat{a}_q \hat{a}_q^\dagger - q\,\hat{a}_q^\dagger \hat{a}_q = q^{-\hat{n}} \tag{88}$$

considered by Biedenharn in 1989 in his famous paper [397], which gave rise (together with several other publications [398]) to the 'boom' in the field of 'q-deformed' states in quantum optics.

Squeezing properties of *q-coherent states* and different types of *q-squeezed states* were studied in [399, 400]. For example, in [399] the q-squeezed state was defined as a solution to equation $\left(\hat{a}_q - \alpha\hat{a}_q^\dagger\right)|\alpha\rangle_{q-sq} = 0$ (cf (16) and (17)). It has the following structure:

$$|\alpha\rangle_{q-sq} = \mathcal{N}\sum_{n=0}^{\infty}\alpha^n\left(\frac{[2n-1]_q!!}{[2n]_q!!}\right)^{1/2}|2n\rangle. \qquad (89)$$

Two families of coherent states for the *difference analogue* of the harmonic oscillator have been studied in [401], and one of them coincided with the q-coherent states. The 'quasicoherent' state $\exp_q(z\hat{a}_q^\dagger)\exp_q(z^*\hat{a}_q^\dagger)|0\rangle$ was considered in [402]. Coherent states of the two-parameter quantum algebra $su_{pq}(2)$ have been introduced in [403], on the basis of the definition:

$$[x]_{pq} = (q^x - p^{-x})/(q - p^{-1}).$$

Analogous construction, characterized by the deformations of the form

$$\hat{a}\hat{a}^\dagger = q_1^2\hat{a}^\dagger\hat{a} + q_2^{2\hat{n}} = q_2^2\hat{a}^\dagger\hat{a} + q_1^{2\hat{n}},$$
$$[n] = (q_1^{2n} - q_2^{2n})/(q_1^2 - q_2^2),$$

has been studied in [404] under the name '*Fibonacci oscillator*.' *Even and odd q-coherent states* have been studied in [405]. The *q-binomial states* were considered in [406]. Multimode q-coherent states were studied in [407]. Various families of 'q-states', including q-analogues of the 'standard sets' of nonclassical states, have been studied in [186, 408]. Interrelations between 'para'- and 'q-coherent' states were elucidated in [389, 409].

9.3. Generalized k-photon and fractional photon states

The usual coherent states are generated from the vacuum by the displacement operator $\hat{U}_1 = \exp(z\hat{a}^\dagger - z^*\hat{a})$, whereas the squeezed states are generated from the vacuum by the squeeze operator $\hat{U}_2 = \exp(z\hat{a}^{\dagger 2} - z^*\hat{a}^2)$. It seems natural to suppose that one could define a much more general class of states, acting on the vacuum by the operator $\hat{U}_k = \exp(z\hat{a}^{\dagger k} - z^*\hat{a}^k)$. However, such a simple definition leads to certain troubles [410], for instance, the vacuum expectation value $\langle 0|\hat{U}_k|0\rangle$ has zero radius of convergence as a power series with respect to z, for any $k > 2$. Although this phenomenon was considered as a mathematical artefact in [411], many people preferred to modify the operators $\hat{a}^{\dagger k}$ and $\hat{a}^k$ in the argument of the exponential in such a way that no questions on the convergence would arise.

One possibility was studied in a series of papers [412]. Instead of a simple power $\hat{a}^{\dagger k}$ in $\hat{U}_k$, it was proposed to use the *k-photon generalized boson operator* (introduced in [384])

$$\hat{A}_{(k)}^\dagger = \left(\left[\left[\frac{\hat{n}}{k}\right]\right]\frac{(\hat{n}-k)!}{\hat{n}!}\right)^{1/2}(\hat{a}^\dagger)^k \qquad \hat{n} = \hat{a}^\dagger\hat{a}, \quad (90)$$

which satisfies the relations (note that $\hat{A}_{(1)} = \hat{a}$, but $\hat{A}_{(k)} \neq \hat{a}^k$ for $k \geqslant 2$):

$$\left[\hat{A}_{(k)},\, \hat{A}_{(k)}^\dagger\right] = 1, \qquad \left[\hat{n},\, \hat{A}_{(k)}\right] = -k\hat{A}_{(k)}. \qquad (91)$$

The related concept of coherent and squeezed *fractional photon states* was used in [413].

Bužek and Jex [414] used Hermitian superpositions of the operators $\hat{A}_{(k)}$ and $\hat{A}_{(k)}^\dagger$ in order to define the kth-order squeezing in the frameworks of the Walls–Zoller scheme (30). The multiphoton squeezing operator can be defined as $\hat{S}_{(k)} = \exp[z\hat{A}_{(k)}^\dagger - z^*\hat{A}_{(k)}]$. As a matter of fact, we have an infinite series of the products of the operators $(\hat{a}^\dagger)^l\hat{a}^{l+k}$ and their Hermitially conjugated partners in the argument of the exponential function. The generic multiphoton squeezed state can be written as $|\alpha, z\rangle_{(k)} = \hat{D}(\alpha)\hat{S}_{(k)}(z)|\psi\rangle$ (in the cited papers the initial state $|\psi\rangle$ was assumed to be the vacuum state). An alternative definition (in the simplest case of $\alpha = \psi = 0$)

$$|z; k\rangle = \exp(-|z|^2/2)\sum_{n=0}^{\infty}\frac{z^n}{n!}\hat{A}_{(k)}^{\dagger n}|0\rangle \qquad (92)$$

results in the superposition of the states with $0, k, 2k, \ldots$ photons of the form:

$$|z; k\rangle = \exp(-|z|^2/2)\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|kn\rangle. \qquad (93)$$

9.4. Nonlinear coherent states

The NLCS were defined in [415, 416] as the right-hand eigenstates of the product of the boson annihilation operator $\hat{a}$ and some function $f(\hat{n})$ of the number operator $\hat{n}$:

$$f(\hat{n})\hat{a}|\alpha, f\rangle = \alpha|\alpha, f\rangle. \qquad (94)$$

Actually, such states have been known for many years under other names. The first example is the *phase state* (43) or its generalization (45) (known nowadays as the *negative binomial state* or the $SU(1,1)$ group coherent state), for which $f(n) = [(1+\kappa)/(1+\kappa+n)]^{1/2}$. The decomposition of the NLCS over the Fock basis has the form (80), consequently, NLCS coincide with 'para-coherent states' of [387]. Many nonclassical states turn out to be eigenstates of some 'nonlinear' generalizations of the annihilation operator. It has already been mentioned that the 'Barut–Girardello states' belong to the family (80). Another example is the *photon-added state* (73), which corresponds to the nonlinearity function $f(n) = 1 - m/(1+n)$ [417]. The physical meaning of NLCS was elucidated in [415, 416], where it was shown that such states may appear as stationary states of the centre-of-mass motion of a trapped ion [415], or may be related to some nonlinear processes (such as a hypothetical 'frequency blue shift' in high intensity photon beams [416]). Even and odd NLCS, introduced in [418], were studied in [419], and applications to the ion in the Paul trap were considered in [420]. Further generalizations, namely, NLCS on the circle, were given in [421] (also with applications to the trapped ions). Nonclassical properties of NLCS and their generalizations have been studied in [422, 423].

10. Concluding remarks

We see that the nomenclature of nonclassical states studied by theoreticians for seventy five years (most of them appeared in the last thirty years) is rather impressive. Now the main problem is to create them in laboratory and to verify that the desired state was indeed obtained. Therefore to conclude this review, I present a few references to studies devoted to experimental aspects and applications in other areas of physics (different from quantum optics). This list is very incomplete, but the reader can find more extensive discussions in the papers cited later.

10.1. Generation and detection of nonclassical states

The first proposals on different schemes of generating 'the most nonclassical' n-photon (Fock) states appeared in [424]. The problem of creating Fock states and their arbitrary superpositions in a cavity (in particular, via the interaction between the field and atoms passing through the cavity), named *quantum state engineering* in [425], was studied, e.g., in [425,426]. Different methods of producing 'cat' states were proposed in [427]. The use of *beamsplitters* to create various types of nonclassical states was considered in [354, 428]. The problem of generating the states with 'holes' in the photon-number distribution was analysed in [429]. A possibility of creating nonclassical states in a cavity with *moving mirrors* (which can mimic a Kerr-like medium) was studied in [430] (for a detailed review of studies on the classical and quantum electrodynamics in cavities with moving boundaries, with the emphasis on the dynamical Casimir effect, see [431]). The results of the first experiments were described in [432] (nonclassical states of the electromagnetic field inside a cavity) and [433] (nonclassical motional states of trapped ions). For details and other proposals see, e.g., [6, 434, 435]. Various aspects of the problem of *detecting* quantum states and their 'recognition' or reconstruction were treated in [436–438].

Different schemes of the conditional generation of special states (in cavities, via continuous measurements, etc) were discussed, e.g., in [439].

Several specific kinds of quantum states became popular in the last decade. *Dark states* [440] are certain superpositions of the atomic eigenstates, whose typical common feature is the existence of some sharp 'dips' in the spectra of absorption, fluorescence, etc, due to the destructive quantum interference of transition amplitudes between different energy levels involved. These states are connected with the NLCS [415]. For reviews and references see, e.g., [441,442]. *Greenberger–Horne–Zeilingler states* (or *GHZ-states*) [443], which are certain states of three or more correlated spin-$\frac{1}{2}$ particles, are popular in the studies related to the EPR-paradox, Bell inequalities, quantum teleportation, and so on. For methods of their generation and other references see, e.g., [444].

10.2. Applications of nonclassical states in different areas of physics

The squeezed and 'cat' states in *high energy physics* were considered in [445]. Applications of the squeezed and other nonclassical states to *cosmological problems* were studied in [446]. Squeezed and 'cat' states in *Josephson junctions* were considered in [447]. Squeezed states of *phonons* and other bosonic excitations (*polaritons*, *excitons*, etc) in condensed matter were studied in [448]. Spin-coherent states were used in [449]. Nonclassical states in *molecules* were studied in [450]. Nonclassical states of the *Bose–Einstein condensate* were considered in [435, 451].

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References

[1] Schrödinger E 1926 Der stetige Übergang von der Mikro- zur Makromechanik *Naturwissenschaften* **14** 664–6
[2] Kennard E H 1927 Zur Quantenmechanik einfacher Bewegungstypen *Z. Phys.* **44** 326–52
[3] Darwin C G 1927 Free motion in wave mechanics *Proc. Camb. Phil. Soc.* **117** 258–93
[4] Klauder J R and Sudarshan E C G 1968 *Fundamentals of Quantum Optics* (New York: Benjamin)
Klauder J R and Skagerstam B-S (ed) 1985 *Coherent States: Applications in Physics and Mathematical Physics* (Singapore: World Scientific)
Peřina J 1991 *Quantum Statistics of Linear and Nonlinear Optical Phenomena* 2nd edn (Dordrecht: Kluwer)
[5] Loudon R and Knight P L 1987 Squeezed light *J. Mod. Opt.* **34** 709–59
Teich M C and Saleh B E A 1989 Squeezed state of light *Quantum Opt.* **1** 153–91
Zaheer K and Zubairy M S 1990 Squeezed states of the radiation field *Advances in Atomic, Molecular and Optical Physics* vol 28, ed D R Bates and B Bederson (New York: Academic) pp 143–235
Zhang W M, Feng D H and Gilmore R 1990 Coherent states: theory and some applications *Rev. Mod. Phys.* **62** 867–927
[6] Bužek V and Knight P L 1995 Quantum interference, superposition states of light, and nonclassical effects *Progress in Optics* vol 34, ed E Wolf (Amsterdam: North-Holland) pp 1–158
[7] Ali S T, Antoine J-P, Gazeau J-P and Mueller U A 1995 Coherent states and their generalizations: a mathematical overview *Rev. Math. Phys.* **7** 1013–104
[8] Heisenberg W 1927 Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik *Z. Phys.* **43** 172–98
[9] Weyl H 1928 *Theory of Groups and Quantum Mechanics* (New York: Dutton) pp 77, 393–4
[10] Glauber R J 1963 Photon correlations *Phys. Rev. Lett.* **10** 84–6
[11] Fock V 1928 Verallgemeinerung und Lösung der Diracschen statistischen Gleichung *Z. Phys.* **49** 339–57
[12] Schrödinger E 1926 Quantisierung als Eigenwertproblem II *Ann. Phys., Lpz.* **79** 489–527
[13] Iwata G 1951 Non-Hermitian operators and eigenfunction expansions *Prog. Theor. Phys.* **6** 216–26
[14] Schwinger J 1953 The theory of quantized fields III *Phys. Rev.* **91** 728–40
[15] Rashevskiy P K 1958 On mathematical foundations of quantum electrodynamics *Usp. Mat. Nauk* **13** no 3(81) 3–110 (in Russian)
[16] Klauder J R 1960 The action option and a Feynman quantization of spinor fields in terms of ordinary c-numbers *Ann. Phys., NY* **11** 123–68
[17] Bargmann V 1961 On a Hilbert space of analytic functions and an associated integral transform: Part I. *Commun. Pure Appl. Math.* **14** 187–214
[18] Henley E M and Thirring W 1962 *Elementary Quantum Field Theory* (New York: McGraw-Hill) p 15
[19] Feynman R P 1951 An operator calculus having applications in quantum electrodynamics *Phys. Rev.* **84** 108–28
[20] Glauber R J 1951 Some notes on multiple boson processes *Phys. Rev.* **84** 395–400
[21] Bartlett M S and Moyal J E 1949 The exact transition probabilities of a quantum-mechanical oscillator calculated by the phase-space method *Proc. Camb. Phil. Soc.* **45** 545–53

[22] Feynman R P 1950 Mathematical formulation of the quantum theory of electromagnetic interaction *Phys. Rev.* **80** 440–57

[23] Ludwig G 1951 Die erzwungenen Schwingungen des harmonischen Oszillators nach der Quantentheorie *Z. Phys.* **130** 468–76

[24] Husimi K 1953 Miscellanea in elementary quantum mechanics II *Prog. Theor. Phys.* **9** 381–402

[25] Kerner E H 1958 Note on the forced and damped oscillator in quantum mechanics *Can. J. Phys.* **36** 371–7

[26] Sudarshan E C G 1963 Equivalence of semiclassical and quantum mechanical descriptions of statistical light beams *Phys. Rev. Lett.* **10** 277–9

[27] Glauber R J 1963 Coherent and incoherent states of the radiation field *Phys. Rev.* **131** 2766–88

[28] Carruthers P and Nieto M 1965 Coherent states and number-phase uncertainty relation *Phys. Rev. Lett.* **14** 387–9
Klauder J R, McKenna J and Currie D G 1965 On 'diagonal' coherent-state representations for quantum-mechanical density matrices *J. Math. Phys.* **6** 734–9
Carruthers P and Nieto M 1965 Coherent states and the forced quantum oscillator *Am. J. Phys.* **33** 537–44
Cahill K E 1965 Coherent-state representations for the photon density operator *Phys. Rev.* **138** B1566–76

[29] Carruthers P and Nieto M 1968 The phase-angle variables in quantum mechanics *Rev. Mod. Phys.* **40** 411–40

[30] Landau L 1927 Das Dämpfungsproblem in der Wellenmechanik *Z. Phys.* **45** 430–41

[31] Titulaer U M and Glauber R J 1965 Correlation functions for coherent fields *Phys. Rev.* **140** B676–82

[32] Mollow B R and Glauber R J 1967 Quantum theory of parametric amplification: I. *Phys. Rev.* **160** 1076–96

[33] Cahill K E and Glauber R J 1969 Density operators and quasiprobability distributions *Phys. Rev.* **177** 1882–902

[34] Aharonov Y, Falkoff D, Lerner E and Pendleton H 1966 A quantum characterization of classical radiation *Ann. Phys., NY* **39** 498–512

[35] Hillery M 1985 Classical pure states are coherent states *Phys. Lett.* A **111** 409–11

[36] Helstrom C W 1980 Nonclassical states in optical communication to a remote receiver *IEEE Trans. Inf. Theory* **26** 378–82
Hillery M 1985 Conservation laws and nonclassical states in nonlinear optical systems *Phys. Rev.* A **31** 338–42
Mandel L 1986 Nonclassical states of the electromagnetic field *Phys. Scr.* T **12** 34–42

[37] Loudon R 1980 Non-classical effects in the statistical properties of light *Rep. Prog. Phys.* **43** 913–49

[38] Zubairy M S 1982 Nonclassical effects in a two-photon laser *Phys. Lett.* A **87** 162–4
Lugiato L A and Strini G 1982 On nonclassical effects in two-photon optical bistability and two-photon laser *Opt. Commun.* **41** 374–8

[39] Schubert M 1987 The attributes of nonclassical light and their mutual relationship *Ann. Phys., Lpz.* **44** 53–60
Janszky J, Sibilia C, Bertolotti M and Yushin Y 1988 Switch effect of nonclassical light *J. Physique* C **49** 337–9
Gea-Banacloche G 1989 Two-photon absorption of nonclassical light *Phys. Rev. Lett.* **62** 1603–6

[40] Nieto M M 1998 The discovery of squeezed states in 1927 *5th Int. Conf. on Squeezed States and Uncertainty Relations (Balatonfured, 1997) (NASA Conference Publication NASA/CP-1998-206855)* ed D Han, J Janszky, Y S Kim and V I Man'ko (Greenbelt: Goddard Space Flight Center) pp 175–80

[41] Husimi K 1953 Miscellanea in elementary quantum mechanics: I. *Prog. Theor. Phys.* **9** 238–44

[42] Takahasi H 1965 Information theory of quantum-mechanical channels *Advances in Communication Systems. Theory and Applications* vol 1, ed A V Balakrishnan (New York: Academic) pp 227–310

