Direct measurement of the Wigner distribution of a traveling field

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(Received 21 June 2001; published 13 November 2001)

We present a technique for direct measurement of the Wigner distribution function of a traveling field, based on a Mach-Zehnder interferometer with a phase-sensitive element. In our scheme, the problem arising from the non-unity efficiency of the photodetectors is circumvented by the optical intensity of the input field for interferometry.

DOI: 10.1103/PhysRevA.64.065801 PACS number(s): 42.50.Dv, 42.50.Ct, 32.80.Pj, 42.50.Vk

Since the pioneering intensity-correlation stellar interferometer of Hanbury-Brown and Twiss [1], for observing higher-order coherence of a single mode of the radiation field, a broad class of techniques has been developed for measuring properties of the quantum states of light and matter. Earlier investigations of the photon number (or the phase) statistics of the radiation field and collective effects of atomic samples, such as superradiance, photon echoes, and self-induced transparency [2–4], have provided the expertise for some striking accomplishments of present-day physics. Experiments in ionic traps and quantum electrodynamics (with traveling fields manipulated by beam-splitter arrays and stationary fields in high-Q cavities), where the preparation of nonclassical states [5] is pursued for teleportation [6] and logic operations [7], seem to inaugurate a technology based on fundamental quantum phenomena, such as nonlocality and quantum state collapse. In addition to the possibility of the direct measurement of properties of quantum states, the current improvements in experimental techniques have even provided schemes for measuring quasiprobability distributions such as the Glauber-Sudarshan P function, the Q function, and the Wigner function of a state. A method for obtaining the quasiprobability distributions by tomographic inversion of a set of measured probability distributions of quadrature amplitudes was first put forward in [8]. Since then, optical homodyne tomography has been applied successfully to reconstruct experimentally the Wigner function of vacuum and squeezed states of light [9]. Without using homodyne detection, a method for reconstruction of the Wigner function, relying on a convolution of data obtained by photon counting, is presented in [10].

Different from the above-mentioned schemes that reconstruct the Wigner function, in [11], it is shown how this function may be directly measured, in a simple scheme suitable for experiments in cavity quantum electrodynamics and ionic traps. Direct measurement of the Wigner function of the vacuum state, a weak coherent state and a phase diffused coherent state was recently reported by Banaszek *et al.* [12]. The experimental scheme is based on the representation of the Wigner function as an expectation value of a displaced photon number parity operator.

In this paper, we describe a technique for the direct measurement of the Wigner function of a traveling field, adapted from a scheme for quantum nondemolition measurement of photon number via the optical Kerr effect [13]. As in [11], our method also provides a direct physical interpretation of the Wigner distribution, and the problem arising from the nonunity efficiency of the required photodetectors is circumvented by the optical intensity of the input field for interferomentery. Our experimental setup, shown in Fig. 1, consists basically of a Mach-Zehnder interferometer (a pair of 50-50 symmetric beam splitters, BS_1 and BS_2, and a pair of photodetectors, D_1 and D_2) with a phase-sensitive element in the upper arm (consisting of a Kerr medium). A coherent field $|\beta\rangle_a$, in the input mode a, is incident on BS_1. Simultaneously, mode A, prepared in state $|\psi\rangle_A$, whose Wigner function is to be measured, and mode B prepared in a strong coherent state $|\delta\rangle_B$, are superimposed by a symmetric beam splitter BS_3, with transmittance T near unity and reflectance R, in order to produce a displaced signal-field state $|\Psi\rangle_s = \hat{D}(\zeta)|\psi\rangle_A$, in which $\zeta = R\delta$ [14] [with $\hat{D}(\zeta) = \exp(\zeta\hat{a}_s^\dagger - \zeta^*\hat{a}_s)$ denoting the displacement operator]. The signal field $|\Psi\rangle_s$ leaving BS_3 and the probe field a_p leaving BS_1 are coupled through the Kerr medium, where the evolution of their states determined by the unitary operator $\hat{U}_{Kerr} = \exp(i\chi\tau\hat{a}_s^\dagger\hat{a}_s\hat{a}_p^\dagger\hat{a}_p)$ [15]. Finally, two-port homodyne detection, which balances the output modes c and d from BS_2, is achieved by photodetection [4] and the output signal is de-

