

Discrete teleportation protocol of continuum spectra field states

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A discrete protocol for teleportation of superpositions of coherent states of optical-cavity fields is presented. Displacement and parity operators are unconventionally used in Bell-like measurement for field states.

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I. INTRODUCTION

When two spatially delocalized parties A and B share a pair of entangled systems, a “quantum channel” is established allowing information transfer (the state of a third party owned by A) from one party to the other party (A to B). Such a process is well known as teleportation [1]. In fact the information about the third party state achieved by B through the quantum channel is maximal, originating an ambiguity of all possible outcomes, i.e., all allowed states spanning the Hilbert space of the object (system) whose state is to be teleported. The addition of a classical channel reduces the ambiguity. With two bits of classical information sent from party A to party B , the latter party can decide which unitary operation to apply in their physical-system state to recover the state of the system to be teleported. The great concern of Einstein, Podolsky, and Rosen [2] on superluminal information transfer when a nonlocal entangled state, is established, is not manifested by the need of a classical channel, which validate the available information achieved through the quantum channel.

One can describe systematically the necessary elements for efficient teleportation: (i) a quantum channel, i.e., a pair of nonlocally entangled systems; (ii) A Bell-joint measurement of two simultaneously observable quantities [3]; and (iii) a classical channel to transfer the information obtained in the Bell measurement.

Subjected to these conditions many proposals have been made, following the original proposal of Bennett *et al.* [1] considering dichotomic variables. It was first demonstrated experimentally by Bouwmeester *et al.* [4] in a remarkable achievement, in which a four photon coincident detection is necessary for a reasonable photon polarization-state teleportation. More recently much effort has been directed to teleportation of states with continuous spectrum, which basically reduces to a Wigner function representation of states and their respective reconstruction. Braunstein and Kimble [5] proposed it theoretically, and soon after it was achieved experimentally [6].

It is interesting to consider the extension of these ideas to more complicated systems. For example, is every entangled source sufficient to establish an efficient quantum channel; and more important, for any system can a Bell-like measurement be made. Given the existence and control of an en-

tangled pair, the search for simultaneous observables may be a challenge. It is important to notice, that although the original Bennett *et al.* [1] protocol was based on a complete Bell-state measurement, it was only recently achieved experimentally [7] and as far as we know, no real complete implementation of the Bennett protocol has been achieved up to now. Another interesting question, raised first by Popescu [8], which we discuss elsewhere [9] is: What is the exact relation between Bell’s inequalities violation and teleportation? Or in other words: Are there states that violate Bell’s inequalities but cannot be used for teleportation?

In this paper, we pursue questions related to Bell-state measurements. We study a specific system that allows teleportation of the state of a simple harmonic oscillator, but through a protocol resembling closely the dichotomic variable protocol proposed by Bennett *et al.* [1]. Other attempts were made on the discrete formulation of teleportation of oscillator states [10–12], however, these are distinct from the original Bennett *et al.* protocol.

The element to be teleported is a quantum field state. Entanglement of field states can be obtained in optical networks when a nonlinear optical element is present or by linear optical elements when one of the fields is prepared initially in a nonclassical state [13]. Recently, Milburn and Braunstein have addressed the problem of teleportation when a pair of entangled photons is generated in a parametric down-conversion scheme through a two-mode squeezed state [11]. Here instead, the observed field-field correlation for conditioned phase shift in optical cavities [14] is used as a quantum resource.

Birefringence measurements of a single atom strongly coupled to a high-finesse optical resonator were reported [14], with nonlinear phase shifts observed in phase and probe fields for intracavity photon number much less than one. The measured conditional phase shifts were then proposed to be utilized for implementing quantum logic as a quantum-phase gate. The possibility of using these entangled states for teleportation is here analyzed. For that we consider a model Hamiltonian to account for the conditional phase shift and analyze the dynamics of the two-mode states. A realizable setup based on homodyne measurements is proposed for the teleportation of superpositions of coherent states.

