# Trap environment effects over quantum statistics and atom-photon correlations in the collective atomic-recoil laser

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We consider the effects of the trap environment on atomic and optical quantum statistical properties and on atom-photon correlations in the collective atomic-recoil laser. Atomic and optical statistical properties, as well as atom-photon correlations are dependent on the optical-field intensity and phase. In particular, depending on the values of the optical-field intensity and phase, the field statistics vary from coherent to superchaotic.

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Realization of Bose-Einstein condensation in trapped atomic gases [1] has produced new advances in atom optics. Particularly, the interaction of condensates with single-mode quantized light fields has been a challenging topic [2–5], allowing for instance light and matter-wave amplification [3–5] and optical control of atomic statistical properties [3,5].

It is known that the trap environment can modify the properties of ultracold atoms, such as its critical temperature [6]. In Ref. [7], the trap environment effect on the condensate collective atomic-recoil laser (CARL) [8] was included by expanding the matter-wave field in the trap matter-wave modes. Such a situation was called cavity atom optics (CAO), in analogy with the cavity quantum electrodynamics where the spontaneous emission is modified by the presence of the cavity. The dynamics of this model was compared with its counterpart free space model [5], where the matter-wave field was expanded in plane waves. It was found that in the CAO regime, the atomic and optical-field intensities present two regimes of exponential instability. However, the trap effect over the quantum statistical properties and the atom-photon correlations was not considered. These properties are important to characterize the role of trap environments on the control of atomic and optical statistical properties, which can be useful for quantum information processing.

This Brief Report concerns with the effects of trap environment over the quantum statistical properties and the atom-photon correlations in the CARL. By analyzing the second order correlation functions and intensity cross correlations, we verify that the statistical properties of the atomic and optical fields as well as the atom-photon correlations are dependent on the optical-field initial intensity and phase. Furthermore, by setting the optical-field intensity and phase, the fields statistics can be varied from coherent to superchaotic.

Following Ref. [7], we consider a Schrödinger field of noninteracting bosonic two-level atoms interacting with two single-mode running wave optical fields of frequencies $\omega_1$ and $\omega_2$, both being far off resonant from any electronic transition. By two-photon virtual transitions, in which the atoms internal state remains unchanged, the center-of-mass motion may change due to the recoil. In the far-off resonant regime, the excited-state population and spontaneous emission may be neglected, and the ground-state atomic field evolves coherently under the effective Hamiltonian

$$\hat{H}=\int d^3\mathbf{r}\hat{\Psi}^\dagger(\mathbf{r})\left[-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{r})+\hbar\left(\frac{g_1^*g_2}{\Delta}\hat{a}_1^\dagger a_2 e^{-i\mathbf{K}\cdot\mathbf{r}}+\frac{g_2^*g_1}{\Delta}a_2^*\hat{a}_1 e^{i\mathbf{K}\cdot\mathbf{r}}\right)\right]\hat{\Psi}(\mathbf{r})+\hbar(\omega_1-\omega_2)\hat{a}_1^\dagger\hat{a}_1, \qquad (1)$$

where $m$ is the atomic mass, $V(r)$ is the trap potential, $g_1$ and $g_2$ are the probe and pump coupling coefficients, respectively, and $\mathbf{K}=\mathbf{k}_1-\mathbf{k}_2$ is the difference between the probe and pump wave vectors. The operator $\hat{a}_1$ is the photon annihilation operator for the probe mode, taken in the frame rotating at the pump frequency $\omega_2$. The pump is treated classically and assumed to remain undepleted, and the index of refraction of the atomic sample was assumed equal to the vacuum by neglecting spatially independent light shifts potentials.

Now, we distinguish between free propagation and cavity regimes. The free propagation regime is valid for times short enough that atoms at the recoil velocity propagate only over short distances compared to the dimensions of the initial condensate. Following Ref. [5], in this regime, the atomic field is expanded onto momentum side modes, which are simply plane waves with a slowly varying spatial envelope. The cavity regime occurs at much longer time scales, when the recoiling atoms propagate over distances larger than the trap dimensions. In the CAO regime, the atoms probe the trap environment and the atomic-field operator is best expressed in terms of the trap eigenmodes $\{\varphi_n(\mathbf{r})\}$ according to $\hat{\Psi}(\mathbf{r})=\Sigma_{n=0}^{\infty}\varphi_n(\mathbf{r})\hat{c}_n$, where $\hat{c}_n$ is the annihilation operator for atoms in mode $n$.

