Quantum Optics and Quantum Information Group

JOURNAL CLUB

Quantum Phase Transitions in CQE

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Quantum Phase Transitions

In physics, a quantum phase transition (QPT) is a phase transition between different quantum phases (phases of matter at zero temperature). Contrary to classical phase transitions, quantum phase transitions can only be accessed by varying a physical parameter - such as magnetic field or pressure - at absolute zero temperature. The transition describes an abrupt change in the ground state of a many-body system due to its quantum fluctuations. Such quantum phase transitions can be first-order phase transition or continuous.To understand quantum phase transitions, it is useful to contrast them to classical phase transitions (CPT) (also called thermal phase transitions). A CPT describes a discontinuity in the thermodynamic properties of a system. It signals a reorganization of the particles; A typical example is the freezing transition of water describing the transition between liquid and ice. The classical phase transitions are driven by a competition between the energy of a system and the entropy of its thermal fluctuations. A classical system does not have entropy at zero temperature and therefore no phase transition can occur.In contrast, even at zero temperature a quantum-mechanical system has quantum fluctuations and therefore can still support phase transitions. As a physical parameter is varied, quantum fluctuations can drive a phase transition into a different phase of matter. A canonical quantum phase transition is the well-studied superconductor/insulator transition in disordered thin films which separates two quantum phases having different symmetries. Quantum magnets provide another example of QPT.

The Ehrenfest scheme is an inaccurate method of classifying phase transitions, for it does not take into account the case where a derivative of free energy diverges (which is only possible in the thermodynamic limit). For instance, in the ferromagnetic transition, the heat capacity diverges to infinity.In the modern classification scheme, phase transitions are divided into two broad categories, named similarly to the Ehrenfest classes:

- The first-order phase transitions are those that involve a latent heat. During such a transition, a system either absorbs or releases a fixed (and typically large) amount of energy. During this process, the temperature of the system will stay constant as heat is added.

Because energy cannot be instantaneously transferred between the system and its environment, first-order transitions are associated with "mixed-phase regimes" in which some parts of the system have completed the transition and others have not. This phenomenon is familiar to anyone who has boiled a pot of water: the water does not instantly turn into gas, but forms a turbulent mixture of water and water vapor bubbles. Mixed-phase systems are difficult to study, because their dynamics are violent and hard to control. However, many important phase transitions fall in this category, including the solid/liquid/gas transitions and Bose-Einstein condensation.

- The second class of phase transitions are the continuous phase transitions, also called second-order phase transitions. These have no associated latent heat. Examples of second-order phase transitions are the ferromagnetic transition and the superfluid transition.

Several transitions are known as the infinite-order phase transitions. They are continuous but break no symmetries. The most famous example is the Kosterlitz-Thouless transition in the two-dimensional XY model. Many quantum phase transitions in two-dimensional electron gases belong to this class.*

*en.wikipedia.org/wiki/Quantum_phase_transition. en.wikipedia.org/wiki/Phase_transition

Entanglement and Quantum Phase Transition

- *Scaling of entanglement close to a quantum phase transition*,A. Osterloh,L. Amico, G. Falci, R. Fazio, Nature **416**, 608 (2002). Times Cited: 418

Classical phase transitions occur when a physical system reaches a state below a critical temperature characterized by macroscopic order. Quantum phase transitions occur at absolute zero; they are induced by the change of an external parameter or coupling, and are driven by quantum fluctuations.Examples constant include transitions in quantum Hall systems, localization in Si-MOSFETs (metaloxidesiliconeld-effecttransistors) and the superconductor insulator transition in two-dimensional systems. Both classical and quantum critical points are governed by a diverging correlation length,although quantum systems possess additional correlations that do not have a classical counterpart. This phenomenon, known as entanglement, is the resource that enables quantum computation and communication. The role of entanglement at a phase transition is not captured by statistical mechanics a complete classication of the critical many-body state requires the introduction of concepts from quantum information theory.Here we connect the theory of critical phenomena with quantum information by exploring the entangling resources of a system close to its quantum critical point. We demonstrate, for a class of one-dimensional magnetic systems, that entanglement shows scaling behaviour in the vicinity of the transition point.

- *Entanglement in Quantum Critical Phenomena,*G. Vidal, J. I. Latorre, E. Rico, A. Kitaev, Phys. Rev. Lett. **90**, 227902 (2003).Times Cited: 367

Entanglement, one of the most intriguing features of quantum theory and a main resource in quantum information science, is expected to play a crucial role also in the study of quantum phase transitions, where itis responsible for the appearance of long-range correlations. We investigate, through a microscopic calculation, the scaling properties of entanglement in spin chain systems,both near and at a quantum critical point. Our results establish a precise connection between concepts of quantum information, condensed matter physics, and quantum field theory, by showing that the behavior of critical entanglement in spin systems is analogous to that of entropy inconformal field theories. We explore some of the implications of this connection.