[43] Plebański J 1954 Classical properties of oscillator wave packets *Bull. Acad. Pol. Sci.* **2** 213–17

[44] Infeld L and Plebański J 1955 On a certain class of unitary transformations *Acta Phys. Pol.* **14** 41–75

[45] Plebański J 1955 On certain wave packets *Acta Phys. Pol.* **14** 275–93

[46] Plebański J 1956 Wave functions of a harmonic oscillator *Phys. Rev.* **101** 1825–6

[47] Miller M M and Mishkin E A 1966 Characteristic states of the electromagnetic radiation field *Phys. Rev.* **152** 1110–14

[48] Lu E Y C 1971 New coherent states of the electromagnetic field *Lett. Nuovo Cimento* **2** 1241–4

[49] Bialynicki-Birula I 1971 Solutions of the equations of motion in classical and quantum theories *Ann. Phys., NY* **67** 252–73

[50] Jackiw R 1968 Minimum uncertainty product, number-phase uncertainty product, and coherent states *J. Math. Phys.* **9** 339–46

[51] Stoler D 1970 Equivalence classes of minimum uncertainty packets *Phys. Rev.* D **1** 3217–19
Stoler D 1971 Equivalence classes of minimum uncertainty packets: II. *Phys. Rev.* D **4** 1925–6

[52] Bertrand P P, Moy K and Mishkin E A 1971 Minimum uncertainty states $|\gamma\rangle$ and the states $|n\rangle_\gamma$ of the electromagnetic field *Phys. Rev.* D **4** 1909–12

[53] Trifonov D A 1974 Coherent states and uncertainty relations *Phys. Lett.* A **48** 165–6
Stoler D 1975 Most-general minimality-preserving Hamiltonian *Phys. Rev.* D **11** 3033–4
Canivell V and Seglar P 1977 Minimum-uncertainty states and pseudoclassical dynamics *Phys. Rev.* D **15** 1050–4
Canivell V and Seglar P 1977 Minimum-uncertainty states and pseudoclassical dynamics: II. *Phys. Rev.* D **18** 1082–94

[54] Solimeno S, Di Porto P and Crosignani B 1969 Quantum harmonic oscillator with time-dependent frequency *J. Math. Phys.* **10** 1922–8

[55] Malkin I A and Man'ko V I 1970 Coherent states and excitation of N-dimensional nonstationary forced oscillator *Phys. Lett.* A **32** 243–4

[56] Dodonov V V and Man'ko V I 1979 Coherent states and the resonance of a quantum damped oscillator *Phys. Rev.* A **20** 550–60

[57] Dodonov V V and Man'ko V I 1989 Invariants and correlated states of nonstationary quantum systems *Invariants and the Evolution of Nonstationary Quantum Systems* (*Proc. Lebedev Physics Institute vol 183*) ed M A Markov (Commack: Nova Science) pp 71–181

[58] Malkin I A and Man'ko V I 1969 Coherent states of a charged particle in a magnetic field *Sov. Phys. –JETP* **28** 527–32

[59] Feldman A and Kahn A H 1969 Landau diamagnetism from the coherent states of an electron in a uniform magnetic field *Phys. Rev.* B **1** 4584–9
Tam W G 1971 Coherent states and the invariance group of a charged particle in a uniform magnetic field *Physica* **54** 557–72
Varró S 1984 Coherent states of an electron in a homogeneous constant magnetic field and the zero magnetic field limit *J. Phys. A: Math. Gen.* **17** 1631–8

[60] Malkin I A, Man'ko V I and Trifonov D A 1969 Invariants and evolution of coherent states for charged particle in time-dependent magnetic field *Phys. Lett.* A **30** 414–16
Malkin I A, Man'ko V I and Trifonov D A 1970 Coherent states and excitation of nonstationary quantum oscillator and a charge in varying magnetic field *Phys. Rev.* D **2** 1371–80
Dodonov V V, Malkin I A and Man'ko V I 1972 Coherent states of a charged particle in a time-dependent uniform electromagnetic field of a plane current *Physica* **59** 241–56

[61] Holz A 1970 N-dimensional anisotropic oscillator in a uniform time-dependent electromagnetic field *Lett. Nuovo Cimento* **4** 1319–23

[62] Kim H Y and Weiner J H 1973 Gaussian-wave packet dynamics in uniform magnetic and quadratic potential fields *Phys. Rev.* B **7** 1353–62

[63] Malkin I A, Man'ko V I and Trifonov D A 1973 Linear adiabatic invariants and coherent states *J. Math. Phys.* **14** 576–82

[64] Picinbono B and Rousseau M 1970 Generalizations of Gaussian optical fields *Phys. Rev.* A **1** 635–43

[65] Holevo A S 1975 Some statistical problems for quantum Gaussian states *IEEE Trans. Inf. Theory* **21** 533–43
Helstrom C W 1976 *Quantum Detection and Estimation Theory* (New York: Academic)
Holevo A S 1982 *Probabilistic and Statistical Aspects of Quantum Theory* (Amsterdam: North-Holland)

[66] Fujiwara I 1952 Operator calculus of quantized operator *Prog. Theor. Phys.* **7** 433–48
Lewis H R Jr and Riesenfeld W B 1969 An exact quantum theory of the time-dependent harmonic oscillator and of a charged particle in a time-dependent electromagnetic field *J. Math. Phys.* **10** 1458–73

[67] Dodonov V V and Man'ko V I 1989 Evolution of multidimensional systems. Magnetic properties of ideal gases of charged particles *Invariants and the Evolution of Nonstationary Quantum Systems* (*Proc. Lebedev Physics Institute vol 183*) ed M A Markov (Commack: Nova Science) pp 263–414

[68] Heller E J 1975 Time-dependent approach to semiclassical dynamics *J. Chem. Phys.* **62** 1544–55

[69] Bagrov V G, Buchbinder I L and Gitman D M 1976 Coherent states of a relativistic particle in an external electromagnetic field *J. Phys. A: Math. Gen.* **9** 1955–65
Dodonov V V, Malkin I A and Man'ko V I 1976 Coherent states and Green functions of relativistic quadratic systems *Physica* A **82** 113–33
Kaiser G 1977 Phase-space approach to relativistic quantum mechanics: 1. Coherent-state representation for massive scalar particles *J. Math. Phys.* **18** 952–9

[70] Yuen H P 1976 Two-photon coherent states of the radiation field *Phys. Rev.* A **13** 2226–43

[71] Yuen H P and Shapiro J H 1978 Optical communication with two-photon coherent states—part I: Quantum-state propagation and quantum-noise reduction *IEEE Trans. Inf. Theory* **24** 657–68

[72] Hollenhorst J N 1979 Quantum limits on resonant-mass gravitational-radiation detectors *Phys. Rev.* D **19** 1669–79

[73] Caves C M, Thorne K S, Drever R W P, Sandberg V D and Zimmermann M 1980 On the measurement of a weak classical force coupled to a quantum mechanical oscillator: I. Issue of principle *Rev. Mod. Phys.* **52** 341–92

[74] Dodonov V V, Man'ko V I and Rudenko V N 1980 Nondemolition measurements in gravity wave experiments *Sov. Phys. –JETP* **51** 443–50

[75] Caves C M 1981 Quantum-mechanical noise in an interferometer *Phys. Rev.* D **23** 1693–708

[76] Grishchuk L P and Sazhin M V 1984 Squeezed quantum states of a harmonic oscillator in the problem of gravitational-wave detector *Sov. Phys. –JETP* **57** 1128–35
Bondurant R S and Shapiro J H 1984 Squeezed states in phase-sensing interferometers *Phys. Rev.* D **30** 2548–56

[77] Walls D F 1983 Squeezed states of light *Nature* **306** 141–6

[78] Brosa U and Gross D H E 1980 Squeezing of quantal fluctuations *Z. Phys.* A **294** 217–20

[79] Milburn G and Walls D F 1981 Production of squeezed states in a degenerate parametric amplifier *Opt. Commun.* **39** 401–4

[80] Becker W, Scully M O and Zubairy M S 1982 Generation of squeezed coherent states via a free-electron laser *Phys. Rev. Lett.* **48** 475–7

[81] Mandel L 1982 Squeezed states and sub-Poissonian photon statistics *Phys. Rev. Lett.* **49** 136–8

[82] Meystre P and Zubairy M S 1982 Squeezed states in the Jaynes–Cummings model *Phys. Lett.* A **89** 390–2

[83] Lugiato L A and Strini G 1982 On the squeezing obtainable in parametric oscillators and bistable absorption *Opt. Commun.* **41** 67–70

[84] Mandel L 1982 Squeezing and photon antibunching in harmonic generation *Opt. Commun.* **42** 437–9

[85] Tanaś R and Kielich S 1983 Self-squeezing of light propagating through non-linear optically isotropic media *Opt. Commun.* **45** 351–6

[86] Milburn G J 1984 Interaction of a two-level atom with squeezed light *Opt. Acta* **31** 671–9

[87] Stoler D 1974 Photon antibunching and possible ways to observe it *Phys. Rev. Lett.* **33** 1397–400
Peřina J, Peřinová V and Kodousek J 1984 On the relations of antibunching, sub-Poissonian statistics and squeezing *Opt. Commun.* **49** 210–14

[88] Kimble H J, Dagenais M and Mandel L 1977 Photon antibunching in resonance fluorescence *Phys. Rev. Lett.* **39** 691–5

[89] Paul H 1982 Photon anti-bunching *Rev. Mod. Phys.* **54** 1061–102
Smirnov D F and Troshin A S 1987 New phenomena in quantum optics: photon antibunching, sub-Poisson photon statistics, and squeezed states *Sov. Phys.–Usp.* **30** 851–74
Teich M C and Saleh B E A 1988 Photon bunching and antibunching *Progress in Optics* vol 16, ed E Wolf (Amsterdam: North-Holland) pp 1–104

[90] Yuen H P and Shapiro J H 1979 Generation and detection of two-photon coherent states in degenerate four-wave mixing *Opt. Lett.* **4** 334–6
Bondurant R S, Kumar P, Shapiro J H and Maeda M 1984 Degenerate four-wave mixing as a possible source of squeezed-state light *Phys. Rev.* A **30** 343–53
Milburn G J, Walls D F and Levenson M D 1984 Quantum phase fluctuations and squeezing in degenerate four-wave mixing *J. Opt. Soc. Am.* B **1** 390–4

[91] Walls D F and Zoller P 1981 Reduced quantum fluctuations in resonance fluorescence *Phys. Rev. Lett.* **47** 709–11

[92] Ficek Z, Tanaś R and Kielich S 1983 Squeezed states in resonance fluorescence of two interacting atoms *Opt. Commun.* **46** 23–6
Arnoldus H F and Nienhuis G 1983 Conditions for sub-Poissonian photon statistics and squeezed states in resonance fluorescence *Opt. Acta* **30** 1573–86
Loudon R 1984 Squeezing in resonance fluorescence *Opt. Commun.* **49** 24–8

[93] Sibilia C, Bertolotti M, Peřinová V, Peřina J and Lukš A 1983 Photon antibunching effect and statistical properties of single-mode emission in free-electron lasers *Phys. Rev.* A **28** 328–31
Dattoli G and Richetta M 1984 FEL quantum theory: comments on Glauber coherence, antibunching and squeezing *Opt. Commun.* **50** 165–8
Rai S and Chopra S 1984 Photon statistics and squeezing properties of a free-electron laser *Phys. Rev.* A **30** 2104–7

[94] Yurke B 1987 Squeezed-state generation using a Josephson parametric amplifier *J. Opt. Soc. Am.* B **4** 1551–7

[95] Kozierowski M and Kielich S 1983 Squeezed states in harmonic generation of a laser beam *Phys. Lett.* A **94** 213–16
Lugiato L A, Strini G and DeMartini F 1983 Squeezed states in second harmonic generation *Opt. Lett.* **8** 256–8
Friberg S and Mandel L 1984 Production of squeezed states by combination of parametric down-conversion and harmonic generation *Opt. Commun.* **48** 439–42

[96] Zubairy M S, Razmi M S K, Iqbal S and Idress M 1983 Squeezed states in a multiphoton absorption process *Phys. Lett.* A **98** 168–70
Loudon R 1984 Squeezing in two-photon absorption *Opt. Commun.* **49** 67–70

[97] Wódkiewicz K and Zubairy M S 1983 Effect of laser fluctuations on squeezed states in a degenerate parametric amplifier *Phys. Rev.* A **27** 2003–7

Carmichael H J, Milburn G J and Walls D F 1984 Squeezing in a detuned parametric amplifier *J. Phys. A: Math. Gen.* **17** 469–80

Collett M J and Gardiner C W 1984 Squeezing of intracavity and traveling-wave light fields produced in parametric amplification *Phys. Rev.* A **30** 1386–91

[98] Slusher R E, Hollberg L W, Yurke B, Mertz J C and Valley J F 1985 Observation of squeezed states generated by four-wave mixing in an optical cavity *Phys. Rev. Lett.* **55** 2409–12

[99] Shelby R M, Levenson M D, Perlmutter S H, DeVoe R G and Walls D F 1986 Broad-band parametric deamplification of quantum noise in an optical fiber *Phys. Rev. Lett.* **57** 691–4

[100] Wu L A, Kimble H J, Hall J L and Wu H 1986 Generation of squeezed states by parametric down conversion *Phys. Rev. Lett.* **57** 2520–3

[101] Yamamoto Y and Haus H A 1986 Preparation, measurement and information capacity of optical quantum states *Rev. Mod. Phys.* **58** 1001–20

Loudon R and Knight P L (ed) 1987 *Squeezed Light J. Mod. Opt.* **34** N 6/7 (special issue)

Kimble H J and Walls D F (ed) 1987 *Squeezed States of the Electromagnetic Field J. Opt. Soc. Am* B **4** N10 (feature issue)

[102] Schrödinger E 1930 Zum Heisenbergschen Unschärfeprinzip *Ber. Kgl. Akad. Wiss. Berlin* **24** 296–303

Robertson H P 1930 A general formulation of the uncertainty principle and its classical interpretation *Phys. Rev.* **35** 667

[103] Dodonov V V and Man'ko V I 1989 Generalization of uncertainty relation in quantum mechanics *Invariants and the Evolution of Nonstationary Quantum Systems* (*Proc. Lebedev Physics Institute, vol 183*) ed M A Markov (Commack: Nova Science) pp 3–101

[104] Trifonov D A 2000 Generalized uncertainty relations and coherent and squeezed states *J. Opt. Soc. Am.* A **17** 2486–95

[105] Dodonov V V, Kurmyshev E V and Man'ko V I 1980 Generalized uncertainty relation and correlated coherent states *Phys. Lett.* A **79** 150–2

[106] Jannussis A D, Brodimas G N and Papaloucas L C 1979 New creation and annihilation operators in the Gauss plane and quantum friction *Phys. Lett.* A **71** 301–3

Fujiwara I and Miyoshi K 1980 Pulsating states for quantal harmonic oscillator *Prog. Theor. Phys.* **64** 715–18

Rajagopal A K and Marshall J T 1982 New coherent states with applications to time-dependent systems *Phys. Rev.* D **26** 2977–80

Remaud B, Dorso C and Hernandez E S 1982 Coherent state propagation in open systems *Physica* A **112** 193–213

[107] Heller E 1991 Wavepacket dynamics and quantum chaology *Chaos and Quantum Physics* ed M-J Giannoni, A Voros and J Zinn-Justin (Amsterdam: Elsevier) pp 547–663

[108] Yuen H P 1983 Contractive states and the standard quantum limit for monitoring free-mass positions *Phys. Rev. Lett.* **51** 719–22

Storey P, Sleator T, Collett M and Walls D 1994 Contractive states of a free atom *Phys. Rev.* A **49** 2322–8

Walls D 1996 Quantum measurements in atom optics *Aust. J. Phys.* **49** 715–43

[109] Lerner P B, Rauch H and Suda M 1995 Wigner-function calculations for the coherent superposition of matter wave *Phys. Rev.* A **51** 3889–95

[110] Barvinsky A O and Kamenshchik A Y 1995 Preferred basis in quantum theory and the problem of classicalization of the quantum universe *Phys. Rev.* D **52** 743–57

[111] Dodonov V V, Kurmyshev E V and Man'ko V I 1988 Correlated coherent states *Classical and Quantum Effects in Electrodynamics* (*Proc. Lebedev Physics Institute, vol 176*) ed A A Komar (Commack: Nova Science) pp 169–99

[112] Dodonov V V, Klimov A B and Man'ko V I 1993 Physical effects in correlated quantum states *Squeezed and Correlated States of Quantum Systems* (*Proc. Lebedev Physics Institute, vol 205*) ed M A Markov (Commack: Nova Science) pp 61–107

Chumakov S M, Kozierowski M, Mamedov A A and Man'ko V I 1993 Squeezed and correlated light sources *Squeezed and Correlated States of Quantum Systems* (*Proc. Lebedev Physics Institute, vol 205*) ed M A Markov (Commack: Nova Science) pp 110–61

[113] Campos R A 1999 Quantum correlation coefficient for position and momentum *J. Mod. Opt.* **46** 1277–94

[114] Bykov V P 1991 Basic properties of squeezed light *Sov. Phys.–Usp.* **34** 910–24

Reynaud S, Heidmann A, Giacobino E and Fabre C 1992 Quantum fluctuations in optical systems *Progress in Optics* vol 30, ed E Wolf (Amsterdam: North-Holland) pp 1–85

Mizrahi S S and Daboul J 1992 Squeezed states, generalized Hermite polynomials and pseudo-diffusion equation *Physica* A **189** 635–50