We consider that the condensate mode $\hat{c}_0$ is initially highly populated with $N$ mean number of atoms, allowing treat it as a $c$-number. Also is neglected condensate depletion, which is valid for short times so that the side mode populations remain small compared to $N$. These conditions allow the replacement $\hat{c}_0\approx\sqrt{N}$. For simplicity, we assume that the condensate mode couples only to the $m$th trap mode. Thus, we obtain the following effective Hamiltonian:

$$\hat{H}=\hbar\omega_m\hat{c}_m^\dagger\hat{c}_m+\hbar\delta\hat{a}^\dagger\hat{a}+\hbar\chi_m(\hat{a}^\dagger\hat{c}_m^\dagger+\hat{a}^\dagger\hat{c}_m+\hat{c}_m^\dagger\hat{a}+\hat{c}_m\hat{a}), \qquad (2)$$

 

where $\hbar\omega_m$ is the energy of the $m$th trap mode, $\delta=\omega_1-\omega_2$ is the detuning between the pump and probe optical fields, $\chi_m=\sqrt{N}A_{0m}|g_1||g_2||a_2|/|\Delta|$ is the coupling constant between atomic and optical fields, $\hat{a}=(g_1g_2^*a_2^*\Delta/|g_1||g_2||a_2||\Delta|)\hat{a}_1$ is the probe annihilation operator times a phase factor related to the phase of the pump laser and the sign of the detuning, and $A_{0m}=\int d^3\mathbf{r}\varphi_0^*(\mathbf{r})e^{i\mathbf{K}\cdot\mathbf{r}}\varphi_m(\mathbf{r})$ is the matrix element for the optical transition that was assumed a real number. Terms such as $\hat{a}^\dagger\hat{c}_m^\dagger$ in Eq. (2) correspond to the generation of correlated atom-photon pairs. This is analogous to the optical parametric amplifier (OPA), except that in that case it is correlated photon pairs that are generated.

The Heisenberg equations of motion for the field operators follow from Hamiltonian (2), resulting the $4\times4$ system of equations

$$\frac{d}{dt}\begin{pmatrix}\hat{c}_m\\ \hat{c}_m^\dagger\\ \hat{a}\\ \hat{a}^\dagger\end{pmatrix}=i\begin{pmatrix}-1 & 0 & -\chi_m & -\chi_m\\ 0 & 1 & \chi_m & \chi_m\\ -\chi_m & -\chi_m & -\delta & 0\\ \chi_m & \chi_m & 0 & \delta\end{pmatrix}\begin{pmatrix}\hat{c}_m\\ \hat{c}_m^\dagger\\ \hat{a}\\ \hat{a}^\dagger\end{pmatrix}, \tag{3}$$

where we introduced the dimensionless quantities $t\equiv\omega_m t$, $\delta\equiv\delta/\omega_m$, and $\chi_m\equiv\chi_m/\omega_m$.

The solution of the linear system (3) can be written as

$$\hat{x}_i(t)=\sum_{j=1}^{4}\sum_{k=1}^{4}G_{ij}^{(k)}(t)e^{i\omega_k t}\hat{x}_j(0), \tag{4}$$