REVIEW

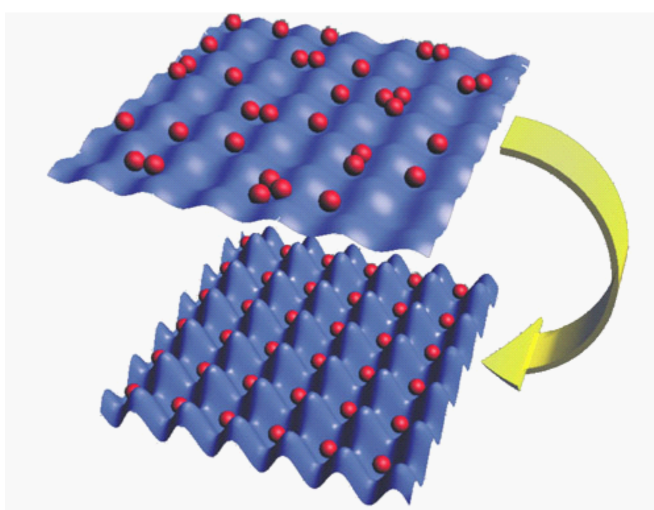
- *Entanglement in Many-Body Systems,*Amico, Luigi; Fazio, Rosario; Osterloh, Andreas; Vedral, Vlatko, quant-ph/0703044v2.Times Cited: 58

The recent interest in aspects common to quantum information and condensed matter has prompted a prosperous activity at the border of these disciplines that were far distant until few years ago. Numerous interesting questions have been addressed so far. Here we review an important part of this field, the properties of the entanglement in many-body systems. We discuss the zero and finite temperature properties of entanglement in interacting spin, fermionic and bosonic model systems. Both bipartite and multipartite entanglement will be considered. At equilibrium we emphasize on how entanglement is connected to the phase diagram of the underlying model. The behavior of entanglement can be related, via certain witnesses, to thermodynamic quantities thus offering interesting possibilities for an experimental test. Out of equilibrium we discuss how to generate and manipulate entangled states by means of many-body Hamiltonians.

- *Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond,* M.Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen; U. Sen, Advances in Physics **56:2** 243 (2007) Times Cited: 42

We review recent developments in the physics of ultracold atomic and molecular gases in optical lattices. Such systems are nearly perfect realisations of various kinds of Hubbard models, and as such may very well serve to mimic condensed matter phenomena. We show how these systems may be employed as quantum simulators to answer some challenging open questions of condensed matter, and even high energy physics. After a short presentation of the models and the methods of treatment of such systems, we discuss in detail, which challenges of condensed matter physics can be addressed with (i) disordered ultracold lattice gases, (ii) frustrated ultracold gases, (iii) spinor lattice gases, (iv) lattice gases in "artificial" magnetic fields, and, last but not least, (v) quantum information processing in lattice gases. For completeness, also some recent progress related to the above topics with trapped cold gases will be discussed.

QPT and CQE



- *Quantum phase transitions of light*, A. Greentree, C. Tahan, J. Cole, L. C. L. Hollenberg, Nature Physics **2**, 856, (2006). Times Cited: 32

As physics and engineering extend their reach to the control of single excitations of nature, we gain the ability to explore and even design the interaction of matter and energy in fundamentally new ways. One of the most interesting opportunities this presents is controllable interactions between many quantum particles — such as electrons — which is traditionally the realm of condensed matter physics. The questions we asked ourselves were these: Can we also do this with light? Can it be useful? We show that the answer is YES.

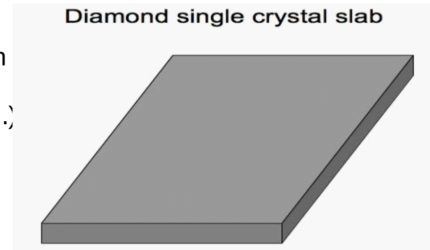
In recent decades, condensed matter and atomic experiments have reached a length-scale and temperature regime of interacting systems where new quantum and collective phenomena emerge. Witness the remarkable progress in cold atom optical lattices that physically manifest the Bose-Hubbard model. Finding such interacting quantum particle analogues based on systems of photons, however, is problematic, as photons typically do not interact with each other and are governed by a no conservation principle, they can be created or destroyed at will. Here, we introduce a physical system of photons that exhibits strongly correlated dynamics on a meso-scale. By adding photons to a two-dimensional array of coupled optical cavities, each containing a single two-level atom in the photon-blockade regime, we form atom-photon ‘molecules’ termed dressed states or polaritons. Our results for zero temperature show for the first time a characteristic Mott insulator to superfluid quantum phase transition in these systems. Moreover, the Mott-insulator phase with exactly n photons on each site can be quite robust. A cavity’s impressive out-coupling potential and the ability to separate the photon part of the dressed photon-atom composite quickly may lead to actual devices and tunable quantum simulators based on these quantum many-body effects.