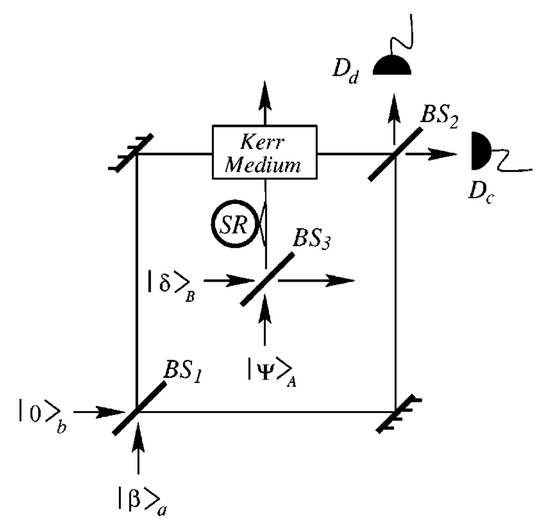

FIG. 1. Sketch of the experimental setup for a direct measurement of the Wigner function of a traveling field. In addition to the beam splitters BS_1, BS_2, and BS_3, the Kerr medium and detectors D_c and D_d, the figure shows a storage ring SR for matching the modes A and a in the Kerr medium.

termined by the number operator $\hat{n}_{cd}=\hat{d}^\dagger\hat{d}-\hat{c}^\dagger\hat{c}$.

The general relationships between the input and output operators $\hat{\alpha},\hat{\beta}$, arising from the unitary operator $\hat{U}=\exp[i\theta(\hat{\alpha}_{in}^\dagger\hat{\beta}_{in}+\hat{\beta}_{in}^\dagger\hat{\alpha}_{in})]$ describing the action of an ideal BS, are written as

$$\hat{\alpha}_{out}=T\hat{\alpha}_{in}+R\hat{\beta}_{in}, \tag{1a}$$

$$\hat{\beta}_{out}=T\hat{\beta}_{in}+R\alpha_{in}, \tag{1b}$$

where $T=\cos(\theta)$ and $R=i\sin(\theta)$ are the beam-splitter transmission and reflection coefficients. (For a 50-50 symmetric beam splitter, where $\theta=\pi/4$, $T=|R|=1/\sqrt{2}$.) These coefficients, and so the operators, depend on the frequency of the fields and here a monochromatic source is considered. As the input field b is the vacuum state, it is easily verified from Eqs. (1a) and (1b) the relation

$$\begin{aligned}\langle\hat{n}_{cd}\rangle&=\eta\langle\hat{a}_p^\dagger\hat{a}_p\rangle\langle\cos[(\chi\tau)\hat{a}_s^\dagger\hat{a}_s]\rangle\\&=\eta|\beta|^2\sum_n e^{in\pi}\langle n|\hat{D}(\zeta)\hat{\rho}_s\hat{D}^{-1}(\zeta)|n\rangle,\end{aligned} \tag{2}$$

where the interaction paramenter in the Kerr medium has been adjusted so that $\chi\tau=\pi$ [16], $\rho_s=|\psi\rangle_{ss}\langle\psi|$ and η indicates the efficiency of the photodetectors included in the calculations through Langevin operators. We note that the efficiency factor η attenuates the measured intensity $\langle\hat{n}_{cd}\rangle$, but this can be circumvented by increasing the intensity of the input state $|\beta\rangle_a$. Introducing output fields α_{out} (modes c and d), allowing for the efficiency of detection of the input fields α_{in} (probe and reference modes a_p' and a_r) reaching the photodetectors, we write [17]

$$\hat{\alpha}_{out}=\sqrt{\eta}\hat{\alpha}_{in}+\hat{\mathcal{L}}_\alpha. \tag{3}$$

The input fields and the noise sources are required to be independent so that the input operators must commute with the output Langevin operators, i.e., $[\hat{\alpha}_{in},\hat{\mathcal{L}}_\alpha]=[\hat{\alpha}_{in},\hat{\mathcal{L}}_\alpha^\dagger]=0$. Imposition of the bosonic commutation relations on the output operators then leads to the noise-operator commutation relation $[\hat{\mathcal{L}}_\alpha,\hat{\mathcal{L}}_\alpha^\dagger]=1-\eta$. For optical frequencies, the photodetector states may be very well approximated by the vacuum state even at room temperature, so that $\hat{\mathcal{L}}_\alpha|0\rangle=\hat{\alpha}_{in}|0\rangle=0$ and, from the input-output relation (3), it also follows that $\hat{\alpha}_{out}|0\rangle=0$. Finally, the quantum averages of the Langevin operators vanish, $\langle\hat{\mathcal{L}}_\alpha\rangle=\langle\hat{\mathcal{L}}_\alpha^\dagger\rangle=0$, and the only nonzero ground-state expectation value for the products of pairs of noise operators is $\langle\hat{\mathcal{L}}_\alpha\hat{\mathcal{L}}_\alpha^\dagger\rangle=1-\eta$. We note that we have not considered losses in the beam splitters, since their damping constants are very small, less than 2% in BK7 crystals, while the efficiency for single-photon detectors is about 70%. However, we note that the error sources arising from both photoabsorption in the beam splitters and the efficiency of detectors may easily be included by following the reasoning in Ref. [17].