This paper is organized as follows. In Sec. II we present the generation of a coherent entangled state of two fields and analyze its dynamical structure. In Sec. III we analyze which kind of operators can be used for simultaneous measurement; more specifically we encounter product combinations of dis-

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placement and parity operators acting as the Bell operators. In Sec. IV we analyze the discrete protocol for teleportation of superpositions of coherent states. In Sec. VI we discuss the meaning of parity and displacement measurements and propose a setup where the Bell-state measurement is considerably simplified to homodyne detection if the target and Alice mode are first entangled. In Sec. VII we present a conclusion enclosing the paper.

II. CONDITIONAL DYNAMICS

Let us consider the dynamics generated by the following Hamiltonian:

$$H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b + \hbar \chi a^\dagger a b^\dagger b, \quad (1)$$

where a and b are annihilation operators for two distinct harmonic-oscillator modes, respectively and χ is a coupling constant. Such a Hamiltonian for optical systems describes a four-wave mixing process, when the constant χ is then proportional to the third-order susceptibility [15,16]. It can also, for instance, describe two distinct modes interaction in Bose condensates [18]. For our purpose here, it describes the effective interaction of output pump and probe fields of an optical-cavity mediated by a two-level atom, in the dispersive limit. A strong field-field coupling at the few photons limit, induced by nonresonant interactions between the fields and Cs atom beams was observed experimentally [14]. An alternative scheme based on adiabatic transformation of a nonlinear Hamiltonian is described in Cochrane, Milburn, and Munro [17]. If the pumping and probe fields are prepared in coherent states, $|\alpha\rangle_a$ and $|\beta\rangle_b$, respectively, the evolution operator $U(t) = e^{-iHt/\hbar}$ acts over these states as

$$\begin{aligned} |\psi(t)\rangle &= U(t)|\alpha\rangle_a|\beta\rangle_b \\ &= e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{(\alpha e^{-i\omega_a t})^m}{\sqrt{m!}} |m\rangle_a |\beta e^{-i\omega_b t} e^{-i\chi m t}\rangle_b, \end{aligned} \quad (2)$$

which for $t = \pi/\chi$, turns out to be the entangled state given by

$$\begin{aligned} |\psi(\pi/\chi)\rangle &\equiv |\alpha_+ e^{-i\pi\omega_a/\chi}\rangle_a |\beta e^{-i\pi\omega_b/\chi}\rangle_b + |\alpha_- e^{-i\pi\omega_a/\chi}\rangle_a \\ &\quad - \beta e^{-i\pi\omega_b/\chi}\rangle_b = |\alpha e^{-i\pi\omega_a/\chi}\rangle_a |\beta_+ e^{-i\pi\omega_b/\chi}\rangle_b \\ &\quad + |-\alpha e^{-i\pi\omega_a/\chi}\rangle_a |\beta_- e^{-i\pi\omega_b/\chi}\rangle_b, \end{aligned} \quad (3)$$

where $|\lambda_\pm e^{-i\pi\omega_l/\chi}\rangle_l = (|\lambda e^{-i\pi\omega_l/\chi}\rangle_l \pm |-\lambda e^{-i\pi\omega_l/\chi}\rangle_l)/2$ for $l = a, b$ and $\lambda = \alpha, \beta$, respectively. Choosing properly the modes frequency, ω_a and ω_b a set of approximately orthogonal states can be generated that is summarized as follows:

ω_a	ω_b	$ \psi(\pi/\chi)\rangle$
2χ	2χ	$ \Phi_+\rangle$
2χ	χ	$ \Phi_-\rangle$
χ	2χ	$ \Psi_+\rangle$
χ	χ	$ \Psi_-\rangle$

where the final states given in the third column are

$$|\Phi_\pm\rangle = |\alpha\rangle_a |\beta_\pm\rangle_b \pm |-\alpha\rangle_a |\beta_\mp\rangle_b, \quad (4)$$

$$|\Psi_\pm\rangle = |\alpha\rangle_a |\beta_\mp\rangle_b \pm |-\alpha\rangle_a |\beta_\pm\rangle_b. \quad (5)$$