where we defined $\hat{x}_1=\hat{c}_m$, $\hat{x}_2=\hat{c}_m^\dagger$, $\hat{x}_3=\hat{a}$, and $\hat{x}_4=a^\dagger$ for convenience, and $\omega_k$ are the system eigenfrequencies. For nondegenerate eigenfrequencies $G_{ij}^{(k)}(t)=[\mathbf{U}]_{ik}[\mathbf{U}^{-1}]_{kj}$, where $[\mathbf{U}]_{ik}$ is the $i$ th component of the $k$th eigenvector of the matrix at right-hand side of Eq. (3). Following Ref. [7], the stability analysis shows three distinct regimes. (i) The eigenfrequencies are purely real, the system being stable. (ii) Two purely real and two purely imaginary eigenfrequencies of the form $\{\omega_1=\Omega,\omega_2=-\Omega,\omega_3=i\Gamma,\omega_4=-i\Gamma\}$, where $\Omega$ and $\Gamma$ are both real quantities. There is only one exponentially growing solution at the imaginary frequency $\omega_4$ and the system is unstable. (iii) The eigenfrequencies are complex numbers of the form $\{\omega_1=\Omega+i\Gamma,\omega_2=-\Omega+i\Gamma,\omega_3=\Omega-i\Gamma,\omega_4=-\Omega-i\Gamma\}$. This case presents two exponentially growing solutions, $\omega_3$ and $\omega_4$, which grow at the same rate $\Gamma$, but rotate at equal and opposite frequencies $\pm\Omega$, producing a beating in the exponential growth of the fields intensities. (iv) The eigenfrequencies are not imaginary and are degenerate at the critical values $\delta=0$, and $\delta=4\chi_m^2$ and $(1-\delta^2)^2/|\delta|=16\chi_m^2$ $(\delta<0)$, which define the threshold separating stable from exponentially unstable solutions. The coefficient $G_{ij}^{(k)}(t)$ is a polynomial in $t$ of degree one less than the degree of degeneracy. The fields amplitudes acquire a linear time dependence and the system becomes unstable. We are interested in unstable regimes (ii), (iii), and (iv), which show stationary statistical characteristics as we see below.

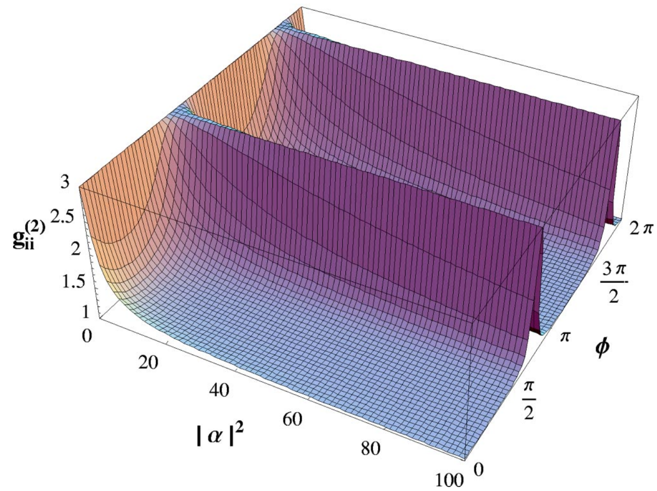


FIG. 1. Plot of the long-time value of atomic and optical-field second-order correlation function as a function of $|\alpha|^2$ and $\phi$ in regime (ii). The parameters are set at $\delta=1$ and $\chi_m=1$.

Let us first consider the atomic and optical-field statistical properties, which can be characterized by the normalized equal-time second-order correlation function defined by [10]

$$g_{ii}^{(2)}(t)=\frac{\langle\hat{x}_i^\dagger(t)\hat{x}_i^\dagger(t)\hat{x}_i(t)\hat{x}_i(t)\rangle}{\langle\hat{x}_i^\dagger(t)\hat{x}_i(t)\rangle^2},\quad i=1,3. \tag{5}$$

We assume that the atomic mode begins in a vacuum state, whereas the light field is initially in a coherent state $|\alpha\rangle$ with complex amplitude $\alpha=|\alpha|e^{-i\phi}$. The calculated expression $g_{ii}^{(2)}(t)$ is dependent on the optical-field intensity $|\alpha|^2$ and phase $\phi$, as well as of the system parameters. Unfortunately, the analytical expression does not provide so much insight. We begin by considering values of parameters in regime (ii). After a transient, the exponential with eigenfrequencies $\omega_4$ will dominate in Eq. (4) and both, $g_{11}^{(2)}(t)$ and $g_{33}^{(2)}(t)$, attain at long time the same constant value. The long-time value of $g_{ii}^{(2)}(t)$ is plotted in Fig. 1 as function of $|\alpha|^2$ and $\phi$, which shows a strong sensitivity to the initial phase and intensity of the light field. We see that by varying the light field intensity and phase, it is possible to continuously change the field statistics from coherent ($g_{ii}^{(2)}=1$) to chaotic ($g_{ii}^{(2)}=2$) to superchaotic ($g_{ii}^{(2)}>2$). The generation of superchaotic light was first analyzed in Ref. [9] by considering a two-photon emission process. The situation considered here is rather interesting because it allows to produce a superchaotic atomic source.