How to make photons behave: Building a photonic super-lattice†

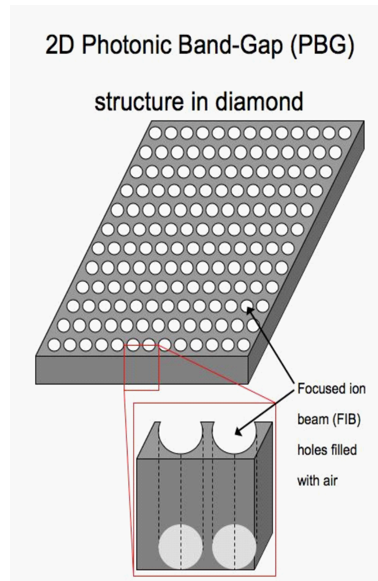
Although the physics we describe in our paper is quite universal and can be realized in many systems, we give one specific example: that of Nitrogen-Vacancy color centres in a diamond photonic band-gap cavity superlattice. This is a fancy way of saying we want to make a lattice of photon traps each with a two-level system (the NV-center or atom) inside. An NV-centre is one way to make a two-level system where a photon will drive transitions between the two levels - so we are making a solid state equivalent of a single atom in a quantum optical cavity.

†http://www.tahan.com/charlie/research/physics/light/QPTlight.php

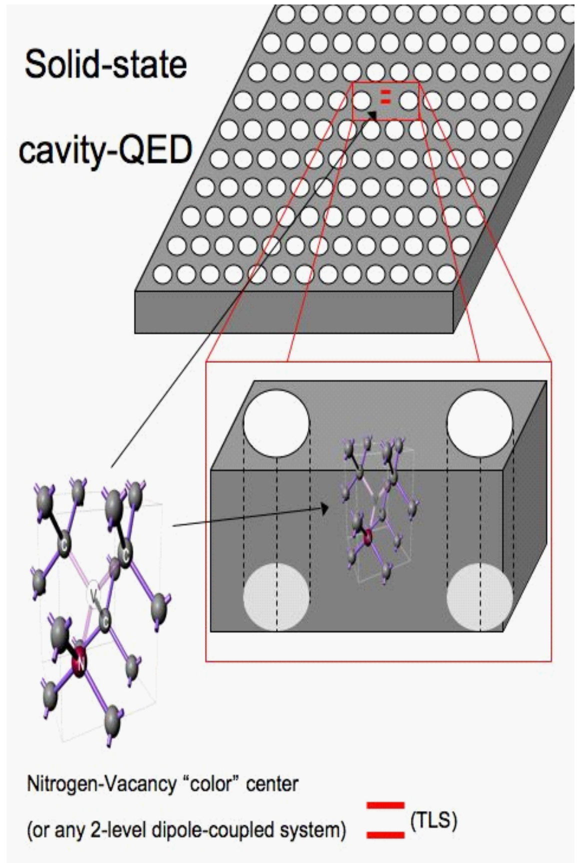
Step 1: Start with a slab of ultrapure diamond crystal whose height (the smallest dimension) is on order of the wavelength of light (photons) you want to trap inside it (200 - 700 nm).
(In other words, make a lambda/2 waveguide in the growth direction.)



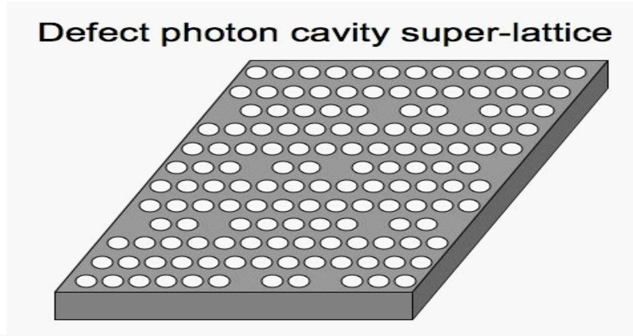
Imaginary Step 2: Create a photonic band gap material by drilling holes periodically in your slate with the lattice spacing *a* given by the wavelength of light you want to use.