Next, we observe that the Wigner distribution function, whichever the input state $|\psi\rangle_A$ described by the density operator ρ_A, may be written as [11,18]

$$\begin{aligned}W(\zeta)&=2\mathrm{Tr}(\hat{\rho}_A\hat{D}(\zeta)e^{i\pi\hat{A}^\dagger\hat{A}}\hat{D}^{-1}(\zeta))\\&=2\sum_n e^{in\pi}\langle n|\hat{D}(\zeta)\hat{\rho}_s\hat{D}^{-1}(\zeta)|n\rangle,\end{aligned} \tag{4}$$

which, apart from constant factors, turns out to be the output signal for balanced homodyne detection, i.e.,

$$W(\zeta)=\frac{2}{\eta|\beta|^2}\langle\hat{n}_{cd}\rangle. \tag{5}$$

So, by measuring $\langle\hat{n}_{cd}\rangle$, one may obtain the value of the Wigner function at a given point $\zeta(=R\delta)$ of phase space, defined by the amplitude and phase of the coherent field $|\delta\rangle_B$ injected through BS_3. By scanning the phase-space point by point, we obtain the Wigner function of the desired state.

Assuming perfect photodetectors ($\eta=1$) and substituting the convenient coherent field $|\beta\rangle_a$ injected through the input mode a on BS_1 by a single-photon field $|1\rangle_a$, a result is obtained that is similar to that obtained in [11] for direct measurement of the Wigner function in CQED and trapped-ion experiments. As stressed in [11], such a result gives a physical meaning to the well-known property of the Wigner function that its value is limited between $+2$ and -2. In fact, in this case, the Wigner function is given by the difference between the probabilities P_c and P_d of detecting the photon $|1\rangle_a$ in modes c or d, i.e., $W(\zeta)=2(P_c-P_d)$. We observe that the Wigner function will not need to be modified when considering the real situation of nonideal detectors. In this situation, we just neglect the events where the photon $|1\rangle_a$ is absorbed or scattered by the detectors, since they do not contribute to probabilities P_c and P_d, differently from Banaszek *et al.* [12], in whose scheme the photocounting plays a crucial role.

In addition to the above-mentioned errors arising from the imperfect photodetection and the absorptive beam splitters, there are other experimental nonidealities that seem to be important. In fact, the two modes interfering at the beam splitters are never matched perfectly and the effects of the mode mismatch may be discussed within the multimode theory [12]. Moreover, the noise arising from the local oscillator $|\delta\rangle_B$ must be taken into account [19] and when considering a single-photon field $|1\rangle_a$ instead of the more convenient coherent field $|\beta\rangle_a$, we have to account for the fact that we do not have perfect single-photon sources. The commonly cited method of parametric fluorescence only approximates a single-photon source, and this approximation must be evaluated for its effect on the Wigner function measurement. The interaction between both light pulses crossing the Kerr medium is also subject to errors associated to intensity-fluctuating coherent probe beam. However, when considering light pulses of tiny energies crossing the Kerr medium, the method proposed in Ref. [16] for achieving a nonlinear phase shift on the order of π permits the interaction to be maintained for a very long time without dissipation. Finally, it is worth mentioning that the errors arising from fluctuating parameters could be taken into account through the recent proposed phenomenological-operator technique [20].

In summary, we have proposed a feasible method for a direct measurement of the Wigner function of a traveling field based on a scheme for the quantum nondemolition measurement of photon number via the optical Kerr effect [13]. In spite of all error sources common to the other reported techniques for measuring the Wigner distribution of a traveling field, in our scheme, the problem arising from the nonunity efficiency of the photodetectors may be circumvented by the optical intensity of the coherent input field for interferometry (or even eliminated at the expense of substituting this coherent input field by an nonideal single-photon field).

We wish to thank the CNPq and FAPESP, Brazilian support agencies.

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