In regime (iii), the atomic and optical correlation functions attain at long times a stationary oscillating values, which are not necessarily the same. In order to explore the long-time optical-field intensity and phase sensitivity, it is plotted in Figs. 2(a) and 2(b) the value of $g_{11}^{(2)}$ and $g_{33}^{(2)}$, respectively, as a function of $|\alpha|^2$ and $\phi$ by considering a fixed large value of time. We see, by comparing Fig. 1 with Fig. 2 that the later one shows less sensitivity to the optical-field phase. In regime (iv), the correlations attain at long time a steady value with amplitude decreasing oscillations around

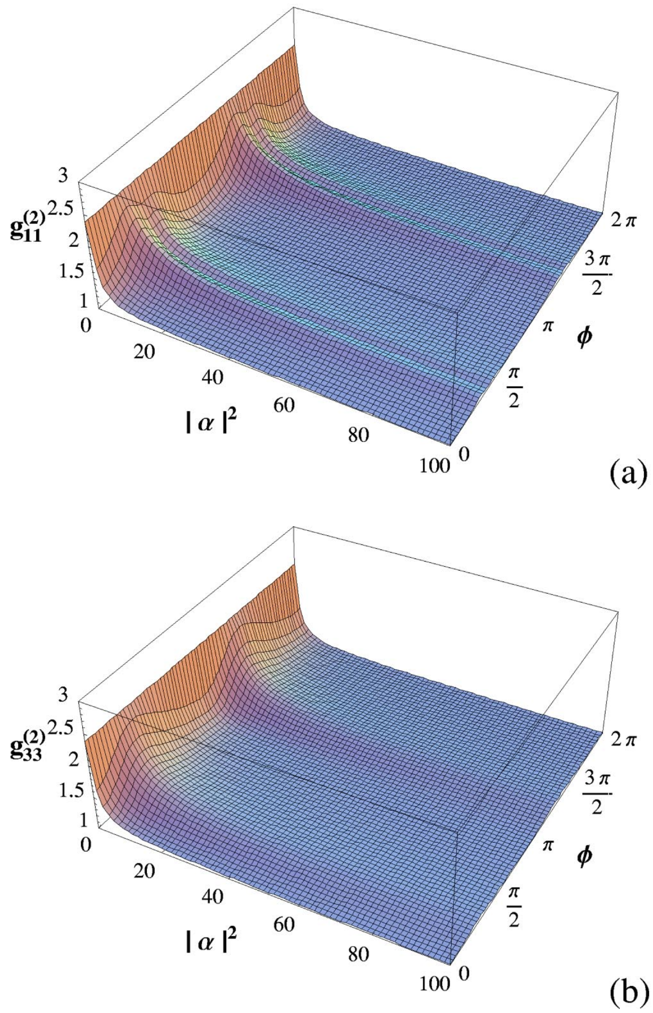


FIG. 2. Same as in Fig. 1 in regime (iii) and setting $t=8$. (a) Atomic field; (b) optical field. The parameters are set at $\delta=-1$ and $\chi_m=1$.

a constant value for $\delta=0$ and $\delta=4\chi_m^2$, and oscillating for $(1-\delta^2)^2/|\delta|=16\chi_m^2$ $(\delta<0)$. For $\delta=0$ and $\delta=4\chi_m^2$, an analytical expression for the asymptotic value of $g_{ii}^{(2)}(t)$ can be found by maintaining only those terms with linear time dependence in the fields amplitudes. Both $g_{11}^{(2)}$ and $g_{33}^{(2)}$ attains the same value given by

$$g_{ii}^{(2)}=1+2\frac{[1+\delta_c]\left[1+\delta_c+8|\alpha|^2\cos^2\left(\phi-\frac{\pi\delta_c}{8\chi_m^2}\right)\right]}{\left[1+\delta_c+4|\alpha|^2\cos^2\left(\phi-\frac{\pi\delta_c}{8\chi_m^2}\right)\right]^2}, \tag{6}$$

where $\delta_c=0$ or $\delta_c=4\chi_m^2$. From Eq. (6), we see that $1\leqslant g_{ii}^{(2)}\leqslant 3$, which confirms the presence of superchaotic fluctuations. The behavior of $g_{ii}^{(2)}(t)$ at $(1-\delta^2)^2/|\delta|=16\chi_m^2$ $(\delta<0)$ is similar to that found in regime (iii); however, no simple analytical expression was obtained.