Notice that with a reformulation of the last set of states they are written, respectively, as

$$|\Phi_+\rangle = |\beta\rangle_a |\alpha_+\rangle_b + |-\beta\rangle_a |\alpha_-\rangle_b = |\Phi'_+\rangle, \quad (6)$$

$$|\Phi_-\rangle = |\beta\rangle_a |\alpha_-\rangle_b + |-\beta\rangle_a |\alpha_+\rangle_b = |\Psi'_+\rangle, \quad (7)$$

$$|\Psi_+\rangle = |\beta\rangle_a |\beta_+\rangle_b - |-\beta\rangle_a |\alpha_-\rangle_b = |\Phi'_-\rangle, \quad (8)$$

$$|\Psi_-\rangle = |\beta\rangle_a |\alpha_-\rangle_b - |-\beta\rangle_a |\alpha_+\rangle_b = |\Psi'_-\rangle, \quad (9)$$

i.e., if we permute the order and rewrite the state, $|\Phi_+\rangle$ and $|\Psi_-\rangle$ show perfect symmetry, while $|\Phi_-\rangle$ goes to $|\Psi'_+\rangle$ and $|\Psi_+\rangle$ goes to $|\Phi'_-\rangle$. This asymmetry differentiates this kind of state from qubits written in Bell basis [1,5]. Another point is that, actually these states are not perfectly orthogonal, but this can be remedied if we take large-amplitude fields, $|\alpha|, |\beta| \gg 1$. To shorten the notation, from now on, we will specify the states $|\lambda_\pm\rangle_l$ as $|\pm\rangle_l$.

III. PARITY AND DISPLACEMENT OPERATORS MEASUREMENTS AS RESOURCES FOR TELEPORTATION

Our goal is to find a set of simultaneous observables for the state $|\Phi_\pm\rangle$ and $|\Psi_\pm\rangle$. It turns out that these operators are exactly the displacement and parity operators. It may be interesting to notice that displacement and parity operators have already been combined in the literature as an alternative definition of the Wigner function [19,20]. It is straightforward to check that the parity operators $P_a = e^{i\pi a^\dagger a}$ and $P_b = e^{i\pi b^\dagger b}$ act as

$$P_a |\Phi_\pm\rangle = \pm |\Psi_\pm\rangle, \quad (10)$$

$$P_a |\Psi_\pm\rangle = \pm |\Phi_\pm\rangle, \quad (11)$$

$$P_b |\Phi_\pm\rangle = |\Phi_\mp\rangle, \quad (12)$$

$$P_b |\Psi_\pm\rangle = -|\Psi_\mp\rangle. \quad (13)$$

The parity operator by itself cannot be used for our purposes, since the above states are not its eigenvectors. We can, however, build another set of operators, observing that the displacement operator $D_a(\epsilon) = e^{\epsilon a^\dagger - \epsilon^* a}$ acts on $|\Phi_\pm\rangle$ as

$$\begin{aligned} D_a(\epsilon) |\Phi_\pm\rangle &= e^{i\text{Im}(\epsilon\alpha^*)} |\alpha + \epsilon\rangle_a | \rangle_b \\ &\quad \pm e^{-i\text{Im}(\epsilon\alpha^*)} |-\alpha + \epsilon\rangle_a |-\rangle_b \\ &= \cos[\text{Im}(\epsilon\alpha^*)] (|\alpha + \epsilon\rangle_a | \rangle_b \pm |-\alpha \\ &\quad + \epsilon\rangle_a |-\rangle_b) + i \sin[\text{Im}(\epsilon\alpha^*)] (|\alpha + \epsilon\rangle_a | \rangle_b \\ &\quad \mp |-\alpha + \epsilon\rangle_a |-\rangle_b). \end{aligned} \quad (14)$$

For very small displacements such as $|\epsilon| \ll |\alpha|$ and assuming hereafter, without loss of generality, that α is a real number and ϵ is a pure imaginary number, it follows:

$$D_a(\epsilon)|\Phi_{\pm}\rangle \approx \cos(\epsilon\alpha)|\Phi_{\pm}\rangle + i \sin(\epsilon\alpha)|\Phi_{\mp}\rangle, \quad (15)$$

and similarly

$$D_a(\epsilon)|\Psi_{\pm}\rangle \approx \cos(\epsilon\alpha)|\Psi_{\pm}\rangle + i \sin(\epsilon\alpha)|\Psi_{\mp}\rangle. \quad (16)$$