Wünsche A 1992 Eigenvalue problem for arbitrary linear combinations of a boson annihilation and creation operator *Ann. Phys., Lpz.* **1** 181–97

Freyberger M and Schleich W 1994 Phase uncertainties of a squeezed state *Phys. Rev.* A **49** 5056–66

Arnoldus H F and George T F 1995 Squeezing in resonance fluorescence and Schrödinger's uncertainty relation *Physica* A **222** 330–46

Bužek V and Hillery M 1996 Operational phase distributions via displaced squeezed states *J. Mod. Opt.* **43** 1633–51

Weigert S 1996 Spatial squeezing of the vacuum and the Casimir effect *Phys. Lett.* A **214** 215–20

Bialynicki-Birula I 1998 Nonstandard introduction to squeezing of the electromagnetic field *Acta Phys. Pol.* B **29** 3569–90

[115] Yuen H P and Shapiro J H 1980 Optical communication with two-photon coherent states—part III: Quantum measurements realizable with photoemissive detectors *IEEE Trans. Inf. Theory* **26** 78–92

Gulshani P and Volkov A B 1980 Heisenberg-symplectic angular-momentum coherent states in two dimensions *J. Phys. A: Math. Gen.* **13** 3195–204

Gulshani P and Volkov A B 1980 The cranked oscillator coherent states *J. Phys. G: Nucl. Phys.* **6** 1335–46

Caves C M 1982 Quantum limits on noise in linear amplifiers *Phys. Rev.* D **26** 1817–39

Schumaker B L 1986 Quantum-mechanical pure states with Gaussian wave-functions *Phys. Rep.* **135** 317–408

Jannussis A and Bartzis V 1987 General properties of the squeezed states *Nuovo Cimento* B **100** 633–50

Bishop R F and Vourdas A 1987 A new coherent paired state with possible applications to fluctuation-dissipation phenomena *J. Phys. A: Math. Gen.* **20** 3727–41

Kim Y S and Yeh L 1992 $E(2)$-symmetric two-mode sheared states *J. Math. Phys.* **33** 1237–46

Yeoman G and Barnett S M 1993 Two-mode squeezed gaussons *J. Mod. Opt.* **40** 1497–530

Arvind, Dutta B, Mukunda N and Simon R 1995 Two-mode quantum systems: invariant classification of squeezing transformations and squeezed states *Phys. Rev.* A **52** 1609–20

Hacyan S 1996 Squeezed states and uncertainty relations in rotating frames and Penning trap *Phys. Rev.* A **53** 4481–7

Selvadoray M and Kumar M S 1997 Phase properties of correlated two-mode squeezed coherent states *Opt. Commun.* **136** 125–34

Fiurasek J and Peřina J 2000 Phase properties of two-mode Gaussian light fields with application to Raman scattering *J. Mod. Opt.* **47** 1399–417

Slater P B 2000 Essentially all Gaussian two-party quantum states are a priori nonclassical but classically correlated *J. Opt. B: Quantum Semiclass. Opt.* **2** L19–24

Lindblad G 2000 Cloning the quantum oscillator *J. Phys. A: Math. Gen.* **33** 5059–76

Abdalla M S and Obada A S F 2000 Quantum statistics of a new two mode squeeze operator model *Int. J. Mod. Phys.* B **14** 1105–28

[116] Mølmer K and Slowíkowski W 1988 A new Hilbert-space approach to the multimode squeezing of light *J. Phys. A: Math. Gen.* **21** 2565–71

Slowíkowski W and Mølmer K 1990 Squeezing of free Bose fields *J. Math. Phys.* **31** 2327–33

Huang J and Kumar P 1989 Photon-counting statistics of multimode squeezed light *Phys. Rev.* A **40** 1670–3

Ma X and Rhodes W 1990 Multimode squeeze operators and squeezed states *Phys. Rev.* A **41** 4625–31

Shumovskii A S 1991 Bogolyubov canonical transformation and collective states of bosonic fields *Theor. Math. Phys.* **89** 1323–9

Huang H and Agarwal G S 1994 General linear transformations and entangled states *Phys. Rev.* A **49** 52–60

Simon R, Mukunda N and Dutta B 1994 Quantum-noise matrix for multimode systems: $U(n)$-invariance, squeezing, and normal forms *Phys. Rev.* A **49** 1567–83

Sudarshan E C G, Chiu C B and Bhamathi G 1995 Generalized uncertainty relations and characteristic invariants for the multimode states *Phys. Rev.* A **52** 43–54

Trifonov D A 1998 On the squeezed states for n observables *Phys. Scr.* **58** 246–55

[117] Bastiaans M J 1979 Wigner distribution function and its application to first-order optics *J. Opt. Soc. Am.* **69** 1710–16

Dodonov V V, Man'ko V I and Semjonov V V 1984 The density matrix of the canonically transformed multi-dimensional Hamiltonian in the Fock basis *Nuovo Cimento* B **83** 145–61

Mizrahi S S 1984 Quantum mechanics in the Gaussian wave packet phase space representation *Physica* A **127** 241–64

Peřinová V, Křepelka J, Peřina J, Lukš A and Szlachetka P 1986 Entropy of optical fields *Opt. Acta* **33** 15–32

[118] Dodonov V V and Man'ko V I 1987 Density matrices and Wigner functions of quasiclassical quantum systems *Group Theory, Gravitation and Elementary Particle Physics* (*Proc. Lebedev Physics Institute, vol 167*) ed A A Komar (Commack: Nova Science) pp 7–101

[119] Agarwal G S 1987 Wigner-function description of quantum noise in interferometers *J. Mod. Opt.* **34** 909–21

Janszky J and Yushin Y 1987 Many-photon processes with the participation of squeezed light *Phys. Rev.* A **36** 1288–92

Simon R, Sudarshan E C G and Mukunda N 1987 Gaussian–Wigner distributions in quantum mechanics and optics *Phys. Rev.* A **36** 3868–80

Turner R E and Snider R F 1987 A comparison of local and global single Gaussian approximations to time dynamics: one-dimensional systems *J. Chem. Phys.* **87** 910–20

Mizrahi S S and Galetti D 1988 On the equivalence between the wave packet phase space representation (WPPSR) and the phase space generated by the squeezed states *Physica* A **153** 567–72

Lukš A and Peřinová V 1989 Entropy of shifted Gaussian states *Czech. J. Phys.* **39** 392–407

Mann A and Revzen M 1993 Gaussian density matrices: quantum analogs of classical states *Fortschr. Phys.* **41** 431–46

[120] Dodonov V V, Man'ko O V and Man'ko V I 1994 Photon distribution for one-mode mixed light with a generic Gaussian Wigner function *Phys. Rev.* A **49** 2993–3001

Dodonov V V, Man'ko O V and Man'ko V I 1994 Multi-dimensional Hermite polynomials and photon distribution for polymode mixed light *Phys. Rev.* A **50** 813–17

[121] Caves C M and Drummond P D 1994 Quantum limits on bosonic communication rates *Rev. Mod. Phys.* **66** 481–537

Hall M J W 1994 Gaussian noise and quantum optical communication *Phys. Rev.* A **50** 3295–303

Wünsche A 1996 The complete Gaussian class of quasiprobabilities and its relation to squeezed states and their discrete excitations *Quantum Semiclass. Opt.* **8** 343–79

Holevo A S, Sohma M and Hirota O 1999 Capacity of quantum Gaussian channels *Phys. Rev.* A **59** 1820–8

[122] Bishop R F and Vourdas A 1987 Coherent mixed states and a generalized P representation *J. Phys. A: Math. Gen.* **20** 3743–69

Fearn H and Collett M J 1988 Representations of squeezed states with thermal noise *J. Mod. Opt.* **35** 553–64

Aliaga J and Proto A N 1989 Relevant operators and non-zero temperature squeezed states *Phys. Lett.* A **142** 63–7

Kireev A, Mann A, Revzen M and Umezawa H 1989 Thermal squeezed states in thermo field dynamics and quantum and thermal fluctuations *Phys. Lett.* A **142** 215–21

Oz-Vogt J, Mann A and Revzen M 1991 Thermal coherent states and thermal squeezed states *J. Mod. Opt.* **38** 2339–47

[123] Kim M S, de Oliveira F A M and Knight P L 1989 Properties of squeezed number states and squeezed thermal states *Phys. Rev.* A **40** 2494–503

[124] Marian P 1992 Higher order squeezing and photon statistics for squeezed thermal states *Phys. Rev.* A **45** 2044–51

[125] Chaturvedi S, Sandhya R, Srinivasan V and Simon R 1990 Thermal counterparts of nonclassical states in quantum optics *Phys. Rev.* A **41** 3969–74

Dantas C A M and Baseia B 1999 Noncoherent states having Poissonian statistics *Physica* A **265** 176–85

[126] Bekenstein J D and Schiffer M 1994 Universality in grey-body radiance: extending Kirchhoff's law to the statistics of quanta *Phys. Rev. Lett.* **72** 2512–15

Lee C T 1995 Nonclassical effects in the grey-body state *Phys. Rev.* A **52** 1594–600

[127] Wiseman H M 1999 Squashed states of light: theory and applications to quantum spectroscopy *J. Opt. B: Quantum Semiclass. Opt.* **1** 459–63

Mancini S, Vitali D and Tombesi P 2000 Motional squashed states *J. Opt. B: Quantum Semiclass. Opt.* **2** 190–5

[128] Bechler A 1988 Generation of squeezed states in a homogeneous magnetic field *Phys. Lett.* A **130** 481–2

Jannussis A, Vlahas E, Skaltsas D, Kliros G and Bartzis V 1989 Squeezed states in the presence of a time-dependent magnetic field *Nuovo Cimento* B **104** 53–66

[129] Abdalla M S 1991 Statistical properties of a charged oscillator in the presence of a constant magnetic field *Phys. Rev.* A **44** 2040–7

Kovarskiy V A 1992 Coherent and squeezed states of Landau oscillators in a solid. Emission of nonclassical light *Sov. Phys.–Solid State* **34** 1900–2

Baseia B, Mizrahi S and Moussa M H 1992 Generation of squeezing for a charged oscillator and a charged particle in a time dependent electromagnetic field *Phys. Rev.* A **46** 5885–9

Aragone C 1993 New squeezed Landau states *Phys. Lett.* A **175** 377–81

[130] Dodonov V V, Man'ko V I and Polynkin P G 1994 Geometrical squeezed states of a charged particle in a time-dependent magnetic field *Phys. Lett.* A **188** 232–8

[131] Ozana M and Shelankov A L 1998 Squeezed states of a particle in magnetic field *Solid State Phys.* **40** 1276–82

Delgado F C and Mielnik B 1998 Magnetic control of squeezing effects *J. Phys. A: Math. Gen.* **31** 309–20

[132] Hagedorn G A 1980 Semiclassical quantum mechanics: I. The $\hbar \to 0$ limit for coherent states *Commun. Math. Phys.* **71** 77–93

Bagrov V G, Belov V V and Ternov I M 1982 Quasiclassical trajectory-coherent states of a non-relativistic particle in an arbitrary electromagnetic field *Theor. Math. Phys.* **50** 256–61

Dodonov V V, Man'ko V I and Ossipov D L 1990 Quantum evolution of the localized states *Physica* A **168** 1055–72
Combescure M 1992 The squeezed state approach of the semiclassical limit on the time-dependent Schrödinger equation *J. Math. Phys.* **33** 3870–80
Bagrov V G, Belov V V and Trifonov A Y 1996 Semiclassical trajectory-coherent approximation in quantum mechanics: I. High-order corrections to multidimensional time-dependent equations of Schrödinger type *Ann. Phys., NY* **246** 231–90
Hagedorn G A 1998 Raising and lowering operators for semiclassical wave packets *Ann. Phys., NY* **269** 77–104
[133] Karassiov V P and Puzyrevsky V I 1989 Generalized coherent states of multimode light and biphotons *J. Sov. Laser Res.* **10** 229–40
Karassiov V P 1993 Polarization structure of quantum light fields—a new insight: 1. General outlook *J. Phys. A: Math. Gen.* **26** 4345–54
Karassiov V P 1994 Polarization squeezing and new states of light in quantum optics *Phys. Lett.* A **190** 387–92
Karassiov V P 2000 Symmetry approach to reveal hidden coherent structures in quantum optics. General outlook and examples *J. Russ. Laser Res.* **21** 370–410
[134] Korolkova N V and Chirkin A S 1996 Formation and conversion of the polarization-squeezed light *J. Mod. Opt.* **43** 869–78
Lehner J, Leonhardt U and Paul H 1996 Unpolarized light: classical and quantum states *Phys. Rev.* A **53** 2727–35
Lehner J, Paul H and Agarwal G S 1997 Generation and physical properties of a new form of unpolarized light *Opt. Commun.* **139** 262–9
Alodjants A P, Arakelian S M and Chirkin A S 1998 Polarization quantum states of light in nonlinear distributed feedback systems; quantum nondemolition measurements of the Stokes parameters of light and atomic angular momentum *Appl. Phys.* B **66** 53–65
Alodjants A P and Arakelian S M 1999 Quantum phase measurements and non-classical polarization states of light *J. Mod. Opt.* **46** 475–507
Kim Y S 2000 Lorentz group in polarization optics *J. Opt. B: Quantum Semiclass. Opt.* **2** R1–5
[135] Ritze H H and Bandilla A 1987 Squeezing and first-order coherence *J. Opt. Soc. Am.* B **4** 1641–4
[136] Loudon R 1989 Graphical representation of squeezed-state variances *Opt. Commun.* **70** 109–14
[137] Lukš A, Peřinová V and Hradil Z 1988 Principal squeezing *Acta Phys. Pol.* A **74** 713–21
[138] Dodonov V V 2000 Universal integrals of motion and universal invariants of quantum systems *J. Phys. A: Math. Gen.* **33** 7721–38
[139] Hillery M 1989 Squeezing and photon number in Jaynes–Cummings model *Phys. Rev.* A **39** 1556–7
[140] Wódkiewicz K and Eberly J H 1985 Coherent states, squeezed fluctuations, and the $SU(2)$ and $SU(1, 1)$ groups in quantum-optics applications *J. Opt. Soc. Am.* B **2** 458–66
[141] Barnett S M 1987 General criterion for squeezing *Opt. Commun.* **61** 432–6
[142] Hong C K and Mandel L 1985 Generation of higher-order squeezing of quantum electromagnetic field *Phys. Rev.* A **32** 974–82
[143] Kozierowski M 1986 Higher-order squeezing in kth-harmonic generation *Phys. Rev.* A **34** 3474–7
[144] Hillery M 1987 Amplitude-squared squeezing of the electromagnetic field *Phys. Rev.* A **36** 3796–802
[145] Zhang Z M, Xu L, Cai J L and Li F L 1990 A new kind of higher-order squeezing of radiation field *Phys. Lett.* A **150** 27–30
[146] Marian P 1991 Higher order squeezing properties and correlation functions for squeezed number states *Phys. Rev.* A **44** 3325–30
[147] Hillery M 1992 Phase-space representation of amplitude-square squeezing *Phys. Rev.* A **45** 4944–50
[148] Du S D and Gong C D 1993 Higher-order squeezing for the quantized light field: Kth-power amplitude squeezing *Phys. Rev.* A **48** 2198–212
[149] An N B 2001 Multi-directional higher-order amplitude squeezing *Phys. Lett.* A **284** 72–80
[150] Hillery M 1989 Sum and difference squeezing of the electromagnetic field *Phys. Rev.* A **40** 3147–55
[151] Chizhov A V, Haus J W and Yeong K C 1995 Higher-order squeezing in a boson-coupled two-mode system *Phys. Rev.* A **52** 1698–703
An N B and Tinh V 1999 General multimode sum-squeezing *Phys. Lett.* A **261** 34–9
An N B and Tinh V 2000 General multimode difference-squeezing *Phys. Lett.* A **270** 27–40
[152] Nieto M M and Truax D R 1993 Squeezed states for general systems *Phys. Rev. Lett.* **71** 2843–6
[153] Prakash G S and Agarwal G S 1994 Mixed excitation- and deexcitation-operator coherent states for the $SU(1, 1)$ group *Phys. Rev.* A **50** 4258–63
Marian P 1997 Second-order squeezed states *Phys. Rev.* A **55** 3051–8
[154] Yamamoto Y, Imoto N and Machida S 1986 Amplitude squeezing in a semiconductor laser using quantum nondemolition measurement and negative feedback *Phys. Rev.* A **33** 3243–61
[155] Sundar K 1995 Highly amplitude-squeezed states of the radiation field *Phys. Rev. Lett.* **75** 2116–19
[156] Barnett S M and Gilson C R 1990 Quantum electrodynamic bound on squeezing *Europhys. Lett.* **12** 325–8
[157] Mišta L, Peřinová V and Peřina J 1977 Quantum statistical properties of degenerate parametric amplification process *Acta Phys. Pol.* A **51** 739–51
Mišta L and Peřina J 1978 Quantum statistics of parametric amplification *Czech. J. Phys.* B **28** 392–404
[158] Schleich W and Wheeler J A 1987 Oscillations in photon distribution of squeezed states *J. Opt. Soc. Am.* B **4** 1715–22
[159] Vourdas A and Weiner R M 1987 Photon-counting distribution in squeezed states *Phys. Rev.* A **36** 5866–9
[160] Agarwal G S and Adam G 1988 Photon-number distributions for quantum fields generated in nonlinear optical processes *Phys. Rev.* A **38** 750–3
[161] Dodonov V V, Klimov A B and Man'ko V I 1989 Photon number oscillation in correlated light *Phys. Lett.* A **134** 211–16
[162] Chaturvedi S and Srinivasan V 1989 Photon-number distributions for fields with Gaussian Wigner functions *Phys. Rev.* A **40** 6095–8
[163] Schleich W, Walls D F and Wheeler J A 1988 Area of overlap and interference in phase space versus Wigner pseudoprobabilities *Phys. Rev.* A **38** 1177–86
Schleich W, Walther H and Wheeler J A 1988 Area in phase space as determiner of transition probability: Bohr–Sommerfeld bands, Wigner ripples, and Fresnel zones *Found. Phys.* **18** 953–68
[164] Dodonov V V, Man'ko V I and Rudenko V N 1980 Quantum properties of high-Q macroscopic resonators *Sov. J. Quantum Electron.* **10** 1232–8
[165] Walls D F and Barakat R 1970 Quantum-mechanical amplification and frequency conversion with a trilinear Hamiltonian *Phys. Rev.* A **1** 446–53
[166] Brif C 1996 Coherent states for quantum systems with a trilinear boson Hamiltonian *Phys. Rev.* A **54** 5253–61
[167] Caves C M, Zhu C, Milburn G J and Schleich W 1991 Photon statistics of two-mode squeezed states and interference in four-dimensional phase space *Phys. Rev.* A **43** 3854–61
Artoni M, Ortiz U P and Birman J L 1991 Photocount distribution of two-mode squeezed states *Phys. Rev.* A **43** 3954–65