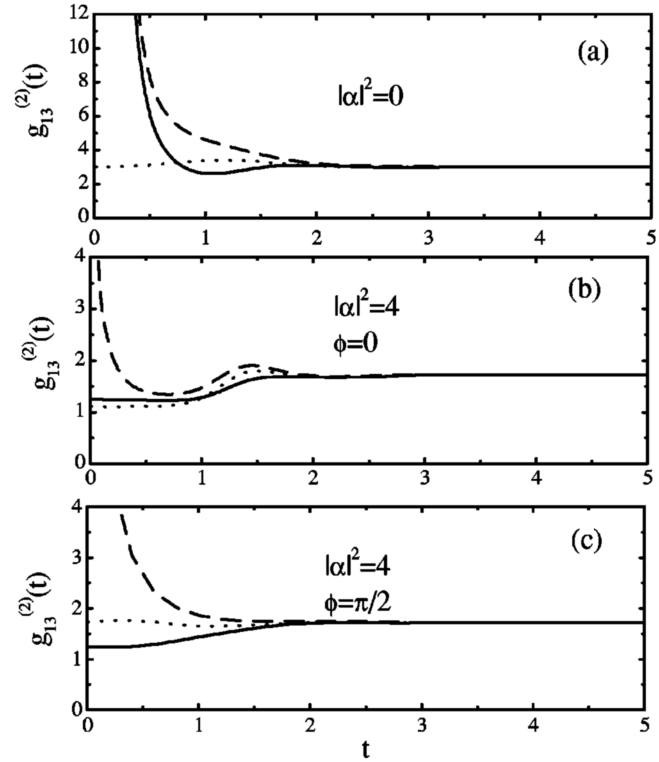


FIG. 3. Solid line: plot of the atom-photon second-order cross-correlation function as a function of time in regime (ii). Dashed line: quantal upper limit. Doted line: classical upper limit. The parameters are set at $\delta=1$ and $\chi_m=1$.

Now we turn to analyze the atom-photon correlations, which be quantified by the two-mode equal-time intensity cross-correlation function [10]

$$g_{ij}^{(2)}(t)=\frac{\langle\hat{x}_i^\dagger(t)\hat{x}_i(t)\hat{x}_j^\dagger(t)\hat{x}_j(t)\rangle}{\langle\hat{x}_i^\dagger(t)\hat{x}_i(t)\rangle\langle\hat{x}_j^\dagger(t)\hat{x}_j(t)\rangle}, \quad i\neq j. \tag{7}$$

Particularly, for classical fields, the two-mode correlation function is bounded by

$$g_{ij}^{(2)}(t)\leqslant\sqrt{g_{ii}^{(2)}(t)g_{jj}^{(2)}(t)}, \tag{8}$$

while quantum fields can violate this inequality, being limited by

$$g_{ij}^{(2)}(t) \leqslant\sqrt{\left[g_{ii}^{(2)}(t)+\frac{1}{\langle\hat{x}_i^\dagger(t)\hat{x}_i(t)\rangle}\right]\left[g_{jj}^{(2)}(t)+\frac{1}{\langle\hat{x}_j^\dagger(t)\hat{x}_j(t)\rangle}\right]}, \tag{9}$$

which reduces to the classical limit at large intensities.