On the other hand, the action of the displacement operator $D_b(\lambda) = e^{\lambda b^\dagger - \lambda^* b}$, where we assume again, without loss of generality, β real and λ imaginary pure numbers, and $|\lambda| \ll |\beta|$,

$$D_b(\lambda)|\Phi_{\pm}\rangle \approx \cos(\lambda\beta)|\Phi_{\pm}\rangle + i \sin(\lambda\beta)|\Psi_{\pm}\rangle, \quad (17)$$

$$D_b(\lambda)|\Psi_{\pm}\rangle \approx \cos(\lambda\beta)|\Psi_{\pm}\rangle + i \sin(\lambda\beta)|\Phi_{\pm}\rangle. \quad (18)$$

It is then straightforward to show that

$$P_b D_a(\epsilon)|\Phi_{\pm}\rangle = \cos(\epsilon\alpha)|\Phi_{\mp}\rangle + i \sin(\epsilon\alpha)|\Phi_{\pm}\rangle, \quad (19)$$

$$P_b D_a(\epsilon)|\Psi_{\pm}\rangle = -[\cos(\epsilon\alpha)|\Psi_{\mp}\rangle + i \sin(\epsilon\alpha)|\Psi_{\pm}\rangle], \quad (20)$$

$$P_a D_b(\lambda)|\Phi_{\pm}\rangle = \pm[\cos(\lambda\beta)|\Psi_{\pm}\rangle + i \sin(\lambda\beta)|\Phi_{\pm}\rangle], \quad (21)$$

$$P_a D_b(\lambda)|\Psi_{\pm}\rangle = \pm[\cos(\lambda\beta)|\Phi_{\pm}\rangle + i \sin(\lambda\beta)|\Psi_{\pm}\rangle]. \quad (22)$$

Now fixing $\epsilon\alpha = (n+1/2)\pi$ for $n=0,1,2,\dots$ and $\lambda\beta = (m+1/2)\pi$ for $m=0,1,2,\dots$, we finally obtain the following eigenvalue equations:

$$P_b D_a(\epsilon)|\Phi_{\pm}\rangle = i(-1)^n|\Phi_{\pm}\rangle, \quad (23)$$

$$P_b D_a(\epsilon)|\Psi_{\pm}\rangle = i(-1)^{n+1}|\Psi_{\pm}\rangle, \quad (24)$$

$$P_a D_b(\lambda)|\Phi_{\pm}\rangle = \pm i(-1)^m|\Phi_{\pm}\rangle, \quad (25)$$

$$P_a D_b(\lambda)|\Psi_{\pm}\rangle = \pm i(-1)^m|\Psi_{\pm}\rangle. \quad (26)$$

As soon as they have the same eigenvector, $P_b D_a(\epsilon)$ and $P_a D_b(\lambda)$ are simultaneous observables (with null variance), and can be used to obtain simultaneous information about the respective quantum state. Once these states are entangled, it is interesting to check if this state is a good resource for teleportation, in which case the state of propagating fields, or even atomic motional states [21] can be teleported. At this point the joint operators described here, play a fundamental role, as is discussed in the following section. It is interesting to note that despite the similarity with dichotomic variables, again it is not possible to match a one-to-one correspondence of those operators described here and the Pauli spin operators. It straightforward to check that while P_b in Eq. (10) could correspond to σ_z , P_a in Eq. (10) would correspond to σ_x , and more interesting, $P_a D_b$ corresponds then to $\sigma_x^a \sigma_x^b$ and $P_b D_a$ corresponds to $\sigma_z^a \sigma_z^b$.