Schrade G, Akulin V M, Man'ko V I and Schleich W 1993 Photon distribution for two-mode squeezed vacuum *Phys. Rev.* A **48** 2398–406

Selvadoray M, Kumar M S and Simon R 1994 Photon distribution in two-mode squeezed coherent states with complex displacement and squeeze parameters *Phys. Rev.* A **49** 4957–67

Arvind B and Mukunda N 1996 Non-classical photon statistics for two-mode optical fields *J. Phys. A: Math. Gen.* **29** 5855–72

Man'ko O V and Schrade G 1998 Photon statistics of two-mode squeezed light with Gaussian Wigner function *Phys. Scr.* **58** 228–34

[168] Marian P and Marian T A 1993 Squeezed states with thermal noise: I. Photon-number statistics *Phys. Rev.* A **47** 4474–86

[169] Dodonov V V, Man'ko O V, Man'ko V I and Rosa L 1994 Thermal noise and oscillations of photon distribution for squeezed and correlated light *Phys. Lett.* A **185** 231–7

Musslimani Z H, Braunstein S L, Mann A and Revzen M 1995 Destruction of photocount oscillations by thermal noise *Phys. Rev.* A **51** 4967–74

[170] Dodonov V V, Dremin I M, Polynkin P G and Man'ko V I 1994 Strong oscillations of cumulants of photon distribution function in slightly squeezed states *Phys. Lett.* A **193** 209–17

[171] Peřina J and Bajer J 1990 Origin of oscillations in photon distribution of squeezed states *Phys. Rev.* A **41** 516–18

Dutta B, Mukunda N, Simon R and Subramaniam A 1993 Squeezed states, photon-number distributions, and $U(1)$ invariance *J. Opt. Soc. Am.* B **10** 253–64

Marchiolli M A, Mizrahi S S and Dodonov V V 1999 Marginal and correlation distribution functions in the squeezed-states representation *J. Phys. A: Math. Gen.* **32** 8705–20

Doebner H D, Man'ko V I and Scherer W 2000 Photon distribution and quadrature correlations in nonlinear quantum mechanics *Phys. Lett.* A **268** 17–24

[172] Boiteux M and Levelut A 1973 Semicoherent states *J. Phys. A: Math. Gen.* **6** 589–96

[173] Roy S M and Singh V 1982 Generalized coherent states and the uncertainty principle *Phys. Rev.* D **25** 3413–16

[174] Satyanarayana M V 1985 Generalized coherent states and generalized squeezed coherent states *Phys. Rev.* D **32** 400–4

[175] Král P 1990 Displaced and squeezed Fock states *J. Mod. Opt.* **37** 889–917

de Oliveira F A M, Kim M S, Knight P L and Bužek V 1990 Properties of displaced number states *Phys. Rev.* A **41** 2645–52

Brisudová M 1991 Nonlinear dissipative oscillator with displaced number states *J. Mod. Opt.* **38** 2505–19

Wünsche A 1991 Displaced Fock states and their connection to quasi-probabilities *Quantum Opt.* **3** 359–83

Tanaś R, Murzakhmetov B K, Gantsog T and Chizhov A V 1992 Phase properties of displaced number states *Quantum Opt.* **4** 1–7

Chizhov A V and Murzakhmetov B K 1993 Photon statistics and phase properties of two-mode squeezed number states *Phys. Lett.* A **176** 33–40

Peřinová V and Křepelka J 1993 Free and dissipative evolution of squeezed and displaced number states in the third-order nonlinear oscillator *Phys. Rev.* A **48** 3881–9

Møller K B, Jørgensen T G and Dahl J P 1996 Displaced squeezed number states: position space representation, inner product, and some applications *Phys. Rev.* A **54** 5378–85

Honegger R and Rieckers A 1997 Squeezing operations in Fock space and beyond *Physica* A **242** 423–38

Lu W-F 1999 Thermalized displaced and squeezed number states in the coordinate representation *J. Phys. A: Math. Gen.* **32** 5037–51

[176] Titulaer U M and Glauber R J 1966 Density operators for coherent fields *Phys. Rev.* **145** 1041–50

[177] Bialynicka-Birula Z 1968 Properties of the generalized coherent state *Phys. Rev.* **173** 1207–9

[178] Stoler D 1971 Generalized coherent states *Phys. Rev.* D **4** 2309–12

[179] Glassgold A E and Holliday D 1965 Quantum statistical dynamics of laser amplifiers *Phys. Rev.* **139** A1717–34

[180] Dodonov V V, Malkin I A and Man'ko V I 1974 Even and odd coherent states and excitations of a singular oscillator *Physica* **72** 597–615

[181] Brif C, Mann A and Vourdas A 1996 Parity-dependent squeezing of light *J. Phys. A: Math. Gen.* **29** 2053–67

[182] Brif C 1996 Two-photon algebra eigenstates. A unified approach to squeezing *Ann. Phys., NY* **251** 180–207

[183] Bužek V, Vidiella-Barranco A and Knight P L 1992 Superpositions of coherent states—squeezing and dissipation *Phys. Rev.* A **45** 6570–85

[184] Gerry C C 1993 Nonclassical properties of even and odd coherent states *J. Mod. Opt.* A **40** 1053–71

[185] Dodonov V V, Kalmykov S Y and Man'ko V I 1995 Statistical properties of Schrödinger real and imaginary cat states *Phys. Lett.* A **199** 123–30

[186] Spiridonov V 1995 Universal superpositions of coherent states and self-similar potentials *Phys. Rev.* A **52** 1909–35

[187] Marian P and Marian T A 1997 Generalized characteristic functions for a single-mode radiation field *Phys. Lett.* A **230** 276–82

[188] Liu Y-X 1999 Atomic odd–even coherent state *Nuovo Cimento* B **114** 543–53

[189] Ansari N A and Man'ko V I 1994 Photon statistics of multimode even and odd coherent light *Phys. Rev.* A **50** 1942–5

Gerry C C and Grobe R 1995 Nonclassical properties of correlated two-mode Schrödinger cat states *Phys. Rev.* A **51** 1698–701

Dodonov V V, Man'ko V I and Nikonov D E 1995 Even and odd coherent states for multimode parametric systems *Phys. Rev.* A **51** 3328–36

Gerry C C and Grobe R 1997 Two-mode $SU(2)$ and $SU(1,1)$ Schrödinger-cat states *J. Mod. Opt.* **44** 41–53

Trifonov D A 1998 Barut-Girardello coherent states for $u(p,q)$ and $sp(N,R)$ and their macroscopic superpositions *J. Phys. A: Math. Gen.* **31** 5673–96

[190] Lee C T 1996 Shadow states and shaded states *Quantum Semiclass. Opt.* **8** 849–60

[191] Dodonov V V and Mizrahi S S 1998 Stationary states in saturated two-photon processes and generation of phase-averaged mixtures of even and odd quantum states *Acta Phys. Slovaca* **48** 349–60

[192] Susskind L and Glogower J 1964 Quantum mechanical phase and time operator *Physics* **1** 49–62

[193] Paul H 1974 Phase of a microscopic electromagnetic field and its measurement *Fortschr. Phys.* **22** 657–89

[194] Dirac P A M 1966 *Lectures on Quantum Field Theory* (New York: Academic) p 18

[195] Lerner E C 1968 Harmonic-oscillator phase operators *Nuovo Cimento* B **56** 183–6

[196] Wigner E P 1951 Do the equations of motion determine the quantum mechanical commutation relations *Phys. Rev.* **77** 711–12

[197] Eswaran K 1970 On generalized phase operators for the quantum harmonic-oscillator *Nuovo Cimento* B **70** 1–11

[198] Lerner E C, Huang H W and Walters G E 1970 Some mathematical properties of oscillator phase operator *J. Math. Phys.* **11** 1679–84

Ifantis E K 1972 States, minimising the uncertainty product of the oscillator phase operator *J. Math. Phys.* **13** 568–75

[199] Aharonov Y, Lerner E C, Huang H W and Knight J M 1973 Oscillator phase states, thermal equilibrium and group representations *J. Math. Phys.* **14** 746–56

[200] Aharonov Y, Huang H W, Knight J M and Lerner E C 1971 Generalized destruction operators and a new method in group theory *Lett. Nuovo Cimento* **2** 1317–20

[201] Onofri E and Pauri M 1972 Analyticity and quantization *Lett. Nuovo Cimento* **3** 35–42

[202] Sukumar C V 1989 Revival Hamiltonians, phase operators and non-Gaussian squeezed states *J. Mod. Opt.* **36** 1591–605

[203] Marhic M E and Kumar P 1990 Squeezed states with a thermal photon distribution *Opt. Commun.* **76** 143–6

[204] Shapiro J H and Shepard S R 1991 Quantum phase measurement: a system-theory perspective *Phys. Rev.* A **43** 3795–818

[205] Mathews P M and Eswaran K 1974 Simultaneous uncertainties of the cosine and sine operators *Nuovo Cimento* B **19** 99–104

[206] Vourdas A 1992 Analytic representations in the unit disk and applications to phase state and squeezing *Phys. Rev.* A **45** 1943–50

[207] Sudarshan E C G 1993 Diagonal harmonious state representation *Int. J. Theor. Phys.* **32** 1069–76

[208] Brif C and Ben-Aryeh Y 1994 Phase-state representation in quantum optics *Phys. Rev.* A **50** 3505–16

[209] Brif C 1995 Photon states associated with the Holstein–Primakoff realisation of the $SU(1,1)$ Lie algebra *Quantum Semiclass. Opt.* **7** 803–34

[210] Dodonov V V and Mizrahi S S 1995 Uniform nonlinear evolution equations for pure and mixed quantum states *Ann. Phys., NY* **237** 226–68

[211] Lee C T 1997 Application of Klyshko's criterion for nonclassical states to the micromaser pumped by ultracold atoms *Phys. Rev.* A **55** 4449–53

[212] Baseia B, Dantas C M A and Moussa M H Y 1998 Pure states having thermal photon distribution revisited: generation and phase-optimization *Physica* A **258** 203–10

[213] Wünsche A 2001 A class of phase-like states *J. Opt. B: Quantum Semiclass. Opt.* **3** 206–18

[214] Bergou J and Englert B-G 1991 Operators of the phase. Fundamentals *Ann. Phys., NY* **209** 479–505
Schleich W P and Barnett S M (ed) 1993 *Quantum phase and phase dependent measurements Phys. Scr.* T **48** 1–142 (special issue)
Lukš A and Peřinová V 1994 Presumable solutions of quantum phase problem and their flaws *Quantum Opt.* **6** 125–67
Lynch R 1995 The quantum phase problem: a critical review *Phys. Rep.* **256** 367–437
Tanaś R, Miranowicz A and Gantsog T 1996 Quantum phase properties of nonlinear optical phenomena *Progress in Optics* vol 35, ed E Wolf (Amsterdam: North-Holland) pp 355–446
Peřinová V, Lukš A and Peřina J 1998 *Phase in Optics* (Singapore: World Scientific)

[215] Pegg D T and Barnett S M 1997 Tutorial review: quantum optical phase *J. Mod. Opt.* **44** 225–64

[216] Bonifacio R, Kim D M and Scully M O 1969 Description of many-atom system in terms of coherent boson states *Phys. Rev.* **187** 441–5

[217] Atkins P W and Dobson J C 1971 Angular momentum coherent states *Proc. Roy. Soc.* A **321** 321–40
Mikhailov V V 1971 Transformations of angular momentum coherent states under coordinate rotations *Phys. Lett.* A **34** 343–4
Makhviladze T M and Shelepin L A 1972 Coherent states of rotating bodies *Sov. J. Nucl. Phys.* **15** 601–2
Doktorov E V, Malkin I A and Man'ko V I 1975 Coherent states and asymptotic behavior of symmetric top wave-functions *Int. J. Quantum Chem.* **9** 951–68
Janssen D 1977 Coherent states of quantum-mechanical top *Sov. J. Nucl. Phys.* **25** 479–84
Gulshani P 1979 Generalized Schwinger boson realization and the oscillator-like coherent states of the rotation groups and the asymmetric top *Can. J. Phys.* **57** 998–1021

[218] Radcliffe J M 1971 Some properties of coherent spin states *J. Phys. A: Math. Gen.* **4** 313–23

[219] Arecchi F T, Courtens E, Gilmore R and Thomas H 1972 Atomic coherent states in quantum optics *Phys. Rev.* A **6** 2211–37

[220] Gilmore R 1972 Geometry of symmetrized states *Ann. Phys., NY* **74** 391–463
Lieb E H 1973 Classical limit on quantum spin systems *Commun. Math. Phys.* **31** 327–40

[221] Klauder J R 1963 Continuous representation theory: II. Generalized relation between quantum and classical dynamics *J. Math. Phys.* **4** 1058–73

[222] Várilly J C and Gracia-Bondía J M 1989 The Moyal representation for spin *Ann. Phys., NY* **190** 107–48

[223] Kitagawa M and Ueda M 1993 Squeezed spin states *Phys. Rev.* A **47** 5138–43

[224] Rozmej P and Arvieu R 1998 Clones and other interference effects in the evolution of angular-momentum coherent states *Phys. Rev.* A **58** 4314–29

[225] Perelomov A M 1972 Coherent states for arbitrary Lie group *Commun. Math. Phys.* **26** 222–36

[226] Hioe F T 1974 Coherent states and Lie algebras *J. Math. Phys.* **15** 1174–7

[227] Ducloy M 1975 Application du formalisme des états cohérentes de moment angulaire a quelques problèmes de physique atomique *J. Physique* **36** 927–41

[228] Karasev V P and Shelepin L A 1980 Theory of generalized coherent states of the group SU_n *Theor. Math. Phys.* **45** 879–86
Gitman D M and Shelepin A L 1993 Coherent states of $SU(N)$ groups *J. Phys. A: Math. Gen.* **26** 313–27
Nemoto K 2000 Generalized coherent states for $SU(n)$ systems *J. Phys. A: Math. Gen.* **33** 3493–506

[229] Isham C J and Klauder J R 1991 Coherent states for n-dimensional Euclidean groups $E(n)$ and their application *J. Math. Phys.* **32** 607–20
Puri R R 1994 $SU(m,n)$ coherent states in the bosonic representation and their generation in optical parametric processes *Phys. Rev.* A **50** 5309–16

[230] Cao C Q and Haake F 1995 Coherent states and holomorphic representations for multilevel atoms *Phys. Rev.* A **51** 4203–10

[231] Barut A O and Girardello L 1971 New 'coherent' states associated with noncompact groups *Commun. Math. Phys.* **21** 41–55

[232] Horn D and Silver R 1971 Coherent production of pions *Ann. Phys., NY* **66** 509–41

[233] Bhaumik D, Bhaumik K and Dutta-Roy B 1976 Charged bosons and the coherent state *J. Phys. A: Math. Gen.* **9** 1507–12

[234] Bhaumik D, Nag T and Dutta-Roy B 1975 Coherent states for angular momentum *J. Phys. A: Math. Gen.* **8** 1868–74

[235] Skagerstam B-S 1979 Coherent-state representation of a charged relativistic boson field *Phys. Rev.* D **19** 2471–6

[236] Agarwal G S 1988 Nonclassical statistics of fields in pair coherent states *J. Opt. Soc. Am.* B **5** 1940–7

[237] Gerry C C 1991 Correlated two-mode $SU(1,1)$ coherent states: nonclassical properties *J. Opt. Soc. Am.* B **8** 685–90

[238] Prakash G S and Agarwal G S 1995 Pair excitation–deexcitation coherent states *Phys. Rev.* A **52** 2335–41

[239] Liu X M 2001 Even and odd charge coherent states and their non-classical properties *Phys. Lett.* A **279** 123–32

[240] Yurke B, McCall S L and Klauder J R 1986 $SU(2)$ and $SU(1,1)$ interferometers *Phys. Rev.* A **33** 4033–54

[241] Bishop R F and Vourdas A 1986 Generalised coherent states and Bogoliubov transformations *J. Phys. A: Math. Gen.* **19** 2525–36

[242] Beckers J and Debergh N 1989 On generalized coherent states with maximal symmetry for the harmonic oscillator *J. Math. Phys.* **30** 1732–8