In order to explore the general characteristics of the atom-photon correlations, we consider only the exponentially unstable regimes (ii) and (iii). Considering regime (ii), the sequence in Fig. 3 shows the atom-photon correlation function $g_{13}^{(2)}(t)$ as a function of time and by considering different values of light field intensity and phase. For comparison, in

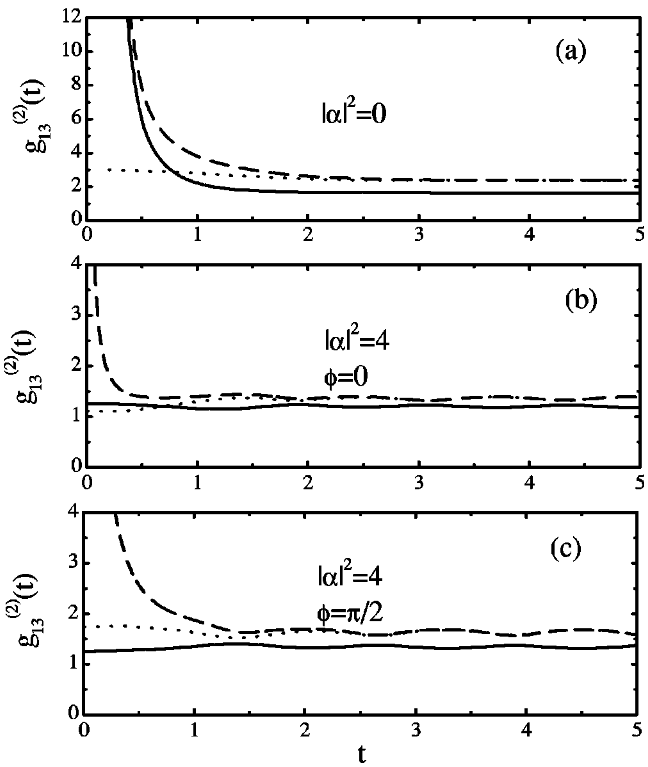


FIG. 4. Same as in Fig. 3 in regime (iii). The parameters are set at $\delta=-1$ and $\chi_m=1$.

the same figure, it is also plotted the classical upper limit given by Eq. (8) (dotted line) and the quantum upper limit given by Eq. (9) (dashed line). Figure 3(a) shows the spontaneous case $(\alpha=0)$, in which the field instabilities are triggered by the noise from vacuum fluctuations. We see that only at short times, the violation of the classical inequality is close to the maximum limit consistent with quantum mechanics. In the case that $\alpha\neq 0$, the correlations are also phase dependent. Figures 3(b) and 3(c) show a situation with same intensity $|\alpha|^2=4$ , but taking two different values of phase. We see that even at short times, a nonzero intensity reduces the correlations for values close to the classical ones. The phase dependence is evident by comparing Figs. 3(b) and 3(c), where the later one presents no violation of the classical inequality. At long times, the correlations attain the classical limit and the fields become uncorrelated $(g_{ij}^{(2)}=1)$ or correlated $(g_{ij}^{(2)}>1)$ depending on the optical-field intensity and phase.

Regime (iii) presents, at short and intermediate times, similar characteristics to regime (ii). The main difference is the long-time limit where the correlations attain a stationary value below the classical upper limit. Figures 4(a)–4(c) illustrate the behavior of $g_{13}^{(2)}(t)$ as a function of time, and its dependence to the optical-field intensity and phase.

Note that in the two-mode OPA, the difference of population between the two modes is a constant of motion [10], revealing that in the spontaneous case $(\alpha=0)$ the two-mode correlation function shows the maximum violation of the classical inequality consistent with quantum mechanics. There is no such constant of motion in the model considered in this paper because both the processes of emission and absorption of photons transfer atoms to the same quantum state, thus justifying that the violation of the classical inequality in Figs. 3(a) and 4(a) is not maximized for all times. Furthermore, although the violation of the classical inequality indicates nonclassicality, it is not possible to determine whether this represents an entanglement of the two modes or if each mode is in a nonclassical state. Further indication of entanglement is given by the presence of chaotic intensity fluctuations in the individual modes, which means that each of its components is in a mixed state, although the atom-photon system is in a pure state.

In summary, we studied the effects of the trap environment on the quantum statistical properties and on the atom-photon correlations in the CARL. The atomic and optical-field statistical properties and the atom-photon correlations are sensitive to the initial intensity and phase of the light field. This result is contrasting when compared with the counterpart free space regime model [5] whose statistical properties are only dependent on the intensity of the light field. Furthermore, from the viewpoint of optical control of statistical properties of atoms, while in the free space regime the variation is limited between coherent and chaotic, the CAO regime allows a a superchaotic statistics by setting the optical-field intensity and phase.

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