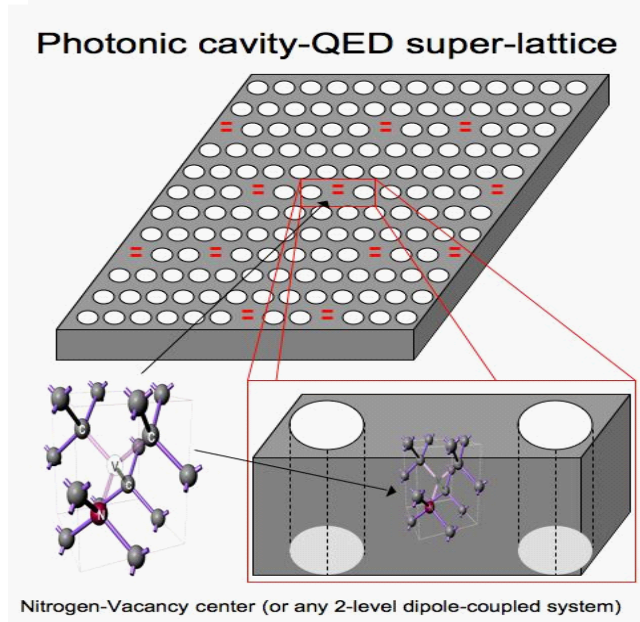
Imaginary Step 3: Implant a nitrogen atom precisely in the center of the PBG defect (horizontally in the 2D lattice and midway in the diamond crystal vertically) creating a nitrogen-vacancy complex (meaning one nitrogen atom and one "hole" where there is neither a carbon atom from the the original diamond crystal nor a nitrogen atom). Now we have created a solid-state equivalent of a cavity-QED device (where an atom is placed in an extremely high Q mirrored optical cavity to increase photon-atom interaction). The two-level system (TLS) creates a non-linearity, causing an interaction between the photons (a repulsion). The fewer the number of photons, the more strongly interacting the photons are (in reverse: the cavity becomes more classical and less quantum as you add more photons to the cavity).



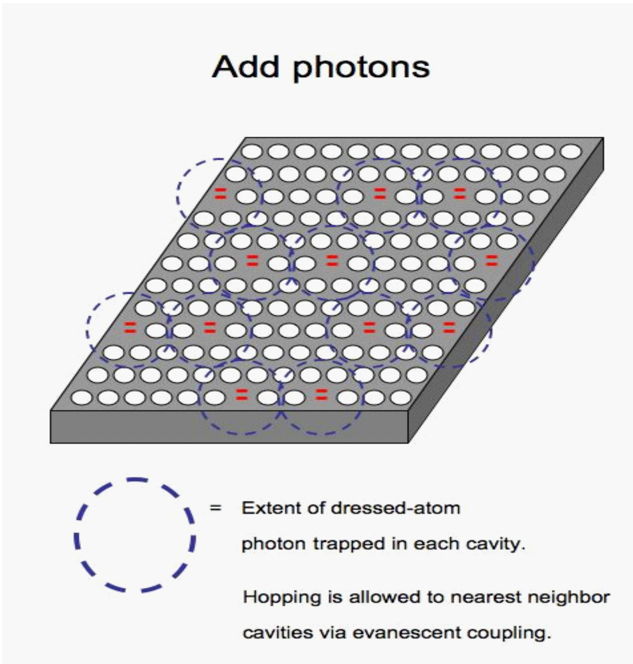
Step 2: Make the defect cavity super-lattice for real by drilling holes on a lattice everywhere except where we want the cavities to be. We make the defect cavities close enough such that there can be a possibility of photon hopping between nearest neighbors.



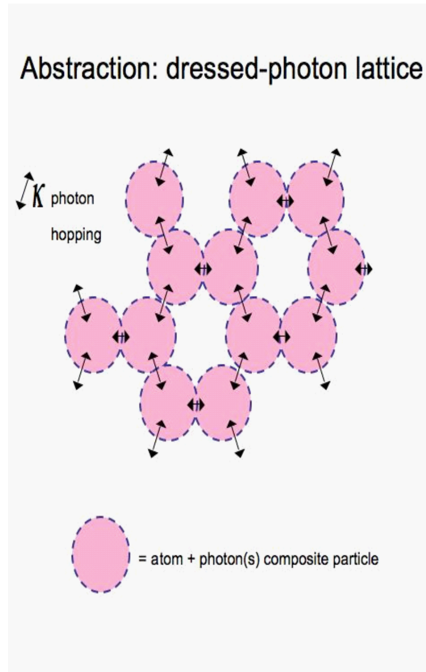
Step 3: Implant nitrogen atoms in each defect to create NV centers in each PBG defect optical cavity. Now we have an interconnected array of cavity-QED sites.



Step 4: Add photons (say with a coherent laser pulse) and wait until equilibrium is reached.



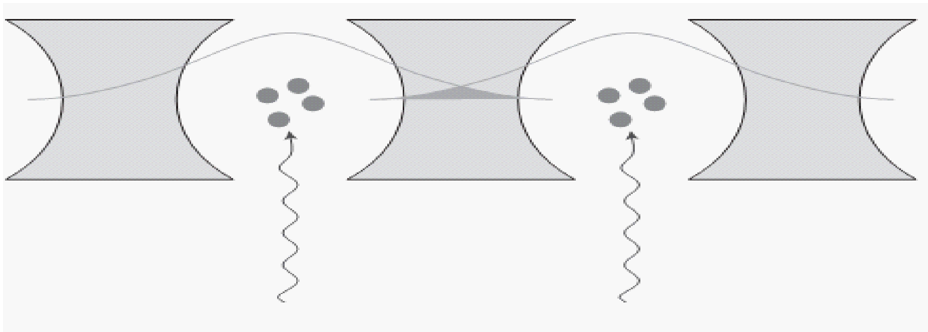
Step 5: Find the place in parameter space where the many-body system forms a Mott insulator state. (Note that the pink circles are abstractions and would better be represented by Gaussians with a tale that overlaps with nearest neighbors.)