[243] Vourdas A 1990 $SU(2)$ and $SU(1, 1)$ phase states *Phys. Rev.* A **41** 1653–61
[244] Bužek V 1990 $SU(1, 1)$ squeezing of $SU(1, 1)$ generalized coherent states *J. Mod. Opt.* **37** 303–16
Ban M 1993 Lie-algebra methods in quantum optics: the Liouville-space formulation *Phys. Rev.* A **47** 5093–119
Wünsche A 2000 Symplectic groups in quantum optics *J. Opt. B: Quantum Semiclass. Opt.* **2** 73–80
[245] Brif C, Vourdas A and Mann A 1996 Analytic representation based on SU(1,1) coherent states and their applications *J. Phys. A: Math. Gen.* **29** 5873–85
[246] Gnutzmann S and Kus M 1998 Coherent states and the classical limit on irreducible $SU(3)$ representations *J. Phys. A: Math. Gen.* **31** 9871–96
[247] Kowalski K, Rembieliński J and Papaloucas L C 1996 Coherent states for a quantum particle on a circle *J. Phys. A: Math. Gen.* **29** 4149–67
González J A and del Olmo M A 1998 Coherent states on the circle *J. Phys. A: Math. Gen.* **31** 8841–57
Kowalski K and Rembieliński J 2000 Quantum mechanics on a sphere and coherent states *J. Phys. A: Math. Gen.* **33** 6035–48
[248] Fu H C and Sasaki R 1998 Probability distributions and coherent states of B_r, C_r and D_r algebras *J. Phys. A: Math. Gen.* **31** 901–25
Basu D 1999 The coherent states of the $SU(1, 1)$ group and a class of associated integral transforms *Proc. R. Soc.* A **455** 975–89
[249] Solomon A I 1999 Quantum optics: group and non-group methods *Int. J. Mod. Phys.* B **13** 3021–38
Sunilkumar V, Bambah B A, Jagannathan R, Panigrahi P K and Srinivasan V 2000 Coherent states of nonlinear algebras: applications to quantum optics *J. Opt. B: Quantum Semiclass. Opt.* **2** 126–32
[250] Kitagawa M and Yamamoto Y 1986 Number-phase minimum uncertainty state with reduced number uncertainty in a Kerr nonlinear interferometer *Phys. Rev.* A **34** 3974–88
[251] Yamamoto Y, Machida S, Imoto N, Kitagawa M and Bjork G 1987 Generation of number-phase minimum-uncertainty states and number states *J. Opt. Soc. Am.* B **4** 1645–61
[252] Luo S L 2000 Minimum uncertainty states for Dirac's number-phase pair *Phys. Lett.* A **275** 165–8
[253] Marburger J H and Power E A 1980 Minimum-uncertainty states of systems with many degrees of freedom *Found. Phys.* **10** 865–74
Milburn G J 1984 Multimode minimum uncertainty squeezed states *J. Phys. A: Math. Gen.* **17** 737–45
[254] Trifonov D A 1993 Completeness and geometry of Schrödinger minimum uncertainty states *J. Math. Phys.* **34** 100–10
[255] Hradil Z 1990 Noise minimum states and the squeezing and antibunching of light *Phys. Rev.* A **41** 400–7
[256] Hradil Z 1991 Extremal properties of near-photon-number eigenstate fields *Phys. Rev.* A **44** 792–5
[257] Yu D and Hillery M 1994 Minimum uncertainty states for amplitude-squared squeezing: general solutions *Quantum Opt.* **6** 37–56
Puri R R and Agarwal G S 1996 $SU(1, 1)$ coherent states defined via a minimum-uncertainty product and an equality of quadrature variances *Phys. Rev.* A **53** 1786–90
Weigert S 1996 Landscape of uncertainty in Hilbert space for one-particle states *Phys. Rev.* A **53** 2084–8
Fan H Y and Sun Z H 2000 Minimum uncertainty $SU(1, 1)$ coherent states for number difference—two-mode phase squeezing *Mod. Phys. Lett.* **14** 157–66
[258] Brif C and Mann A 1997 Generation of single-mode $SU(1, 1)$ intelligent states and an analytic approach to their quantum statistical properties *Quantum Semiclass. Opt.* **9** 899–920
[259] Aragone C, Guerri G, Salamó S and Tani J L 1974 Intelligent spin states *J. Phys. A: Math. Gen.* **7** L149–51
[260] Aragone C, Chalbaud E and Salamó S 1976 On intelligent spin states *J. Math. Phys.* **17** 1963–71
Delbourgo R 1977 Minimal uncertainty states for the rotation and allied groups *J. Phys. A: Math. Gen.* **10** 1837–46
Bacry H 1978 Eigenstates of complex linear combinations of $J_1 J_2 J_3$ for any representation of $SU(2)$ *J. Math. Phys.* **19** 1192–5
Bacry H 1978 Physical significance of minimum uncertainty states of an angular momentum system *Phys. Rev.* A **18** 617–19
Rashid M A 1978 The intelligent states: I. Group-theoretic study and the computation of matrix elements *J. Math. Phys.* **19** 1391–6
Van den Berghe G and De Meyer H 1978 On the existence of intelligent states associated with the non-compact group $SU(1, 1)$ *J. Phys. A: Math. Gen.* **11** 1569–78
[261] Arvieu R and Rozmej P 1999 Geometrical properties of intelligent spin states and time evolution of coherent states *J. Phys. A: Math. Gen.* **32** 2645–52
[262] Trifonov D A 1994 Generalized intelligent states and squeezing *J. Math. Phys.* **35** 2297–308
Trifonov D A 1997 Robertson intelligent states *J. Phys. A: Math. Gen.* **30** 5941–57
[263] Gerry C C and Grobe R 1995 Two-mode intelligent $SU(1, 1)$ states *Phys. Rev.* A **51** 4123–31
Gerry C C and Grobe R 1997 Intelligent photon states associated with the Holstein–Primakoff realisation of the $SU(1, 1)$ Lie algebra *Quantum Semiclass. Opt.* **9** 59–67
Peřinová V, Lukš A and Krepelka J 2000 Intelligent states in $SU(2)$ and $SU(1, 1)$ interferometry *J. Opt. B: Quantum Semiclass. Opt.* **2** 81–9
Liu N L, Sun Z H and Huang L S 2000 Intelligent states of the quantized radiation field associated with the Holstein–Primakoff realisation of $su(2)$ *J. Phys. A: Math. Gen.* **33** 3347–60
[264] Bergou J A, Hillery M and Yu D 1991 Minimum uncertainty states for amplitude-squared squeezing: Hermite polynomial states *Phys. Rev.* A **43** 515–20
[265] Datta S and D'Souza R 1996 Generalised quasiprobability distribution for Hermite polynomial squeezed states *Phys. Lett.* A **215** 149–53
[266] Fan H Y and Ye X 1993 Hermite polynomial states in two-mode Fock space *Phys. Lett.* A **175** 387–90
[267] Dodonov V V and Man'ko V I 1994 New relations for two-dimensional Hermite polynomials *J. Math. Phys.* **35** 4277–94
Dodonov V V 1994 Asymptotic formulae for two-variable Hermite polynomials *J. Phys. A: Math. Gen.* **27** 6191–203
Dattoli G and Torre A 1995 Phase space formalism: the generalized harmonic-oscillator functions *Nuovo Cimento* B **110** 1197–212
Wünsche A 2000 General Hermite and Laguerre two-dimensional polynomials *J. Phys. A: Math. Gen.* **33** 1603–29
Kok P and Braunstein S L 2001 Multi-dimensional Hermite polynomials in quantum optics *J. Phys. A: Math. Gen.* **34** 6185–95
[268] Nieto M M and Simmons L M Jr 1978 Coherent states for general potentials *Phys. Rev. Lett.* **41** 207–10
[269] Nieto M M and Simmons L M Jr 1979 Coherent states for general potentials: I. Formalism *Phys. Rev.* D **20** 1321–31
Nieto M M and Simmons L M Jr 1979 Coherent states for general potentials: II. Confining one-dimensional examples *Phys. Rev.* D **20** 1332–41
Nieto M M and Simmons L M Jr 1979 Coherent states for general potentials: III. Nonconfining one-dimensional examples *Phys. Rev.* D **20** 1342–50
[270] Jannussis A, Filippakis P and Papaloucas L C 1980 Commutation relations and coherent states *Lett. Nuovo Cimento* **29** 481–4
Jannussis A, Papatheou V, Patargias N and Papaloucas L C 1981 Coherent states for general potentials *Lett. Nuovo Cimento* **31** 385–9

[271] Gutschik V P, Nieto M M and Simmons L M Jr 1980 Coherent states for the 'isotonic oscillator' *Phys. Lett.* A **76** 15–18
[272] Ghosh G 1998 Generalized annihilation operator coherent states *J. Math. Phys.* **39** 1366–72
[273] Wang J-S, Liu T-K and Zhan M-S 2000 Nonclassical properties of even and odd generalized coherent states for an isotonic oscillator *J. Opt. B: Quantum Semiclass. Opt.* **2** 758–63
[274] Nieto M M and Simmons L M Jr 1979 Eigenstates, coherent states, and uncertainty products for the Morse oscillator *Phys. Rev.* A **19** 438–44
[275] Levine R D 1983 Representation of one-dimensional motion in a Morse potential by a quadratic Hamiltonian *Chem. Phys. Lett.* **95** 87–90
Gerry C C 1986 Coherent states and a path integral for the Morse oscillator *Phys. Rev.* A **33** 2207–11
Kais S and Levine R D 1990 Coherent states for the Morse oscillator *Phys. Rev.* A **41** 2301–5
[276] Cooper I L 1992 A simple algebraic approach to coherent states for the Morse oscillator *J. Phys. A: Math. Gen.* **25** 1671–83
Crawford M G A and Vrscay E R 1998 Generalized coherent states for the Pöschl–Teller potential and a classical limit *Phys. Rev.* A **57** 106–13
Avram N M, Drăgănescu G E and Avram C N 2000 Vibrational coherent states for Morse oscillator *J. Opt. B: Quantum Semiclass. Opt.* **2** 214–19
Molnár B, Benedict M G and Bertrand J 2001 Coherent states and the role of the affine group in the quantum mechanics of the Morse potential *J. Phys. A: Math. Gen.* **34** 3139–51
El Kinani A H and Daoud M 2001 Coherent states à la Klauder–Perelomov for the Pöschl–Teller potentials *Phys. Lett.* A **283** 291–9
[277] Samsonov B F 1998 Coherent states of potentials of soliton origin *J. Exp. Theor. Phys.* **87** 1046–52
[278] El Kinani A H and Daoud M 2001 Generalized intelligent states for an arbitrary quantum system *J. Phys. A: Math. Gen.* **34** 5373–87
El Kinani A H and Daoud M 2001 Generalized intelligent states for nonlinear oscillators *Int. J. Mod. Phys.* B **15** 2465–83
[279] Klauder J R 1993 Coherent states without groups: quantization on nonhomogeneous manifolds *Mod. Phys. Lett.* A **8** 1735–8
Klauder J R 1995 Quantization without quantization *Ann. Phys., NY* **237** 147–60
Gazeau J P and Klauder J R 1999 Coherent states for systems with discrete and continuous spectrum *J. Phys. A: Math. Gen.* **32** 123–32
[280] Klauder J R 1996 Coherent states for the hydrogen atom *J. Phys. A: Math. Gen.* **29** L293–8
[281] Sixdeniers J M, Penson K A and Solomon A I 1999 Mittag–Leffler coherent states *J. Phys. A: Math. Gen.* **32** 7543–63
[282] Penson K A and Solomon A I 1999 New generalized coherent states *J. Math. Phys.* **40** 2354–63
[283] Fox R F 1999 Generalized coherent states *Phys. Rev.* A **59** 3241–55
Fox R F and Choi M H 2000 Generalized coherent states and quantum-classical correspondence *Phys. Rev.* A **61** 032107
[284] Antoine J P, Gazeau J P, Monceau P, Klauder J R and Penson K A 2001 Temporally stable coherent states for infinite well and Pöschl–Teller potentials *J. Math. Phys.* **42** 2349–87
Klauder J R, Penson K A and Sixdeniers J M 2001 Constructing coherent states through solutions of Stieltjes and Hausdorff moment problems *Phys. Rev.* A **64** 013817
Hollingworth J M, Konstadopoulou A, Chountasis S, Vourdas A and Backhouse N B 2001 Gazeau–Klauder coherent states in one-mode systems with periodic potential *J. Phys. A: Math. Gen.* **34** 9463–74
[285] Brown L S 1973 Classical limit of the hydrogen atom *Am. J. Phys.* **41** 525–30
[286] Gaeta Z D and Stroud C R Jr 1990 Classical and quantum-mechanical dynamics of a quasiclassical state of the hydrogen atom *Phys. Rev.* A **42** 6308–13
[287] Bialynicki-Birula I, Kaliński M and Eberly J H 1994 Lagrange equilibrium points in celestial mechanics and nonspreading wave packets for strongly driven Rydberg electrons *Phys. Rev. Lett.* **73** 1777–80
Nieto M M 1994 Rydberg wave packets are squeezed states *Quantum Opt.* **6** 9–14
Bluhm R, Kostelecky V A and Tudose B 1995 Elliptical squeezed states and Rydberg wave packets *Phys. Rev.* A **52** 2234–44
Cerjan C, Lee E, Farrelly D and Uzer T 1997 Coherent states in a Rydberg atom: quantum mechanics *Phys. Rev.* A **55** 2222–31
Hagedorn G A and Robinson S L 1999 Approximate Rydberg states of the hydrogen atom that are concentrated near Kepler orbits *Helv. Phys. Acta* **72** 316–40
[288] Fock V 1935 Zur Theorie des Wasserstoffatoms *Z. Phys.* **98** 145–54
[289] Bargmann V 1936 Zur Theorie des Wasserstoffatoms. Bemerkungen zur gleichnamigen Arbeit von V Fock *Z. Phys.* **99** 576–82
[290] Malkin I A and Man'ko V I 1965 Symmetry of the hydrogen atom *Sov. Phys. –JETP Lett.* **2** 146–8
[291] Barut A O and Kleinert H 1967 Transition probabilities of the hydrogen atom from noncompact dynamical groups *Phys. Rev.* **156** 1541–5
[292] Mostowski J 1977 On the classical limit of the Kepler problem *Lett. Math. Phys.* **2** 1–5
Gerry C C 1984 On coherent states for the hydrogen atom *J. Phys. A: Math. Gen.* **17** L737–40
Gay J-C, Delande D and Bommier A 1989 Atomic quantum states with maximum localization on classical elliptical orbits *Phys. Rev.* A **39** 6587–90
Nauenberg M 1989 Quantum wave packets on Kepler elliptic orbits *Phys. Rev.* A **40** 1133–6
de Prunelé E 1990 $SO(4,2)$ coherent states and hydrogenic atoms *Phys. Rev.* A **42** 2542–9
McAnally D S and Bracken A J 1990 Quasi-classical states of the Coulomb system and and $SO(4,2)$ *J. Phys. A: Math. Gen.* **23** 2027–47
[293] Kustaanheimo P and Stiefel E 1965 Perturbation theory of Kepler motion based on spinor regularization *J. Reine. Angew. Math.* **218** 204–19
[294] Moshinsky M, Seligman T H and Wolf K B 1972 Canonical transformations and radial oscillator and Coulomb problems *J. Math. Phys.* **13** 901–7
[295] Gerry C C 1986 Coherent states and the Kepler–Coulomb problem *Phys. Rev.* A **33** 6–11
Bhaumik D, Dutta-Roy B and Ghosh G 1986 Classical limit of the hydrogen atom *J. Phys. A: Math. Gen.* **19** 1355–64
Nandi S and Shastry C S 1989 Classical limit of the two-dimensional and the three-dimensional hydrogen atom *J. Phys. A: Math. Gen.* **22** 1005–16
[296] Thomas L E and Villegas-Blas C 1997 Asymptotics of Rydberg states for the hydrogen atom *Commun. Math. Phys.* **187** 623–45
Bellomo P and Stroud C R 1999 Classical evolution of quantum elliptic states *Phys. Rev.* A **59** 2139–45
Toyoda T and Wakayama S 1999 Coherent states for the Kepler motion *Phys. Rev.* A **59** 1021–4
Nouri S 1999 Generalized coherent states for the d-dimensional Coulomb problem *Phys. Rev.* A **60** 1702–5
Crawford M G A 2000 Temporally stable coherent states in energy-degenerate systems: the hydrogen atom *Phys. Rev.* A **62** 012104
Michel L and Zhilinskii B I 2001 Rydberg states of atoms and molecules. Basic group theoretical and topological analysis *Phys. Rep.* **341** 173–264

[297] Ginzburg V L and Tamm I E 1947 On the theory of spin *Zh. Eksp. Teor. Fiz.* **17** 227–37 (in Russian)

[298] Yukawa H 1953 Structure and mass spectrum of elementary particles: 2. Oscillator model *Phys. Rev.* **91** 416–17

Markov M A 1956 On dynamically deformable form factors in the theory of elementary particles *Nuovo Cimento* (Suppl.) **3** 760–72

Ginzburg V L and Man'ko V I 1965 Relativistic oscillator models of elementary particles *Nucl. Phys.* **74** 577–88

Feynman R P, Kislinger M and Ravndal F 1971 Current matrix elements from a relativistic quark model *Phys. Rev.* D **3** 2706–32

Kim Y S and Noz M E 1973 Covariant harmonic oscillators and quark model *Phys. Rev.* D **8** 3521–7

Kim Y S and Wigner E P 1988 Covariant phase-space representation for harmonic oscillator *Phys. Rev.* A **38** 1159–67

[299] Atakishiev N M, Mir-Kasimov R M and Nagiev S M 1981 Quasipotential models of a relativistic oscillator *Theor. Math. Phys.* **44** 592–602

[300] Ternov I M and Bagrov V G 1983 On coherent states of relativistic particles *Ann. Phys., Lpz.* **40** 2–9