IV. DISCRETE PROTOCOL

For the discrete protocol we prepare the entangled state and the target state $|\psi\rangle_T$ of a third party. We consider our pair of entangled modes prepared in the $|\Phi_{\pm}\rangle$ state. Let the target state be a superposition of coherent states

$$|\psi\rangle_T = c_a|\gamma\rangle + c_b|-\gamma\rangle, \quad (27)$$

with $|c_a|^2 + |c_b|^2 = 1$. Then the initial state of the system will be

$$|\psi\rangle_T(|\alpha\rangle_a|+\rangle_b + |-\alpha\rangle_a|-\rangle_b). \quad (28)$$

If we write $|\gamma\rangle$ and $|-\gamma\rangle$ in terms of $|+\rangle_T$ and $|-\rangle_T$, the total state can be written as

$$\begin{aligned} & |\Phi_{+}\rangle_{aT}(c_a|\beta\rangle + c_b|-\beta\rangle) + |\Phi_{-}\rangle_{aT}(c_a|-\beta\rangle + c_b|\beta\rangle) \\ & + |\Psi_{+}\rangle_{aT}(c_a|\beta\rangle - c_b|-\beta\rangle) + |\Psi_{-}\rangle_{aT}(c_a|-\beta\rangle - c_b|\beta\rangle). \end{aligned} \quad (29)$$

Now, in the Bell-measurement process on system $A+T$, each one of the four terms in the above state has one-fourth of chance to be detected, collapsing instantaneously the state of the party B . Each one of these states are eigenvector of the Bell operators $P_T D_a(\epsilon)$, and $P_a D_T(\lambda)$ with eigenvalues $\{i, i, -i, -i\}$ and $\{i, -i, i, -i\}$, respectively. These eigenvalues are complex as the displacement operators are unitary. The measurement of such an operator would need to be described by an appropriate, generalized measurement or positive operator-valued measurement, similar to the description of the complex amplitude in heterodyne measurement. How this measurement can be used to transfer the classical information to Bob is described below. As usual, without such a classical information path it is impossible for Bob to determine its state by any other way than guessing, obtaining the classical limit for teleportation of one-fourth. Let us consider that the first measurement made is described by the operator $P_T D_a(\epsilon)$. If the outcome is $+i$ the state is $|\Phi_{\pm}\rangle$ and if it is $-i$ then the state is $|\Psi_{\pm}\rangle$. Then the second measurement described by $P_a D_T(\lambda)$ is made. If the first measurement made was $+i$ then the second measurement will give $+i$ for the state $|\Phi_{+}\rangle$ and $-i$ for the state $|\Phi_{-}\rangle$. Now, if the first measurement made was $-i$ then the second measurement will give $+i$ for the state $|\Psi_{+}\rangle$ and $-i$ for the state $|\Psi_{-}\rangle$. In possession of this information, Bob can effect the necessary inverse transformations once he has one of the following states:

$$|\psi\rangle_b, \quad (30)$$

$$P_b|\psi\rangle_b, \quad (31)$$

$$iD_b(\mu)|\psi\rangle_b, \quad (32)$$

$$iP_b D_b(\mu)|\psi\rangle_b, \quad (33)$$

for $\mu\beta = \pi/2$, completing the teleportation protocol. Notice that, despite that the parties states are essentially coherent states and the entanglement in a continuous basis, the tele-