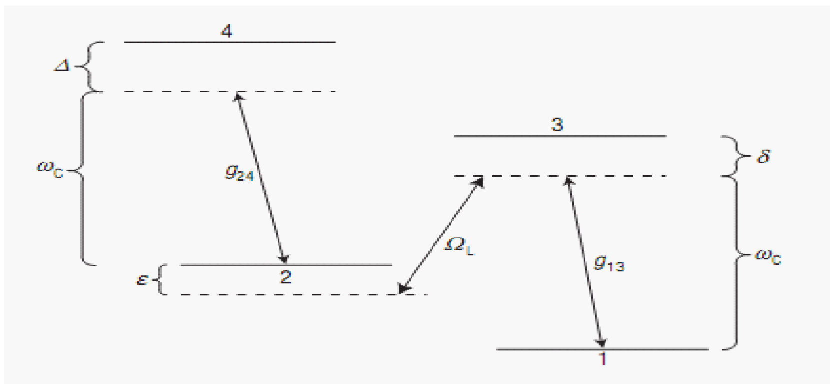


- *Strongly Interacting Polaritons in Coupled Arrays of Cavities*, M. J. Hartmann, F.G. S. L.Brandão; M. B. Plenio, Nature Physics **2**, 849 (2006).Times Cited: 37

The experimental observation of quantum phenomena in strongly correlated many particle systems is difficult because of the short length- and timescales involved. Obtaining at the same time detailed control of individual constituents appears even more challenging and thus to date inhibits employing such systems as quantum computing devices. Substantial progress to overcome these problems has been achieved with cold atoms in optical lattices, where a detailed control of collective properties is feasible but it is very difficult to address and hence control or measure individual sites. Here we show, that polaritons, combined atom and photon excitations, in an array of cavities such as a photonic crystal or coupled toroidal micro-cavities, can form a strongly interacting many body system, where individual particles can be controlled and measured. All individual building blocks of the proposed setting have already been experimentally realised, thus demonstrating the potential of this device as a quantum simulator. With the possibility to create attractive on-site potentials the scheme allows for the creation of highly entangled states and a phase with particles much more delocalised than in superfluids.

$$H_{eff} = \kappa \sum_{R} (p_R^\dagger)^2 (p_R)^2 + J \sum_{\langle R,R' \rangle} (p_R^\dagger p_{R'} + h.c.).$$





*Polaritonic characteristics of insulator and superfluid phases in a coupled-cavity array,*E. K. Irish, C. D. Ogden, M. S. Kim, Phys. Rev. A **77**, 033801 (2008).Times Cited: 2

Recent studies of quantum phase transitions in coupled atom-cavity arrays have focused on the similarities between such systems and the Bose-Hubbard model. However, the bipartite nature of the atom-cavity systems that make up the array introduces some differences. In order to examine the unique features of the coupled-cavity system, the behavior of a simple two-site model is studied over a wide range of parameters. Four regions are identified, in which the ground state of the system may be classified as a polaritonic insulator, a photonic superfluid, an atomic insulator, or a polaritonic superfluid.

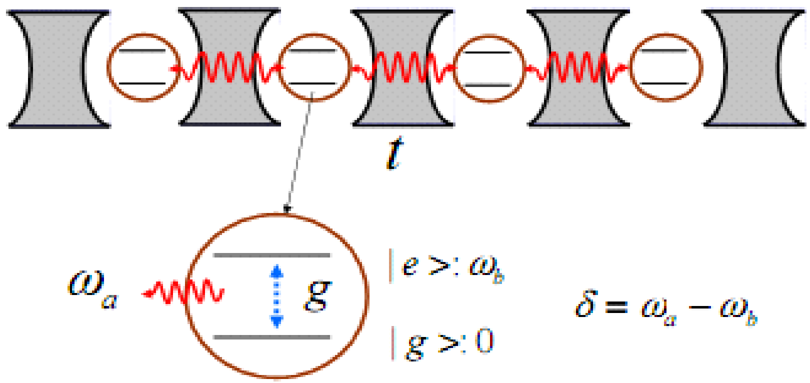
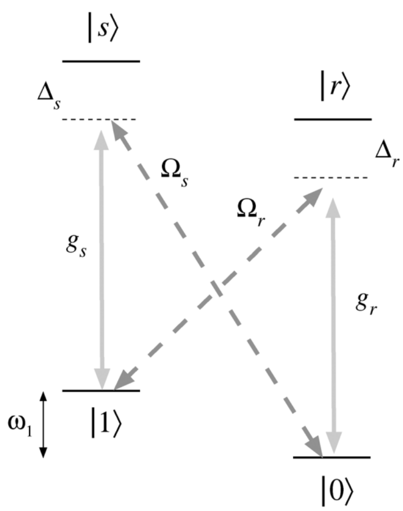


TABLE I. Characteristics of the four types of ground states in the coupled-cavity system.