[301] Mir-Kasimov R M 1991 $SU_q(1,1)$ and the relativistic oscillator *J. Phys. A: Math. Gen.* **24** 4283–302

[302] Moshinsky M and Szczepaniak A 1989 The Dirac oscillator *J. Phys. A: Math. Gen.* **22** L817–19

[303] Itô D, Mori K and Carriere E 1967 An example of dynamical systems with linear trajectory *Nuovo Cimento* A **51** 1119–21

Cho Y M A 1974 Potential approach to spinor quarks *Nuovo Cimento* A **23** 550–6

[304] Nogami Y and Toyama F M 1996 Coherent state of the Dirac oscillator *Can. J. Phys.* **74** 114–21

Rozmej P and Arvieu R 1999 The Dirac oscillator. A relativistic version of the Jaynes–Cummings model *J. Phys. A: Math. Gen.* **32** 5367–82

[305] Aldaya V and Guerrero J 1995 Canonical coherent states for the relativistic harmonic oscillator *J. Math. Phys.* **36** 3191–9

[306] Tang J 1996 Coherent states and squeezed states of massless and massive relativistic harmonic oscillators *Phys. Lett.* A **219** 33–40

[307] Gol'fand Y A and Likhtman E P 1971 Extension of the algebra of Poincare group generators and violation of P invariance *JETP Lett.* **13** 323–6

[308] Witten E 1981 Dynamical breaking of supersymmetry *Nucl. Phys.* B **188** 513–54

[309] Salomonson P and van Holten J W 1982 Fermionic coordinates and supersymmetry in quantum mechanics *Nucl. Phys.* B **196** 509–31

Cooper F and Freedman B 1983 Aspects of supersymmetric quantum mechanics *Ann. Phys., NY* **146** 262–88

De Crombrugghe M and Rittenberg V 1983 Supersymmetric quantum mechanics *Ann. Phys., NY* **151** 99–126

[310] Bars I and Günaydin M 1983 Unitary representations of non-compact supergroups *Commun. Math. Phys.* **91** 31–51

[311] Balantekin A B, Schmitt H A and Barrett B R 1988 Coherent states for the harmonic oscillator representations of the orthosymplectic supergroup $Osp(1/2N, R)$ *J. Math. Phys.* **29** 1634–9

Orszag M and Salamó S 1988 Squeezing and minimum uncertainty states in the supersymmetric harmonic oscillator *J. Phys. A: Math. Gen.* **21** L1059–64

[312] Aragone C and Zypman F 1986 Supercoherent states *J. Phys. A: Math. Gen.* **19** 2267–79

[313] Fukui T and Aizawa N 1993 Shape-invariant potentials and an associated coherent state *Phys. Lett.* A **180** 308–13

Fernández D J, Hussin V and Nieto L M 1994 Coherent states for isospectral oscillator Hamiltonians *J. Phys. A: Math. Gen.* **27** 3547–64

Bagrov V G and Samsonov B F 1996 Coherent states for anharmonic oscillator Hamiltonians with equidistant and quasi-equidistant spectra *J. Phys. A: Math. Gen.* **29** 1011–23

Fu H C and Sasaki R 1996 Generally deformed oscillator, isospectral oscillator system and Hermitian phase operator *J. Phys. A: Math. Gen.* **29** 4049–64

Fernández D J and Hussin V 1999 Higher-order SUSY, linearized nonlinear Heisenberg algebras and coherent states *J. Phys. A: Math. Gen.* **32** 3603–19

Junker G and Roy P 1999 Non-linear coherent states associated with conditionally exactly solvable problems *Phys. Lett.* A **257** 113–19

Samsonov B F 2000 Coherent states for transparent potentials *J. Phys. A: Math. Gen.* **33** 591–605

[314] Fatyga B W, Kostelecky V A, Nieto M M and Truax D R 1991 Supercoherent states *Phys. Rev.* D **43** 1403–12

Kostelecky V A, Man'ko V I, Nieto M M and Truax D R 1993 Supersymmetry and a time-dependent Landau system *Phys. Rev.* A **48** 951–63

Kostelecky V A, Nieto M M and Truax D R 1993 Supersqueezed states *Phys. Rev.* A **48** 1045–54

Bérubé-Lauzière Y and Hussin V 1993 Comments of the definitions of coherent states for the SUSY harmonic oscillator *J. Phys. A: Math. Gen.* **26** 6271–5

Bluhm R and Kostelecky V A 1994 Atomic supersymmetry, Rydberg wave-packets, and radial squeezed states *Phys. Rev.* A **49** 4628–40

Bergeron H and Valance A 1995 Overcomplete basis for one-dimensional Hamiltonians *J. Math. Phys.* **36** 1572–92

Jayaraman J, Rodrigues R D and Vaidya A N 1999 A SUSY formulation a là Witten for the SUSY isotonic oscillator canonical supercoherent states *J. Phys. A: Math. Gen.* **32** 6643–52

[315] Stoler D, Saleh B E A and Teich M C 1985 Binomial states of the quantized radiation field *Opt. Acta* **32** 345–55

[316] Lee C T 1985 Photon antibunching in a free-electron laser *Phys. Rev.* A **31** 1213–15

[317] Dattoli G, Gallardo J and Torre A 1987 Binomial states of the quantized radiation-field—comment *J. Opt. Soc. Am.* B **4** 185–7

[318] Simon R and Satyanarayana M V 1988 Logarithmic states of the radiation field *J. Mod. Opt.* **35** 719–25

[319] Joshi A and Lawande S V 1989 The effects of negative binomial field distribution on Rabi oscillations *Opt. Commun.* **70** 21–4

Matsuo K 1990 Glauber–Sudarshan P-representation of negative binomial states *Phys. Rev.* A **41** 519–22

[320] Shepherd T J 1981 A model for photodetection of single-mode cavity radiation *Opt. Acta* **28** 567–83

Shepherd T J and Jakeman E 1987 Statistical analysis of an incoherently coupled, steady-state optical amplifier *J. Opt. Soc. Am.* B **4** 1860–6

[321] Barnett S M and Pegg D T 1996 Phase measurement by projection synthesis *Phys. Rev. Lett.* **76** 4148–50

[322] Pegg D T, Barnett S M and Phillips L S 1997 Quantum phase distribution by projection synthesis *J. Mod. Opt.* **44** 2135–48

Moussa M H Y and Baseia B 1998 Generation of the reciprocal-binomial state *Phys. Lett.* A **238** 223–6

[323] Baseia B, de Lima A F and da Silva A J 1995 Intermediate number-squeezed state of the quantized radiation field *Mod. Phys. Lett.* B **9** 1673–83

[324] Roy B 1998 Nonclassical properties of the even and odd intermediate number squeezed states *Mod. Phys. Lett.* B **12** 23–33

[325] Fu H C and Sasaki R 1997 Negative binomial and multinomial states: probability distribution and coherent state *J. Math. Phys.* **38** 3968–87

[326] Barnett S M 1998 Negative binomial states of the quantized radiation field *J. Mod. Opt.* **45** 2201–5

[327] Fu H, Feng Y and Solomon A I 2000 States interpolating between number and coherent states and their interaction with atomic systems *J. Phys. A: Math. Gen.* **33** 2231–49

[328] Sixdeniers J M and Penson K A 2000 On the completeness of coherent states generated by binomial distribution *J. Phys. A: Math. Gen.* **33** 2907–16

[329] Wang X G and Fu H 2000 Superposition of the λ-parametrized squeezed states *Mod. Phys. Lett.* B **14** 243–50

[330] Joshi A and Lawande S V 1991 Properties of squeezed binomial states and squeezed negative binomial states *J. Mod. Opt.* **38** 2009–22
Agarwal G S 1992 Negative binomial states of the field-operator representation and production by state reduction in optical processes *Phys. Rev.* A **45** 1787–92
Abdalla M S, Mahran M H and Obada A S F 1994 Statistical properties of the even binomial state *J. Mod. Opt.* **41** 1889–902
Vidiella-Barranco A and Roversi J A 1994 Statistical and phase properties of the binomial states of the electromagnetic field *Phys. Rev.* A **50** 5233–41
Vidiella-Barranco A and Roversi J A 1995 Quantum superpositions of binomial states of light *J. Mod. Opt.* **42** 2475–93
Fu H C and Sasaki R 1996 Generalized binomial states: ladder operator approach *J. Phys. A: Math. Gen.* **29** 5637–44
Joshi A and Obada A S F 1997 Some statistical properties of the even and the odd negative binomial states *J. Phys. A: Math. Gen.* **30** 81–97
El-Orany F A A, Mahran M H, Obada A S F and Abdalla M S 1999 Statistical properties of the odd binomial states with dynamical applications *Int. J. Theor. Phys.* **38** 1493–520
El-Orany F A A, Abdalla M S, Obada A S F and Abd Al-Kader G M 2001 Influence of squeezing operator on the quantum properties of various binomial states *Int. J. Mod. Phys.* B **15** 75–100

[331] Tanaś R 1984 Squeezed states of an anharmonic oscillator *Coherence and Quantum Optics* vol 5, ed L Mandel and E Wolf (New York: Plenum) pp 645–8

[332] Milburn G J 1986 Quantum and classical Liouville dynamics of the anharmonic oscillator *Phys. Rev.* A **33** 674–85

[333] Yurke B and Stoler D 1986 Generating quantum mechanical superpositions of macroscopically distinguishable states via amplitude dispersion *Phys. Rev. Lett.* **57** 13–16

[334] Mecozzi A and Tombesi P 1987 Distinguishable quantum states generated via nonlinear birefringence *Phys. Rev. Lett.* **58** 1055–8

[335] Schrödinger E 1935 Die gegenwärtige Situation in der Quantenmechanik *Naturwissenschaften* **23** 807–12

[336] Miranowicz A, Tanaś R and Kielich S 1990 Generation of discrete superpositions of coherent states in the anharmonic oscillator model *Quantum Opt.* **2** 253–65
Tanaś R, Miranowicz A and Kielich S 1991 Squeezing and its graphical representations in the anharmonic oscillator model *Phys. Rev.* A **43** 4014–21
Wilson-Gordon A D, Bužek V and Knight P L 1991 Statistical and phase properties of displaced Kerr states *Phys. Rev.* A **44** 7647–56
Tara K, Agarwal G S and Chaturvedi S 1993 Production of Schrödinger macroscopic quantum superposition states in a Kerr medium *Phys. Rev.* A **47** 5024–9
Peřinová V and Lukš A 1994 Quantum statistics of dissipative nonlinear oscillators *Progress in Optics* vol 33, ed E Wolf (Amsterdam: North-Holland) pp 129–202
Peřinová V, Vrana V, Lukš A and Křepelka J 1995 Quantum statistics of displaced Kerr states *Phys. Rev.* A **51** 2499–515
D'Ariano G M, Fortunato M and Tombesi P 1995 Time evolution of an anharmonic oscillator interacting with a squeezed bath *Quantum Semiclass. Opt.* **7** 933–42
Mancini S and Tombesi P 1995 Quantum dynamics of a dissipative Kerr medium with time-dependent parameters *Phys. Rev.* A **52** 2475–8
Bandilla A, Drobny G and Jex I 1996 Nondegenerate parametric interactions and nonclassical effects *Phys. Rev.* A **53** 507–16
Chumakov S M, Frank A and Wolf K B 1999 Finite Kerr medium: macroscopic quantum superposition states and Wigner functions on the sphere *Phys. Rev.* A **60** 1817–23
Paris M G A 1999 Generation of mesoscopic quantum superpositions through Kerr-stimulated degenerate downconversion *J. Opt. B: Quantum Semiclass. Opt.* **1** 662–7
Kozierowski M 2001 Thermal and squeezed vacuum Jaynes–Cummings models with a Kerr medium *J. Mod. Opt.* **48** 773–81
Klimov A B, Sanchez-Soto L L and Delgado J 2001 Mimicking a Kerr-like medium in the dispersive regime of second-harmonic generation *Opt. Commun.* **191** 419–26

[337] Slosser J J, Meystre P and Braunstein S L 1989 Harmonic oscillator driven by a quantum current *Phys. Rev. Lett.* **63** 934–7

[338] Jaynes E T and Cummings F W Comparison of quantum and semiclassical radiation theories with application to the beam maser *Proc. IEEE* **5** 89–108

[339] Slosser J J and Meystre P 1990 Tangent and cotangent states of the electromagnetic field *Phys. Rev.* A **41** 3867–74

[340] Gerry C C and Grobe R 1994 Statistical properties of squeezed Kerr states *Phys. Rev.* A **49** 2033–9

[341] García-Fernandez P and Bermejo F J 1987 Sub-Poissonian photon statistics and higher-order squeezing behavior of Bernouilli states in the linear amplifier *J. Opt. Soc. Am.* **4** 1737–41

[342] Wódkiewicz K, Knight P L, Buckle S J and Barnett S M 1987 Squeezing and superposition states *Phys. Rev.* A **35** 2567–77

[343] Sanders B C 1989 Superposition of two squeezed vacuum states and interference effects *Phys. Rev.* A **39** 4284–7

[344] Schleich W, Pernigo M and Kien F L 1991 Nonclassical state from two pseudoclassical states *Phys. Rev.* A **44** 2172–87

[345] Sanders B C 1992 Entangled coherent states *Phys. Rev.* A **45** 6811–15

[346] Gerry C C 1997 Generation of Schrödinger cats and entangled coherent states in the motion of a trapped ion by a dispersive interaction *Phys. Rev.* A **55** 2478–81
Rice D A and Sanders B C 1998 Complementarity and entangled coherent states *Quantum Semiclass. Opt.* **10** L41–7
Recamier J, Castaños O, Jáuregi R and Frank A 2000 Entanglement and generation of superpositions of atomic coherent states *Phys. Rev.* A **61** 063808
Massini M, Fortunato M, Mancini S and Tombesi P 2000 Synthesis and characterization of entangled mesoscopic superpositions for a trapped electron *Phys. Rev.* A **62** 041401
Wang X G, Sanders B C and Pan S H 2000 Entangled coherent states for systems with $SU(2)$ and $SU(1,1)$ symmetries *J. Phys. A: Math. Gen.* **33** 7451–67

[347] Moya-Cessa H 1995 Generation and properties of superpositions of displaced Fock states *J. Mod. Opt.* **42** 1741–54
Obada A S F and Omar Z M 1997 Properties of superposition of squeezed states *Phys. Lett.* A **227** 349–56
Dantas C A M, Queroz J R and Baseia B 1998 Superposition of displaced number states and interference effects *J. Mod. Opt.* **45** 1085–96
Ragi R, Baseia B and Bagnato V S 1998 Generalized superposition of two coherent states and interference effects *Int. J. Mod. Phys.* B **12** 1495–529
Obada A S F and Abd Al-Kader G M 1999 Superpositions of squeezed displaced Fock states: properties and generation *J. Mod. Opt.* **46** 263–78
Barbosa Y A, Marques G C and Baseia B 2000 Generalized superposition of two squeezed states: generation and statistical properties *Physica* A **280** 346–61