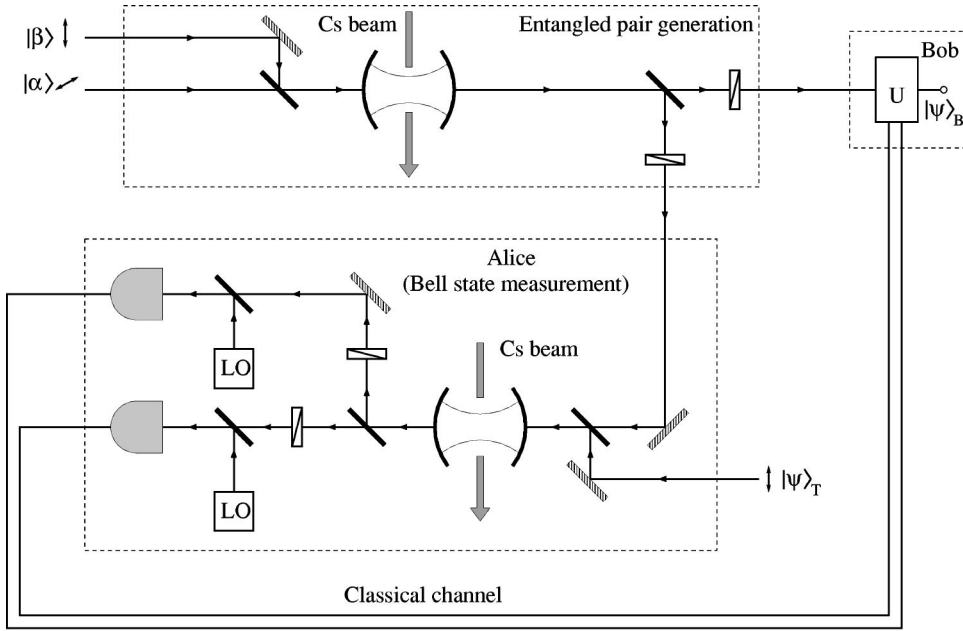


FIG. 1. Schematic of the cavity QED experimental apparatus for teleportation of field states. A Cs beam entangles the vertical (Bob) and horizontal (Alice) pulses. A polarizing beam splitter splits the pulse in two components. The horizontal component is entangled with the target pulse. Results of homodyne measurements made on Alice and target are sent to Bob by classical channels.

portation scheme is analogous to the original dichotomic variables teleportation protocol of Bennett *et al.* [1] even though the joint operator that plays the role of the Bell operator is a unitary non-Hermitian operator. $|\psi\rangle_T$ can vary from a simple coherent state to a coherent superposition, or even in the case in which the superpositions are the even and odd coherent states, for low intensity $|\psi\rangle_T$ corresponds to zero and one Fock states, respectively.

V. MEASUREMENT OF PARITY AND DISPLACEMENT

Although the formal scheme presented in the preceding section allows the complete Bell-state measurements, it was not explained how it could be realized. In fact the measurement process is divided in two stages as there are two operators involved, the parity and displacement operators. The parity is more to be understood as an operation over the joint field state, simultaneously, to the displacement. For field-parity measurements we have to resort to the methods well explored in microwave cavities [22]. An atom is prepared in a superposition $(|e\rangle + |g\rangle)/\sqrt{2}$ and let to interact dispersively with the field. After the interaction the atomic state is rotated again by a $\pi/2$ pulse. Due to the dispersive interaction, only the atom in the state $|e\rangle$ causes a parity flip in the field state and finally the parity of the field can be deduced by the detected atomic state. However, for the operation considered here, the atom has to be prepared in the $|e\rangle$ state and we do not read their final state. In this way excited atoms cause a π shift mode state (e.g., $|\alpha\rangle \rightarrow |-\alpha\rangle$). The displacement measurement is directly given by quadrature ($X = a + a^\dagger$) measurement through homodyne detection. As the parameter ε is known to be very small, the displacement operator, e.g., for the mode A is given by

$$D_a(\varepsilon) = e^{i|\varepsilon|\hat{X}} \approx 1 + i\varepsilon X; \quad [D_a(\varepsilon), X] = 0. \quad (34)$$

Knowing $|\varepsilon| = (n + 1/2)\pi/|\alpha|$, the measurement of \hat{X} gives the displacement. As expected the degree of control in this kind of measurement has to be very high.

The measurement stage can be simplified dramatically if another element is introduced in the protocol, if we actually entangle the target field with the Alice mode by the same scheme used to generate the entangled A - B pair. Allowing the interaction time to be again $t = \pi/\chi$ and setting the mode frequencies as explained in the Sec. III, it is straight to obtain the following entangled state for the joint A - B - T system,

$$\begin{aligned} \frac{1}{2} \{ & |\gamma\rangle|\alpha\rangle(C_a|\beta\rangle + C_b|-\beta\rangle) + |\gamma\rangle|-\alpha\rangle(C_a|\beta\rangle - C_b|-\beta\rangle) \\ & + |-\gamma\rangle|\alpha\rangle(C_a|-\beta\rangle + C_b|\beta\rangle) + |-\gamma\rangle|-\alpha\rangle(-C_a|-\beta\rangle \\ & + C_b|\beta\rangle) \}, \end{aligned} \quad (35)$$