Phase	ΔN_1	Particles	ΔN_{A1}	Regime
Insulator	0	Atoms	0	$\Delta/g<-1, A<\|\Delta\|$
		Polaritons	>0	$\|\Delta\|/g \lesssim 1, A \lesssim \|\Delta\|$
Superfluid	>0	Photons	0	$\Delta<0, A>\|\Delta\| \gtrsim g$; $\Delta>0, A \gtrsim g$
		Polaritons	>0	$\Delta/g<-1, A \approx \|\Delta\|$

- *Proposed realization of the Dicke-model quantum phase transition in an optical cavity QED system*, F. Dimer, B. Estienne, A. S. Parkins, and H. J. Carmichael, Phys. Rev. A **75,**013804 (2007).Times Cited: 4

The Dicke model describing an ensemble of two-state atoms interacting with a single quantized mode of the electromagnetic field with omission of the A^2 term exhibits a zero-temperature phase transition at a critical value of the dipole coupling strength. We propose a scheme based on multilevel atoms and cavity-mediated Raman transitions to realize an effective Dicke model operating in the phase transition regime. Optical light from the cavity carries signatures of the critical behavior, which is analyzed for the thermodynamic limit where the number of atoms is very large.



$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \omega_0 \hat{J}_z + \frac{\lambda}{\sqrt{N}}(\hat{a} + \hat{a}^\dagger)(\hat{J}_+ + \hat{J}_-),$$

with

$$\omega = \frac{N g_r^2}{\Delta_r} + \delta_{\text{cav}},$$

$$\omega_0 = \omega_1 - \omega_1',$$

$$\lambda = \frac{\frac{1}{2}\sqrt{N} g_r \Omega_r}{\Delta_r}.$$

$$\lambda = \lambda_c \equiv \frac{1}{2}\sqrt{\left(\frac{\omega_0}{\omega}\right)(\kappa^2 + \omega^2)}.$$

$$\alpha \equiv \langle \hat{a} \rangle, \quad \beta \equiv \langle \hat{J}_- \rangle, \quad w \equiv \langle \hat{J}_z \rangle,$$

$$\dot{\alpha} = -(\kappa + i\omega)\alpha - i\frac{\lambda}{\sqrt{N}}(\beta + \beta^*),$$

$$\dot{\beta} = -i\omega_0 \beta + 2i\frac{\lambda}{\sqrt{N}}(\alpha + \alpha^*) w,$$

$$\dot{w} = i\frac{\lambda}{\sqrt{N}}(\alpha + \alpha^*)(\beta - \beta^*).$$

$\lambda < \lambda_c$ $\quad \alpha_{\text{ss}} = \beta_{\text{ss}} = 0, \quad w_{\text{ss}} = \frac{\pm N}{2},$

$\lambda > \lambda_c$ $\quad \alpha_{\text{ss}} = \pm\sqrt{N}\frac{\lambda}{\omega - i\kappa}\sqrt{1 - \frac{\lambda_c^4}{\lambda^4}},$

$$\beta_{\text{ss}} = \mp \frac{N}{2}\sqrt{1 - \frac{\lambda_c^4}{\lambda^4}},$$

$$w_{\text{ss}} = -\frac{N}{2}\frac{\lambda_c^2}{\lambda^2}.$$

- *Mott-Insulating and Glassy Phases of Polaritons in 1D Arrays of Coupled Cavities,*D. Rossini, R. Fazio,Phys. Rev. Lett. **99**, 186401 (2007). Times Cited: 1

By means of analytical and numerical methods we analyze the phase diagram of polaritons in onedimensional coupled cavities. We locate the phase boundary, discuss the behavior of the polariton compressibility and visibility fringes across the critical point, and find a nontrivial scaling of the phase boundary as a function of the number of atoms inside each cavity. We also predict the emergence of a polaritonic glassy phase when the number of atoms fluctuates from cavity to cavity.

Polaritonic glass phase.

$$\mathcal{H} = \sum_{i=1}^{L} \mathcal{H}_i^{(a)} - t\sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i) - \mu \sum_{i=1}^{L} n_i.$$

$$\mathcal{H}_i^{(I,a)} = \epsilon(S_i^z + \tfrac{N}{2}) + \omega a_i^\dagger a_i + \beta(S_i^+ a_i + S_i^- a_i^\dagger)$$

- *Dynamical Quantum Phase Transitions in the Dissipative Lipkin-Meshkov-Glick Model with Proposed*

Realization in Optical Cavity QED, S. Morrison and A. S. Parkins, Phys. Rev. Lett. **100**, 040403 (2008).