Liao J, Wang X, Wu L-A and Pan S H 2000 Superpositions of negative binomial states *J. Opt. B: Quantum Semiclass. Opt.* **2** 541–4
El-Orany F A A, Peřina J and Abdalla M S 2000 Phase properties of the superposition of squeezed and displaced number states *J. Opt. B: Quantum Semiclass. Opt.* **2** 545–52
Marchiolli M A, da Silva L F, Melo P S and Dantas C A M 2001 Quantum-interference effects on the superposition of N displaced number states *Physica* A **291** 449–66
[348] Janszky J and Vinogradov A V 1990 Squeezing via one-dimensional distribution of coherent states *Phys. Rev. Lett.* **64** 2771–4
Adam P, Janszky J and Vinogradov A V 1991 Amplitude squeezed and number-phase intelligent states via coherent states superposition *Phys. Lett.* A **160** 506–10
Bužek V and Knight P 1991 The origin of squeezing in a superposition of coherent states *Opt. Commun.* **81** 331–6
Domokos P, Adam P and Janszky J 1994 One-dimensional coherent-state representation on a circle in phase space *Phys. Rev.* A **50** 4293–7
Klimov A B and Chumakov S M 1997 Gaussians on the circle and quantum phase *Phys. Lett.* A **235** 7–14
[349] Agarwal G S and Tara K 1991 Nonclassical properties of states generated by the excitations on a coherent state *Phys. Rev.* A **43** 492–7
[350] Zhang Z and Fan H 1992 Properties of states generated by excitations on a squeezed vacuum state *Phys. Lett.* A **165** 14–18
Kis Z, Adam P and Janszky J 1994 Properties of states generated by excitations on the amplitude squeezed states *Phys. Lett.* A **188** 16–20
Xin Z Z, Duan Y B, Zhang H M, Hirayama M and Matumoto K I 1996 Excited two-photon coherent state of the radiation field *J. Phys. B: At. Mol. Opt. Phys.* **29** 4493–506
Man'ko V I and Wünsche A 1997 Properties of squeezed-state excitations *Quantum Semiclass. Opt.* **9** 381–409
[351] Xin Z Z, Duan Y B, Zhang H M, Qian W J, Hirayama M and Matumoto K I 1996 Excited even and odd coherent states of the radiation field *J. Phys. B: At. Mol. Opt. Phys.* B **29** 2597–606
Dodonov V V, Korennoy Y A, Man'ko V I and Moukhin Y A 1996 Nonclassical properties of states generated by the excitation of even/odd coherent states of light *Quantum Semiclass. Opt.* **8** 413–27
[352] Agarwal G S and Tara K 1992 Nonclassical character of states exhibiting no squeezing or sub-Poissonian statistics *Phys. Rev.* A **46** 485–8
Jones G N, Haight J and Lee C T 1997 Nonclassical effects in the photon-added thermal states *Quantum Semiclass. Opt.* **9** 411–18
[353] Dakna M, Knöll L and Welsch D-G 1998 Photon-added state preparation via conditional measurement on a beam splitter *Opt. Commun.* **145** 309–21
[354] Dakna M, Knöll L and Welsch D-G 1998 Quantum state engineering using conditional measurement on a beam splitter *Eur. Phys. J.* D **3** 295–308
[355] Mehta C L, Roy A K and Saxena G M 1992 Eigenstates of two-photon annihilation operators *Phys. Rev.* A **46** 1565–72
Fan H Y 1993 Inverse in Fock space studied via a coherent-state approach *Phys. Rev.* A **47** 4521–3
Arvind, Dutta B, Mehta C L and Mukunda N 1994 Squeezed states, metaplectic group, and operator Möbius transformations *Phys. Rev.* A **50** 39–61
[356] Roy A K and Mehta C L 1995 Boson inverse operators and associated coherent states *Quantum Semiclass. Opt.* **7** 877–88
[357] Dodonov V V, Marchiolli M A, Korennoy Y A, Man'ko V I and Moukhin Y A 1998 Dynamical squeezing of photon-added coherent states *Phys. Rev.* A **58** 4087–94
Lu H 1999 Statistical properties of photon-added and photon-subtracted two-mode squeezed vacuum state *Phys. Lett.* A **264** 265–9
Wei L F, Wang S J and Xi D P 1999 Inverse q-boson operators and their relation to photon-added and photon-depleted states *J. Opt. B: Quantum Semiclass. Opt.* **1** 619–23
Sixdeniers J M and Penson K A 2001 On the completeness of photon-added coherent states *J. Phys. A: Math. Gen.* **34** 2859–66
Wang X B, Kwek L C, Liu Y and Oh C H 2001 Non-classical effects of two-mode photon-added displaced squeezed states *J. Phys. B: At. Mol. Opt. Phys.* **34** 1059–78
Abdalla M S, El-Orany F A A and Peřina J 2001 Dynamical properties of degenerate parametric amplifier with photon-added coherent states *Nuovo Cimento* B **116** 137–53
Quesne C 2001 Completeness of photon-added squeezed vacuum and one-photon states and of photon-added coherent states on a circle *Phys. Lett.* A **288** 241–50
[358] Bužek V, Jex I and Quang T 1990 k-photon coherent states *J. Mod. Opt.* **37** 159–63
Sun J, Wang J and Wang C 1992 Generation of orthonormalized eigenstates of the operator a^k (for $k \geqslant 3$) from coherent states and their higher-order squeezing *Phys. Rev.* A **46** 1700–2
[359] Jex I and Bužek V 1993 Multiphoton coherent states and the linear superposition principle *J. Mod. Opt.* **40** 771–83
[360] Sun J, Wang J and Wang C 1991 Orthonormalized eigenstates of cubic and higher powers of the annihilation operator *Phys. Rev.* A **44** 3369–72
[361] Hach E E III and Gerry C C 1992 Four photon coherent states. Properties and generation *J. Mod. Opt.* **39** 2501–17
[362] Lynch R 1994 Simultaneous fourth-order squeezing of both quadrature components *Phys. Rev.* A **49** 2800–5
[363] Shanta P, Chaturvedi S, Srinivasan V, Agarwal G S and Mehta C L 1994 Unified approach to multiphoton coherent states *Phys. Rev. Lett.* **72** 1447–50
Nieto M M and Truax D R 1995 Arbitrary-order Hermite generating functions for obtaining arbitrary-order coherent and squeezed states *Phys. Lett.* A **208** 8–16
Nieto M M and Truax D R 1997 Holstein–Primakoff/Bogoliubov transformations and the multiboson system *Fortschr. Phys.* **45** 145–56
[364] Janszky J, Domokos P and Adam P 1993 Coherent states on a circle and quantum interference *Phys. Rev.* A **48** 2213–19
Gagen M J 1995 Phase-space interference approaches to quantum superposition states *Phys. Rev.* A **51** 2715–25
[365] Xin Z Z, Wang D B, Hirayama M and Matumoto K 1994 Even and odd two-photon coherent states of the radiation field *Phys. Rev.* A **50** 2865–9
[366] Jex I, Törmä P and Stenholm S 1995 Multimode coherent states *J. Mod. Opt.* **42** 1377–86
[367] Castaños O, López-Peña R and Man'ko V I 1995 Crystallized Schrödinger cat states *J. Russ. Laser Research* **16** 477–525
[368] Chountasis S and Vourdas A 1998 Weyl functions and their use in the study of quantum interference *Phys. Rev.* A **58** 848–55
Napoli A and Messina A 1999 Generalized even and odd coherent states of a single bosonic mode *Eur. Phys. J.* D **5** 441–5
Nieto M M and Truax D R 2000 Higher-power coherent and squeezed states *Opt. Commun.* **179** 197–213
Ragi R, Baseia B and Mizrahi S S 2000 Non-classical properties of even circular states *J. Opt. B: Quantum Semiclass. Opt.* **2** 299–305
José W D and Mizrahi S S 2000 Generation of circular states and Fock states in a trapped ion *J. Opt. B: Quantum Semiclass. Opt.* **2** 306–14
Souza Silva A L, José W D, Dodonov V V and Mizrahi S S 2001 Production of two-Fock states superpositions from even circular states and their decoherence *Phys. Lett.* A **282** 235–44

Quesne C 2000 Spectrum generating algebra of the C_λ-extended oscillator and multiphoton coherent states *Phys. Lett.* A **272** 313–25

[369] Pegg D T and Barnett S M 1988 Unitary phase operator in quantum mechanics *Europhys. Lett.* **6** 483–7

Barnett S M and Pegg D T 1989 On the Hermitian optical phase operator *J. Mod. Opt.* **36** 7–19

[370] Shapiro J H, Shepard S R and Wong N C 1989 Ultimate quantum limits on phase measurement *Phys. Rev. Lett.* **62** 2377–80

Schleich W P, Dowling J P and Horowicz R J 1991 Exponential decrease in phase uncertainty *Phys. Rev.* A **44** 3365–8

[371] Nath R and Kumar P 1991 Quasi-photon phase states *J. Mod. Opt.* **38** 263–8

[372] Bužek V, Wilson-Gordon A D, Knight P L and Lai W K 1992 Coherent states in a finite-dimensional basis: their phase properties and relationship to coherent states of light *Phys. Rev.* A **45** 8079–94

Kuang L-M and Chen X 1994 Phase-coherent states and their squeezing properties *Phys. Rev.* A **50** 4228–36

Galetti D and Marchiolli M A 1996 Discrete coherent states and probability distributions in finite-dimensional spaces *Ann. Phys., NY* **249** 454–80

[373] Obada A S F, Hassan S S, Puri R R and Abdalla M S 1993 Variation from number-state to chaotic-state fields: a generalized geometric state *Phys. Rev.* A **48** 3174–85

[374] Obada A S F, Yassin O M and Barnett S M 1997 Phase properties of coherent phase and generalized geometric state *J. Mod. Opt.* **44** 149–61

[375] Figurny P, Orlowski A and Wódkiewicz K 1993 Squeezed fluctuations of truncated photon operators *Phys. Rev.* A **47** 5151–7

Baseia B, de Lima A F and Marques G C 1995 Intermediate number-phase states of the quantized radiation field *Phys. Lett.* A **204** 1–6

Opatrný T, Miranowicz A and Bajer J 1996 Coherent states in finite-dimensional Hilbert space and their Wigner representation *J. Mod. Opt.* **43** 417–32

Leoński W 1997 Finite-dimensional coherent-state generation and quantum-optical nonlinear oscillator model *Phys. Rev.* A **55** 3874–8

Lobo A C and Nemes M C 1997 The reference state for finite coherent states *Physica* A **241** 637–48

Roy B and Roy P 1998 Coherent states, even and odd coherent states in a finite-dimensional Hilbert space and their properties *J. Phys. A: Math. Gen.* **31** 1307–17

Zhang Y Z, Fu H C and Solomon A I 1999 Intermediate coherent-phase(PB) states of radiation fields and their nonclassical properties *Phys. Lett.* A **263** 257–62

Wang X 2000 Ladder operator formalisms and generally deformed oscillator algebraic structures of quantum states in Fock space *J. Opt. B: Quantum Semiclass. Opt.* **2** 534–40

[376] Fan H Y, Ye X O and Xu Z H 1995 Laguerre polynomial states in single-mode Fock space *Phys. Lett.* A **199** 131–6

[377] Fu H C and Sasaki R 1996 Exponential and Laguerre polynomial states for $su(1,1)$ algebra and the Calogero–Sutherland model *Phys. Rev.* A **53** 3836–44

[378] Wang X G 2000 Coherent states, displaced number states and Laguerre polynomial states for $su(1,1)$ Lie algebra *Int. J. Mod. Phys.* B **14** 1093–103

[379] Fu H C 1997 Pólya states of quantized radiation fields, their algebraic characterization and non-classical properties *J. Phys. A: Math. Gen.* **30** L83–9

[380] Fu H C and Sasaki R 1997 Hypergeometric states and their nonclassical properties *J. Math. Phys.* **38** 2154–66

Liu N L 1999 Algebraic structure and nonclassical properties of the negative hypergeometric state *J. Phys. A: Math. Gen.* **32** 6063–78

[381] Roy P and Roy B 1997 A generalized nonclassical state of the radiation field and some of its properties *J. Phys. A: Math. Gen.* **30** L719–23

[382] Green H S 1953 A generalized method of field quantization *Phys. Rev.* **90** 270–3

[383] Greenberg O W and Messiah A M L 1965 Selection rules for parafields and the absence of para particles in nature *Phys. Rev.* **138** B1155–67

[384] Brandt R A and Greenberg O W 1969 Generalized Bose operators in the Fock space of a single Bose operator *J. Math. Phys.* **10** 1168–76

[385] Sharma J K, Mehta C L and Sudarshan E C G 1978 Para-Bose coherent states *J. Math. Phys.* **19** 2089–93

[386] Sharma J K, Mehta C L, Mukunda N and Sudarshan E C G 1981 Representation and properties of para-Bose oscillator operators: II. Coherent states and the minimum uncertainty states *J. Math. Phys.* **22** 78–90

[387] Brodimas G, Jannussis A, Sourlas D, Zisis V and Poulopoulos P 1981 Para-Bose operators *Lett. Nuovo Cimento* **31** 177–82

[388] Saxena G M and Mehta C L 1991 Para-squeezed states *J. Math. Phys.* **32** 783–6

[389] Shanta P, Chaturvedi S, Srinivasan V and Jagannathan R 1994 Unified approach to the analogues of single-photon and multiphoton coherent states for generalized bosonic oscillators *J. Phys. A: Math. Gen.* **27** 6433–42

[390] Bagchi B and Bhaumik D 1998 Squeezed states for parabosons *Mod. Phys. Lett.* A **13** 623–30

Jing S C and Nelson C A 1999 Eigenstates of paraparticle creation operators *J. Phys. A: Math. Gen.* **32** 401–9

[391] Iwata G 1951 Transformation functions in the complex domain *Prog. Theor. Phys.* **6** 524–8

[392] Arik M, Coon D D and Lam Y M 1975 Operator algebra of dual resonance models *J. Math. Phys.* **16** 1776–9

[393] Arik M and Coon D D 1976 Hilbert spaces of analytic functions and generalized coherent states *J. Math. Phys.* **17** 524–7

[394] Kuryshkin V V 1976 On some generalization of creation and annihilation operators in quantum theory (μ-quantization) *Deposit no* 3936–76 (Moscow: VINITI) (in Russian)

Kuryshkin V 1980 Opérateurs quantiques généralisés de création et d'annihilation *Ann. Fond. Louis Broglie* **5** 111–25

[395] Jannussis A, Brodimas G, Sourlas D and Zisis V 1981 Remarks on the q-quantization *Lett. Nuovo Cimento* **30** 123–7

[396] Gundzik M G 1966 A continuous representation of an indefinite metric space *J. Math. Phys.* **7** 641–51

Jaiswal A K and Mehta C L 1969 Phase space representation of pseudo oscillator *Phys. Lett.* A **29** 245–6

Agarwal G S 1970 Generalized phase-space distributions associated with a pseudo-oscillator *Nuovo Cimento* B **65** 266–79

[397] Biedenharn L C 1989 The quantum group $SU_q(2)$ and a q-analog of the boson operator *J. Phys. A: Math. Gen.* **22** L873–8

[398] Macfarlane A J 1989 On q-analogs of the quantum harmonic oscillator and the quantum group $SU(2)_q$ *J. Phys. A: Math. Gen.* **22** 4581–8

Kulish P P and Damaskinsky E V 1990 On the q-oscillator and the quantum algebra $su_q(1,1)$ *J. Phys. A: Math. Gen.* **23** L415–19

[399] Solomon A I and Katriel J 1990 On q-squeezed states *J. Phys. A: Math. Gen.* **23** L1209–12

Katriel J and Solomon A I 1991 Generalised q-bosons and their squeezed states *J. Phys. A: Math. Gen.* **24** 2093–105

[400] Celeghini E, Rasetti M and Vitiello G 1991 Squeezing and quantum groups *Phys. Rev. Lett.* **66** 2056–9

Chaturvedi S, Kapoor A K, Sandhya R, Srinivasan V and Simon R 1991 Generalized commutation relations for a single-mode oscillator *Phys. Rev.* A **43** 4555–7

Chiu S-H, Gray R W and Nelson C A 1992 The q-analog quantized radiation field and its uncertainty relations *Phys. Lett.* A **164** 237–42

[401] Atakishiev N M and Suslov S K 1990 Difference analogs of the harmonic oscillator *Teor. Mat. Fiz.* **85** 64–73 (English Transl. 1990 *Theor. Math. Phys.* **85** 1055–62)

[402] Fivel D I 1991 Quasi-coherent states and the spectral resolution of the q-Bose field operator *J. Phys. A: Math. Gen.* **24** 3575–86

[403] Chakrabarti R and Jagannathan R 1991 A (p, q)-oscillator realisation of two-parameter quantum algebras *J. Phys. A: Math. Gen.* **24** L711–18

[404] Arik M, Demircan E, Turgut T, Ekinci L and Mungan M 1992 Fibonacci oscillators *Z. Phys.* C **55** 89–95

[405] Wang F B and Kuang L M 1992 Even and odd q-coherent state representations of the quantum Heisenberg–Weyl algebra *Phys. Lett.* A **169** 225–8

[406] Jing S C and Fan H Y 1994 q-deformed binomial state *Phys. Rev.* A **49** 2277–9

[407] Solomon A I and Katriel J 1993 Multi-mode q-coherent states *J. Phys. A: Math. Gen.* **26** 5443–7

[408] Jurčo B 1991 On coherent states for the simplest quantum groups *Lett. Math. Phys.* **21** 51–8

Quesne C 1991 Coherent states, K-matrix theory and q-boson realizations of the quantum algebra $su_q(2)$ *Phys. Lett.* A **153** 303–7

Zhedanov A S 1991 Nonlinear shift of q-Bose operators and q-coherent states *J. Phys. A: Math. Gen.* **24** L1129–32

Chang Z 1992 Generalized Holstein–Primakoff realizations and quantum group-theoretic coherent states *J. Math. Phys.* **33** 3172–9

Wang F B and Kuang L M 1993 Even and odd q-coherent states and their optical statistics properties *J. Phys. A: Math. Gen.* **26** 293–300

Campos R A 1994 Interpolation between the wave and particle properties of bosons and fermions *Phys. Lett.* A **184** 173–8

Codriansky S 1994 Localized states in deformed quantum mechanics *Phys. Lett.* A **184** 381–4

Aref'eva I Y, Parthasarthy R, Viswanathan K S and Volovich I V 1994 Coherent states, dynamics and semiclassical limit of quantum groups *Mod. Phys. Lett.* A **9** 689–703

Celeghini E, De Martino S, De Siena S, Rasetti M and Vitiello G 1995 Quantum groups, coherent states, squeezing and lattice quantum mechanics *Ann. Phys., NY* **241** 50–67

Mann A and Parthasarathy R 1996 Minimum uncertainty states for the quantum group $SU_q(2)$ and quantum Wigner d-functions *J. Phys. A: Math. Gen.* **29** 427–35

Atakishiyev N M and Feinsilver P 1996 On the coherent states for the q-Hermite polynomials and related Fourier transformation *J. Phys. A: Math. Gen.* **29** 1659–64

Park S U 1996 Equivalence of q-bosons using the exponential phase operator *J. Phys. A: Math. Gen.* **29** 3683–96

Aniello P, Man'ko V, Marmo G, Solimeno S and Zaccaria F 2000 On the coherent states, displacement operators and quasidistributions associated with deformed quantum oscillators *J. Opt. B: Quantum Semiclass. Opt.* **2** 718–25

[409] Chaturvedi S and Srinivasan V 1991 Para-Bose oscillator as a deformed Bose oscillator *Phys. Rev.* A **44** 8024–6

[410] Fisher R A, Nieto M M and Sandberg V D 1984 Impossibility of naively generalizing squeezed coherent states *Phys. Rev.* D **29** 1107–10

[411] Braunstein S L and McLachlan R I 1987 Generalized squeezing *Phys. Rev.* A **35** 1659–67

[412] D'Ariano G, Rasetti M and Vadacchino M 1985 New type of two-photon squeezed coherent states *Phys. Rev.* D **32** 1034–7