where states can be distinguished by simple homodyne detection of modes A and T as schematically described in Fig. 1. We should remark that we choose to deal with states with a real-complex amplitude. In consequence the homodyne detection of each mode gives the phase quadrature X , which is distinguished in each case by positive or negative signals. This information is communicated by a standard classical channel to Bob, who possesses one of the states of Eq. (30) and has to apply the respective inverse unitary operation to obtain the original target state, completing the protocol.

VI. CONCLUSION

We have discussed a teleportation protocol for harmonic-oscillator states based on a different entanglement resource to that usually considered. The standard teleportation resource for an oscillator is a two-mode squeezed state [5,11]. Here we consider a teleportation resource based on entangled coherent states generated, for example, by a Kerr nonlinear-

ity, or by the effective coupling of the probe and pumping field strongly coupled to an Cs atom, as observed experimentally in [14]. The protocol can be made equivalent to the two-qubit scheme originally proposed by Bennett *et al.* [1] and we have explicitly identified the equivalent Bell-basis measurements. In our case these would correspond to generalized measurement but can be realized to a good approximation as measurements of parity and quadrature phase amplitude. It is hoped that this alternative teleportation protocol

will prove useful in elucidating the more general issue of entanglement between two systems with infinite-dimensional Hilbert spaces.

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- [1] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W.K. Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993).
 - [2] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
 - [3] S.L. Braunstein, A. Mann, and M. Revzen, *Phys. Rev. Lett.* **68**, 3259 (1992).
 - [4] D. Bouwmeester J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, *Nature (London)* **390**, 575 (1997).
 - [5] S.L. Braunstein and H.J. Kimble, *Phys. Rev. Lett.* **80**, 869 (1998).
 - [6] A. Furusawa, J.L. Sørensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble, and E.S. Polzik, *Science* **282**, 706 (1998).
 - [7] Y-H Kim, S.P. Kulik, and Y. Shih, e-print quant-ph/0010046.
 - [8] S. Popescu, *Phys. Rev. Lett.* **72**, 797 (1994).
 - [9] M.C. de Oliveira (unpublished).
 - [10] S.J. van Enk, *Phys. Rev. A* **60**, 5095 (1999).
 - [11] G.J. Milburn and S.L. Braunstein, *Phys. Rev. A* **60**, 937 (1999).
 - [12] S.L. Braunstein, G.M. D'Ariano, G.J. Milburn, and M.F. Sacchi, *Phys. Rev. Lett.* **84**, 3486 (2000).
 - [13] M.C. de Oliveira and W.J. Munro (unpublished).
 - [14] Q.A. Turchette, C.J. Hood, W. Lange, H. Mabuchi, and H.J. Kimble, *Phys. Rev. Lett.* **75**, 4710 (1995).
 - [15] G.J. Milburn and D.F. Walls, *Phys. Rev. A* **28**, 2065 (1983).
 - [16] M.D. Reid and D.F. Walls, *Phys. Rev. A* **34**, 4929 (1986).
 - [17] D. Jaksch, H.-J. Briegel, J.I. Cirac, C.W. Gardiner, and P. Zoller, *Phys. Rev. Lett.* **82**, 1975 (1999).
 - [18] P. Cochrane, G.J. Milburn, and W.J. Munro, *Phys. Rev. A* **59**, 2631 (1999).
 - [19] A. Royer, *Phys. Rev. A* **15**, 449 (1977).
 - [20] H. Moya-Cessa and P.L. Knight, *Phys. Rev. A* **48**, 2479 (1993).
 - [21] A.S. Parkins and H.J. Kimble, *J. Opt. B: Quantum Semiclassical Opt.* **1**, 496 (1999).
 - [22] M. Brune, S. Haroche, J.M. Raimond, L. Davidovich, and N. Zagury, *Phys. Rev. A* **45**, 5193 (1992).