- *Collective spin systems in dispersive optical cavity QED: quantum phase transitions,* S. Morrison and A. S. Parkins, arXiv:0711.2325.Times Cited: 0

We present an optical cavity QED configuration that is described by a dissipative version of the Lipkin-Meshkov-Glick model of an infinitely coordinated spin system.This open quantum system exhib its both first- and second-order non equilibrium quantum phase transitions as a single, effective field parameter is varied. Light emitted from the cavity offers measurable signatures of the critical behavior, including that of the spin-spin entanglement.

The LMG model describes a set of N spins half mutually interacting in the anisotropic x- y plane embedded in a perpendicular magnetic field pointing in the z direction.

The corresponding Hamiltonian is written

$$H_{LMG} = -2hJ_z - \frac{2\lambda}{N}(J_x^2 + \gamma J_y^2) \qquad (1)$$

where $\{J_z, J_x, J_y\}$ are collective angular momentum operators for N spin $1/2$ particles, h and $\lambda$are effective magnetic field and spin-spin interaction strengths, respectively, and γ is an anisotropy parameter. This system,in which each spin interacts identically with every other spin, exhibits critical behavior at zero temperature; inparticular, either first- or second-order equilibrium quantum phase transitions may occur, depending on the choiceof and , as the ratio h= is varied across a critical value.Notably, the second-order transition involves a change from a unique ground-state (normal phase) to a pair ofmacroscopically displaced degenerate ground states (broken phase). Entanglement in the system displays the above-mentioned critical behavior, reaching, in particular, a pronounced maximum at the critical point.

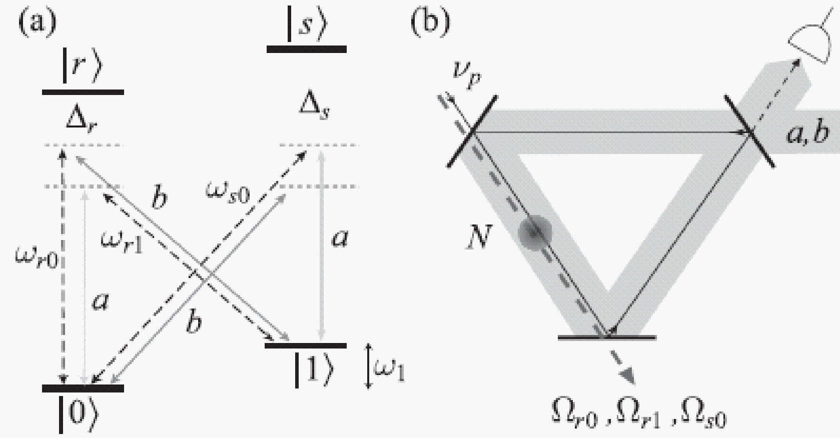


FIG. 1. (a) Atomic level and excitation scheme. (b) Potential ring-cavity setup. The laser fields (dashed lines) are at frequencies that are not supported by the resonator, but can be injected through one of the resonator mirrors so as to be copropagating with the cavity fields through the ensemble.

$$\dot{\rho}_g = -i[H_g, \rho_g] + \kappa_a D[a]\rho_g + \kappa_b D[b]\rho_g, \qquad H_g = \omega_0 J_z + \delta_a a^\dagger a + \delta_b b^\dagger b + 2\delta_a^- J_z a^\dagger a + 2\delta_b^- J_z b^\dagger b$$

$$+ \frac{\lambda_a}{\sqrt{N}} J_x (a + a^\dagger) + \frac{\lambda_b}{\sqrt{N}} (J_- b + J_+ b^\dagger),$$

$$D[A]\rho = 2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A.$$

$$\omega_0 = \frac{|\Omega_{r1}|^2}{4\Delta_r} - \frac{|\Omega_{r0}|^2}{4\Delta_r} - \frac{|\Omega_{s0}|^2}{4\Delta_s} + \omega_1 - \omega_1',$$
$$\delta_a = \omega_a + \omega_1' - \omega_{s0} + N\delta_a^+,$$
$$\delta_b = \omega_b + \omega_1' - \omega_{r0} + N\delta_b^+,$$
$$\delta_a^\pm = \frac{|g_{s1}|^2}{2\Delta_s} \pm \frac{|g_{r0}|^2}{2\Delta_r}, \qquad \delta_b^\pm = \frac{|g_{r1}|^2}{2\Delta_r} \pm \frac{|g_{s0}|^2}{2\Delta_s},$$
$$\lambda_a = \frac{\sqrt{N}\Omega_{r1}^* g_{r0}}{\Delta_r} = \frac{\sqrt{N}\Omega_{s0}^* g_{s1}}{\Delta_s}, \qquad \lambda_b = \frac{\sqrt{N}\Omega_{r0}^* g_{r1}}{2\Delta_r},$$