Katriel J, Solomon A I, D'Ariano G and Rasetti M 1986 Multiboson Holstein–Primakoff squeezed states for $SU(2)$ and $SU(1,1)$ *Phys. Rev.* D **34** 2332–8

D'Ariano G and Rasetti M 1987 Non-Gaussian multiphoton squeezed states *Phys. Rev.* D **35** 1239–47

D'Ariano G, Morosi S, Rasetti M, Katriel J and Solomon A I 1987 Squeezing versus photon-number fluctuations *Phys. Rev.* D **36** 2399–407

[413] Katriel J, Rasetti M and Solomon A I 1987 Squeezed and coherent states of fractional photons *Phys. Rev.* D **35** 1248–54

D'Ariano G M and Sterpi N 1989 Statistical fractional-photon squeezed states *Phys. Rev.* A **39** 1860–8

D'Ariano G M 1990 Amplitude squeezing through photon fractioning *Phys. Rev.* A **41** 2636–44

D'Ariano G M 1992 Number-phase squeezed states and photon fractioning *Int. J. Mod. Phys.* B **6** 1291–354

[414] Bužek V and Jex I 1990 Amplitude kth-power squeezing of k-photon coherent states *Phys. Rev.* A **41** 4079–82

[415] Matos Filho R L and Vogel W 1996 Nonlinear coherent states *Phys. Rev.* A **54** 4560–3

[416] Man'ko V I, Marmo G, Sudarshan E C G and Zaccaria F 1997 f-oscillators and nonlinear coherent states *Phys. Scr.* **55** 528–41

[417] Sivakumar S 1999 Photon-added coherent states as nonlinear coherent states *J. Phys. A: Math. Gen.* **32** 3441–7

[418] Mancini S 1997 Even and odd nonlinear coherent states *Phys. Lett.* A **233** 291–6

[419] Sivakumar S 1998 Even and odd nonlinear coherent states *Phys. Lett.* A **250** 257–62

Roy B and Roy P 1999 Phase properties of even and odd nonlinear coherent states *Phys. Lett.* A **257** 264–8

Sivakumar S 2000 Generation of even and odd nonlinear coherent states *J. Phys. A: Math. Gen.* **33** 2289–97

[420] Manko O V 1997 Symplectic tomography of nonlinear coherent states of a trapped ion *Phys. Lett.* A **228** 29–35

[421] Man'ko V, Marmo G, Porzio A, Solimeno S and Zaccaria F 2000 Trapped ions in laser fields: a benchmark for deformed quantum oscillators *Phys. Rev.* A **62** 053407

[422] Roy B 1998 Nonclassical properties of the real and imaginary nonlinear Schrödinger cat states *Phys. Lett.* A **249** 25–9

Wang X G and Fu H C 1999 Negative binomial states of the radiation field and their excitations are nonlinear coherent states *Mod. Phys. Lett.* B **13** 617–23

Liu X M 1999 Orthonormalized eigenstates of the operator $(\hat{a} f(\hat{n}))^k$ $(k \geqslant 1)$ and their generation *J. Phys. A: Math. Gen.* **32** 8685–9

Wang X G 2000 Two-mode nonlinear coherent states *Opt. Commun.* **178** 365–9

[423] Sivakumar S 2000 Studies on nonlinear coherent states *J. Opt. B: Quantum Semiclass. Opt.* **2** R61–75

Fan H Y and Cheng H L 2001 Nonlinear conserved-charge coherent state and its relation to nonlinear entangled state *J. Phys. A: Math. Gen.* **34** 5987–94

Kis Z, Vogel W and Davidovich L 2001 Nonlinear coherent states of trapped-atom motion *Phys. Rev.* A **64** 033401

Quesne C 2001 Generalized coherent states associated with the C_λ-extended oscillator *Ann. Phys., NY* **293** 147–88

[424] Yuen H P 1986 Generation, detection, and application of high-intensity photon-number-eigenstate fields *Phys. Rev. Lett.* **56** 2176–9

Filipowicz P, Javanainen J and Meystre P 1986 Quantum and semiclassical steady states of a kicked cavity *J. Opt. Soc. Am* B **3** 906–10

Krause J, Scully M O and Walther H 1987 State reduction and $|n\rangle$-state preparation in a high-Q micromaser *Phys. Rev.* A **36** 4546–50

Holmes C A, Milburn G J and Walls D F 1989 Photon-number-state preparation in nondegenerate parametric amplification *Phys. Rev.* A **39** 2493–501

[425] Vogel K, Akulin V M and Schleich W P 1993 Quantum state engineering of the radiation field *Phys. Rev. Lett.* **71** 1816–19

[426] Parkins A S, Marte P, Zoller P and Kimble H J 1993 Synthesis of arbitrary quantum states via adiabatic transfer of Zeeman coherence *Phys. Rev. Lett.* **71** 3095–8

Domokos P, Janszky J and Adam P 1994 Single-atom interference method for generating Fock states *Phys. Rev.* A **50** 3340–4

Law C K and Eberly J H 1996 Arbitrary control of a quantum electromagnetic field *Phys. Rev. Lett.* **76** 1055–8
Brattke S, Varcoe B T H and Walther H 2001 Preparing Fock states in the micromaser *Opt. Express* **8** 131–44
[427] Brune M, Haroche S, Raimond J M, Davidovich L and Zagury N 1992 Manipulation of photons in a cavity by dispersive atom-field coupling. Quantum-nondemolition measurements and generation of 'Schrödinger cat' states *Phys. Rev.* A **45** 5193–214
Tara K, Agarwal G S and Chaturvedi S 1993 Production of Schrödinger macroscopic quantum-superposition states in a Kerr medium *Phys. Rev.* A **47** 5024–9
Gerry C C 1996 Generation of four-photon coherent states in dispersive cavity QED *Phys. Rev.* A **53** 3818–21
de Matos Filho R L and Vogel W 1996 Even and odd coherent states of the motion of a trapped ion *Phys. Rev. Lett.* **76** 608–11
Agarwal G S, Puri R R and Singh R P 1997 Atomic Schrödinger cat states *Phys. Rev.* A **56** 2249–54
Gerry C C and Grobe R 1997 Generation and properties of collective atomic Schrödinger-cat states *Phys. Rev.* A **56** 2390–6
Delgado A, Klimov A B, Retamal J C and Saavedra C 1998 Macroscopic field superpositions from collective interactions *Phys. Rev.* A **58** 655–62
Moya-Cessa H, Wallentowitz S and Vogel W 1999 Quantum-state engineering of a trapped ion by coherent-state superpositions *Phys. Rev.* A **59** 2920–5
[428] Leonhardt U 1993 Quantum statistics of a lossless beam splitter: $SU(2)$ symmetry in phase space *Phys. Rev.* A **48** 3265–77
Ban M 1996 Photon statistics of conditional output states of lossless beam splitter *J. Mod. Opt.* **43** 1281–303
Dakna M, Anhut T, Opatrný T, Knöll L and Welsch D-G 1997 Generating Schrödinger-cat-like states by means of conditional measurements on a beam splitter *Phys. Rev.* A **55** 3184–94
[429] Baseia B, Moussa M H Y and Bagnato V S 1998 Hole burning in Fock space *Phys. Lett.* A **240** 277–81
[430] Mancini S, Man'ko V I and Tombesi P 1997 Ponderomotive control of quantum macroscopic coherence *Phys. Rev.* A **55** 3042–50
Bose S, Jacobs K and Knight P L 1997 Preparation of nonclassical states in cavities with a moving mirror *Phys. Rev.* A **56** 4175–86
Zheng S-B 1998 Preparation of even and odd coherent states in the motion of a cavity mirror *Quantum Semiclass. Opt.* **10** 657–60
[431] Dodonov V V 2001 Nonstationary Casimir effect and analytical solutions for quantum fields in cavities with moving boundaries *Modern Nonlinear Optics, Advances in Chemical Physics* vol 119, Part 1, 2nd edn, ed M W Evans (New York: Wiley) pp 309–94
[432] Haroche S 1995 Mesoscopic coherences in cavity QED *Nuovo Cimento* B **110** 545–56
[433] Meekhof D M, Monroe C, King B E, Itano W M and Wineland D J 1996 Generation of nonclassical motional states of a trapped atom *Phys. Rev. Lett.* **76** 1796–9
Monroe C, Meekhof D M, King B E and Wineland D J 1996 A 'Schrödinger cat' superposition state of an atom *Science* **272** 1131–6
[434] Schleich W and Rempe G (ed) 1995 *Fundamental Systems in Quantum Optics, Appl. Phys.* B **60** N 2/3 (Suppl.) S1–S265
Cirac J I, Parkins A S, Blatt R and Zoller P 1996 Nonclassical states of motion in ion traps *Adv. At. Mol. Opt. Phys.* **37** 237–96
Davidovich L 1996 Sub-Poissonian processes in quantum optics *Rev. Mod. Phys.* **68** 127–73
Karlsson E B and Brändas E (ed) 1998 *Modern studies of basic quantum concepts and phenomena Phys. Scr.* T **76** 1–232
Malbouisson J M C and Baseia B 1999 Higher-generation Schrödinger cat states in cavity QED *J. Mod. Opt.* **46** 2015–41
Massoni E and Orszag M 2000 Phonon–photon translator *Opt. Commun.* **179** 315–21
Lutterbach L G and Davidovich L 2000 Production and detection of highly squeezed states in cavity QED *Phys. Rev.* A **61** 023813
[435] Wineland D J, Monroe C, Itano W M, Leibfried D, King B E and Meekhof D M 1998 Experimental issues in coherent quantum-state manipulation of trapped atomic ions *J. Res. Natl. Inst. Stand. Technol.* **103** 259–328
[436] Leonhardt U and Paul H 1995 Measuring the quantum state of light *Prog. Quantum Electron.* **19** 89–130
Bužek V, Adam G and Drobný G 1996 Reconstruction of Wigner functions on different observation levels *Ann. Phys., NY* **245** 37–97
Breitenbach G and Schiller S 1997 Homodyne tomography of classical and non-classical light *J. Mod. Opt.* **44** 2207-25
Mlynek J, Rempe G, Schiller S and Wilkens M (ed) 1997 *Quantum Nondemolition Measurements, Appl. Phys.* B **64** N2 (special issue)
Schleich W P and Raymer M G (ed) 1997 *Quantum State Preparation and Measurement, J. Mod. Opt.* **44** N11/12 (special issue)
[437] Leonhardt U 1997 *Measuring the Quantum State of Light* (Cambridge: Cambridge University Press)
[438] Welsch D-G, Vogel W and Opatrný T 1999 Homodyne detection and quantum state reconstruction *Progress in Optics* vol 39, ed E Wolf (Amsterdam: North-Holland) 63–211
Bužek V, Drobny G, Derka R, Adam G and Wiedemann H 1999 Quantum state reconstruction from incomplete data *Chaos Solitons Fractols* **10** 981–1074
Santos M F, Lutterbach L G, Dutra S M, Zagury N and Davidovich L 2001 Reconstruction of the state of the radiation field in a cavity through measurements of the outgoing field *Phys. Rev.* A **63** 033813
[439] Garraway B M, Sherman B, Moya-Cessa H, Knight P L and Kurizki G 1994 Generation and detection of nonclassical field states by conditional measurements following two-photon resonant interactions *Phys. Rev.* A **49** 535–47
Peřinová V and Lukš A 2000 Continuous measurements in quantum optics *Progress in Optics* vol 40, ed E Wolf (Amsterdam: North-Holland) 115–269
[440] Cirac J I, Parkins A S, Blatt R and Zoller P 1993 Dark squeezed states of the motion of a trapped ion *Phys. Rev. Lett.* **70** 556–9
[441] Arimondo E 1996 Coherent population trapping in laser spectroscopy *Progress in Optics* vol 35, ed E Wolf (Amsterdam: North-Holland) pp 257–354
[442] Wynands R and Nagel A 1999 Precision spectroscopy with coherent dark states *Appl. Phys.* B **68** 1–25
Kis Z, Vogel W, Davidovich L and Zagury N 2001 Dark $SU(2)$ states of the motion of a trapped ion *Phys. Rev.* A **63** 053410
[443] Greenberger D M, Horne M A, Shimony A and Zeilingler A 1990 Bell's theorem without inequalities *Am. J. Phys.* **58** 1131–43
[444] Englert B-G and Walther H 2000 Preparing a GHZ state, or an EPR state, with the one-atom maser *Opt. Commun.* **179** 283–8
[445] Bambah B A and Satyanarayana M V 1986 Squeezed coherent states and hadronic multiplicity distributions *Prog. Theor. Phys. Suppl.* **86** 377–82
Shih C C 1986 Sub-Poissonian distribution in hadronic processes *Phys. Rev.* D **34** 2720–6
Vourdas A and Weiner R M 1988 Multiplicity distributions and Bose–Einstein correlations in high-energy multiparticle production in the presence of squeezed coherent states *Phys. Rev.* D **38** 2209–17
Ruijgrok T W 1992 Squeezing and $SU(3)$-invariance in multiparticle production *Acta Phys. Pol.* B **23** 629–35

Dremin I M and Hwa R C 1996 Multiplicity distributions of squeezed isospin states *Phys. Rev.* D **53** 1216–23
Dremin I M and Man'ko V I 1998 Particles and nuclei as quantum slings *Nuovo Cimento* A **111** 439–44
Dodonov V V, Dremin I M, Man'ko O V, Man'ko V I and Polynkin P G 1998 Nonclassical field states in quantum optics and particle physics *J. Russ. Laser Res.* **19** 427–63
[446] Sidorov Y V 1989 Quantum state of gravitons in expanding Universe *Europhys. Lett.* **10** 415–18
Grishchuk L P, Haus H A and Bergman K 1992 Generation of squeezed radiation from vacuum in the cosmos and the laboratory *Phys. Rev.* D **46** 1440–9
Albrecht A, Ferreira P, Joyce M and Prokopec T 1994 Inflation and squeezed quantum states *Phys. Rev.* D **50** 4807–20
Hu B L, Kang G and Matacz A 1994 Squeezed vacua and the quantum statistics of cosmological particle creation *Int. J. Mod. Phys.* A **9** 991–1007
Finelli F, Vacca G P and Venturi G 1998 Chaotic inflation from a scalar field in nonclassical states *Phys. Rev.* D **58** 103514
[447] Dodonov V V, Man'ko O V and Man'ko V I 1989 Correlated states in quantum electronics (resonant circuit) *J. Sov. Laser Research* **10** 413–20
Pavlov S T and Prokhorov A V 1991 Correlated and compressed states in a parametrized Josephson junction *Sov. Phys.–Solid State* **33** 1384–6
Man'ko O V 1994 Correlated squeezed states of a Josephson junction *J. Korean. Phys. Soc.* **27** 1–4
Vourdas A and Spiller T P 1997 Quantum theory of the interaction of Josephson junctions with non-classical microwaves *Z. Phys.* B **102** 43–54
Shao B, Zou J and Li Q-S 1998 Squeezing properties in mesoscopic Josephson junction with nonclassical radiation field *Mod. Phys. Lett.* B **12** 623–31
Zou J and Shao B 1999 Superpositions of coherent states and squeezing effects in a mesoscopic Josephson junction *Int. J. Mod. Phys.* B **13** 917–24
[448] Feinberg D, Ciuchi S and de Pasquale F 1990 Squeezing phenomena in interacting electron–phonon systems *Int. J. Mod. Phys.* B **4** 1317–67
An N B 1991 Squeezed excitons in semiconductors *Mod. Phys. Lett.* B **5** 587–91
Artoni M and Birman J L 1991 Quantum optical properties of polariton waves *Phys. Rev.* B **44** 3736–56
De Melo C A R S 1991 Squeezed boson states in condensed matter *Phys. Rev.* B **44** 11 911–17
Artoni M and Birman J L 1994 Non-classical states in solids and detection *Opt. Commun.* **104** 319–24
Sonnek M and Wagner M 1996 Squeezed oscillatory states in extended exciton–phonon systems *Phys. Rev.* B **53** 3190–202
Hu X D and Nori F 1996 Squeezed phonon states: modulating quantum fluctuations of atomic displacements *Phys. Rev. Lett.* **76** 2294–7
Garrett G A, Rojo A G, Sood A K, Whitaker J F and Merlin R 1997 Vacuum squeezing of solids; macroscopic quantum states driven by light pulses *Science* **275** 1638-40
Chai J-H and Guo G-C 1997 Preparation of squeezed-state phonons using the Raman-induced Kerr effect *Quantum Semiclass. Opt.* **9** 921–7
Artoni M 1998 Detecting phonon vacuum squeezing *J. Nonlinear Opt. Phys. Mater.* **7** 241–54
Hu X D and Nori F 1999 Phonon squeezed states: quantum noise reduction in solids *Physica* B **263** 16–29
[449] Solomon A I and Penson K A 1998 Coherent pairing states for the Hubbard model *J. Phys. A: Math. Gen.* **31** L355–60
Altanhan T and Bilge S 1999 Squeezed spin states and Heisenberg interaction *J. Phys. A: Math. Gen.* **32** 115–21
[450] Vinogradov A V and Janszky J 1991 Excitation of squeezed vibrational wave packets associated with Franck–Condon transitions in molecules *Sov. Phys.–JETP* **73** 211–17
Janszky J, Vinogradov A V, Kobayashi T and Kis Z 1994 Vibrational Schrödinger-cat states *Phys. Rev.* A **50** 1777–84
Davidovich L, Orszag M and Zagury N 1998 Quantum diagnosis of molecules: a method for measuring directly the Wigner function of a molecular vibrational state *Phys. Rev.* A **57** 2544–9
[451] Cirac J I, Lewenstein M, Mølmer K and Zoller P 1998 Quantum superposition states of Bose–Einstein condensates *Phys. Rev.* A **57** 1208–18