$$(\kappa_i^2 + \delta_i^2)^{1/2} \gg \lambda_a,\ \lambda_b,\ \omega_0.$$

$$\dot{\rho} = -i[H_{\mathrm{LMG}}^{\gamma=0}, \rho] + \frac{\Gamma_a}{N} D[2J_x]\rho + \frac{\Gamma_b}{N} D[J_+]\rho,$$

$$h = -\omega_0/2, \quad \lambda = 2\lambda_a^2 \delta_a/(\kappa_a^2 + \delta_a^2),$$

$$\Gamma_i = \lambda_i^2 \kappa_i/(\kappa_i^2 + \delta_i^2)\ (i = a,\ b).$$

Specifically, mode a mediates the collective-spin-spin interaction (of strength $\lambda \simeq \lambda_a^2/\delta_a$) associated with the Hamiltonian dynamics, while mode b mediates the collective atomic decay (with rate $\Gamma_b \simeq \lambda_b^2/\kappa_b$).

$$\dot{X} = 2hY - \Gamma_b ZX,$$
$$\dot{Y} = -2hX + 2\lambda ZX - \Gamma_b ZY,$$
$$\dot{Z} = -2\lambda XY + \Gamma_b(X^2 + Y^2),$$

$$(X, Y, Z) \equiv (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle)/j \qquad j = N/2.$$

$$h^c_\pm = [\lambda \pm (\lambda^2 - \Gamma_b^2)^{1/2}]/2$$

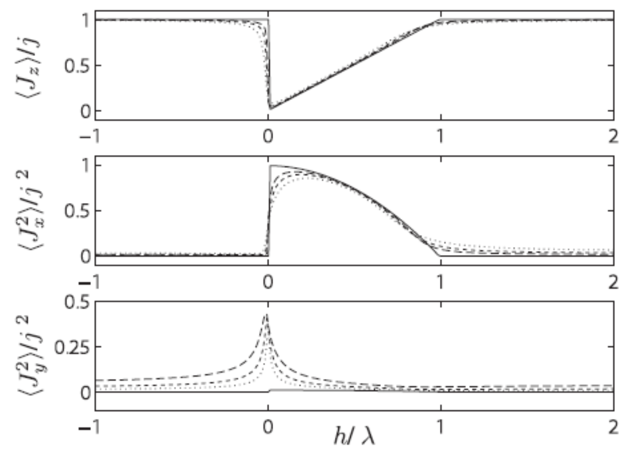


FIG. 2. Semiclassical (solid line) and finite-N steady-state inversion and second-order moments for $\Gamma_a/\lambda = 0.01$, $\Gamma_b/\lambda = 0.2$, and $N = 25$ (dotted line), 50 (short dashed line), 100 (long dashed line).

$h > h^c_+$

$$X_{ss} = \pm\sqrt{\frac{\Lambda^2 - 4h^2}{2\lambda\Lambda}}, \qquad Y_{ss} = \frac{\Gamma_b}{\Lambda} X_{ss}, \qquad Z_{ss} = \frac{2h}{\Lambda},$$

$$\Lambda = \lambda + (\lambda^2 - \Gamma_b^2)^{1/2}$$

Holstein-Primakoff Representation

$$J_z = \frac{N}{2} - c^\dagger c,$$
$$J_+ = \sqrt{N}\sqrt{1 - \frac{c^\dagger c}{N}}\, c\,,$$
$$J_- = \sqrt{N} c^\dagger \sqrt{1 - \frac{c^\dagger c}{N}}\,,$$

$$\dot{\rho} = -i[H_{\mathrm{HP}}, \rho] + \Gamma_+ D[c^\dagger]\rho + \Gamma_- D[c]\rho + \{\Upsilon(2c\rho c - c^2\rho - \rho c^2) + \mathrm{H.c.}\},$$

Entanglement Criteria

$$C_\varphi \equiv 1 - \frac{4}{N}\langle \Delta J_\varphi^2 \rangle - \frac{4}{N^2}\langle J_\varphi \rangle^2 > 0,$$

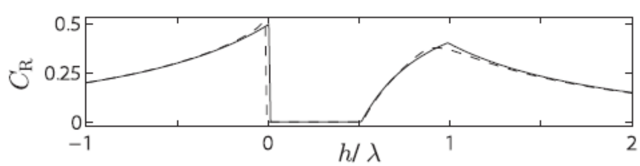


FIG. 4. Maximum entanglement C_R computed from the linearized HP ($N \to \infty$) model (solid) and from (5) for $N = 100$ (dashed), with $\Gamma_a/\lambda = 0.01$ and $\Gamma_b/\lambda = 0.2$.